University of Swaziland

Supplementary Examination, July 2015

B.A.S.S. I , B.Comm I, B. Ed I, D.Comm I (IDE)

Title of Paper : Calculus for Business and Social Science

Course Code : MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

(a) Evaluate the following limits

(i) $\lim_{x\to 1} \frac{\sqrt{x^2+x}}{x}$. [2 marks]

(ii) $\lim_{x\to 3} \frac{x^2-9}{x-3}$. [2 marks]

(iii) $\lim_{x\to\infty} \frac{4+5x+x^2}{7-2x-3x^2}$. [2 marks]

(b) Use the limit definition of the derivative to find the derivative $f'(x)$ of the function

$$f(x) = \frac{1}{x}.$$

[5 marks]

(c) Find the derivatives of the following functions

(i) $f(x) = \frac{2x+1}{(x+1)^2}$. [3 marks]

(ii) $f(x) = x \ln x$. [3 marks]

(d) Find the slope and equation of the tangent line to the graph of

$$f(x) = \frac{1}{x+2}$$

at $x = 1$. [5 marks]

QUESTION 2

(a) Evaluate the following integrals

(i) $\int \left(x^2 + \sin(2x)\right) dx$. [3 marks]

(ii) $\int \cos x e^{\sin x} dx$. [3 marks]

(iii) $\int (2x-2)^5 dx$. [3 marks]

(iv) $\int x^2 e^{-2x} dx$. [4 marks]

(b) Find the area of the region bounded by

$$f(x) = x + 2 \quad \text{and} \quad f(x) = x^2,$$

[5 marks]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 3

(a) Evaluate the following limits

(i) $\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$. [6 marks]

(ii) $\lim_{x \to -1} \frac{x^3 + x^2 - 4x - 4}{x + 1}$. [7 marks]

(b) State the three conditions which guarantee continuity of a function $f(x)$ at the point $x = c$. Using these properties test whether the function

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0; \\ 1, & x = 0 \end{cases}$$

is continuous at the point $x = 0$. [7 marks]

QUESTION 4

Find the derivative of the following functions

(a) $f(x) = 2^{x^2}$. [5 marks]

(b) $f(x) = \left(\frac{x-1}{x}\right)^2$. [5 marks]

(c) $f(x) = (\ln x) \sin^5 2x$. [5 marks]

(d) $f(x) = x^{xe^x}$. [5 marks]

QUESTION 5

(a) Given the function

$$f(x) = x^3 - 6x^2 + 9x + 1,$$

find the

(i) local maximum. [2 marks]

(ii) local minimum. [2 marks]

(iii) point of inflection. [2 marks]

(b) Find the intervals where the curve is

(i) increasing. [2 marks]

(ii) decreasing. [2 marks]

(iii) concave up. [3 marks]

(iv) concave down. [3 marks]

(c) Use all the information obtained in (*a*) and (*b*) to sketch the graph of the function. [4 marks]

QUESTION 6

(a) Given the demand function $D(x)$ and the supply function $S(x)$

$$p = D(x) = 20 - 0.05x, \qquad p = S(x) = 2 + 0.0002x^2$$

find the

(i) equilibrium price [2 marks]

(ii) consumer surplus [5 marks]

(iii) producer surplus [5 marks]

(b) By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, the cardboard may be turned into an open box. if the cardboard is 16 inches long and 10 inches wide, find the dimensions of the box that will yield the maximum volume. [8 marks]

QUESTION 7

(a) Evaluate the following integrals

(i) $\int (x^2 + x + 1)e^x dx.$ [5 marks]

(ii) $\int_0^1 \frac{x-1}{(x+1)^2} dx.$ [5 marks]

(b) The price-demand equation and the cost function for the production of television sets are given, respectively, by

$$p(x) = 300 - \frac{x}{30} \text{ and } C(x) = 150000 + 30x$$

where x is the number of sets that can be sold at a price of p Emalangeni per set and $C(x)$ is the total cost of producing x sets.

(i) Revenue function. [2 marks]

(ii) Find the marginal revenue when $x = 3000$. Interpret your results. [4 marks]

(iii) Find the marginal profit at $x = 1500$. Interpret your results. [4 marks]

END