

University of Swaziland

Final Examination, May 2015

B.A.S.S. I , B.Comm I, D.Comm I (IDE)

Title of Paper : Calculus for Business and Social Science

Course Code : MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
 - a. **SECTION A (COMPULSORY): 40 MARKS**
Answer ALL QUESTIONS.
 - b. **SECTION B: 60 MARKS**
Answer ANY THREE questions.
Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

- (a) The price-demand function and the cost function of calculators are given respectively by

$$p(x) = 100 - \frac{x}{50} \quad \text{and} \quad C(x) = 20000 + 15x,$$

where x is the number of calculators that can be sold at a price of p Emalangeni per calculator and $C(x)$ is the total cost of producing x calculators. Find

- (i) the exact cost of producing the 21st calculator. [2 marks]
 - (ii) $C'(10)$ and interpret your result. [3 marks]
 - (iii) $R'(10)$ and interpret your result. [3 marks]
 - (iv) the number of calculators that maximize profit. [4 marks]
- (b) Evaluate the following limits

- (i) $\lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x + 3}$. [3 marks]
- (ii) $\lim_{x \rightarrow 0} \frac{x^3 - x^2 - x}{x}$. [3 marks]
- (iii) $\lim_{x \rightarrow \infty} \frac{15 - 7x + x^3}{15 + 14x - 2x^3}$. [3 marks]

- (c) Find the derivatives of the following functions

- (i) $f(x) = \sqrt[3]{x} \cos(5x)$. [3 marks]
- (ii) $f(x) = \left(\frac{x^2 + 1}{\ln x} \right)^2$. [3 marks]

QUESTION 2

- (a) Evaluate the following integrals

- (i) $\int \left(2x - \frac{1}{4x} + \sin(7x + 1) \right) dx$. [3 marks]
- (ii) $\int_0^5 \frac{4x^3 + 2x}{x^4 + x^2 - 4} dx$. [5 marks]

- (b) Find the area of the region bounded by the two curves $y = x^2$ and $y = x + 2$. [5 marks]
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SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 3

(a) Evaluate the following limits

(i) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{\sqrt{x} - 2}$. [6 marks]

(ii) $\lim_{x \rightarrow -3} \frac{x^3 + 9x^2 + 26x + 24}{x + 3}$. [7 marks]

(b) State the three conditions which guarantee continuity of a function $f(x)$ at the point $x = c$. Using these properties test whether the function

$$f(x) = \begin{cases} \frac{x^2 - 25}{5 - x}, & x \neq 5; \\ 9, & x = 5 \end{cases}$$

is continuous at the point $x = 5$. [7 marks]

QUESTION 4

Find the derivative of the following functions

(a) $f(x) = 2^{\cos x}$. [5 marks]

(b) $f(x) = \ln[(x^2 + 4)(x - 4)]$. [5 marks]

(c) $f(x) = \left(\frac{x^2 + x}{x}\right)^4$. [5 marks]

(d) $f(x) = e^{3x} \ln(x^3 + 1)$. [5 marks]

QUESTION 5

(a) Given the function

$$y = 2x^3 - 15x^2 + 36x - 10,$$

find the

(i) local maximum. [2 marks]

(ii) local minimum. [2 marks]

(b) Find the intervals where the curve is

(i) increasing. [2 marks]

(ii) decreasing. [2 marks]

(iii) concave up. [3 marks]

(iv) concave down. [3 marks]

- (c) What is a point of inflection? Using this definition find the inflection point for the function $f(x)$. [2 marks]
- (d) Use all the information obtained in (a) – (c) to sketch the graph of the function. [4 marks]
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QUESTION 6

- (a) Given the demand function $D(x)$ and the supply function $S(x)$

$$D(x) = 60 - \frac{x^2}{10}, \quad S(x) = 30 - \frac{63x}{10} + \frac{x^2}{5}$$

find the

- (i) equilibrium quantity, [2 marks]
- (ii) consumer surplus, [5 marks]
- (ii) producer surplus, [5 marks]
- (b) Find the slope and equation of the tangent line to the graph of

$$f(x) = xe^{x^2-1}$$

at $x = 1$.

[8 marks]

QUESTION 7

- (a) Evaluate the following integrals

(i) $\int (x + 3) \cos x dx$, [5 marks]

(ii) $\int_0^5 \frac{x + 2}{x^2 - 5x + 6} dx$. [5 marks]

- (b) A company manufactures x carts per month. If the monthly cost and price-demand functions are given by

$$C(x) = \frac{x^2}{100} + \frac{x}{2} + 8, \quad p(x) = -\frac{x}{200} + 1.$$

Find the following, for each month.

- (i) Average profit function. [2 marks]
- (ii) Find the marginal average cost function. [4 marks]
- (iii) Find the marginal average profit function. [4 marks]
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END