University of Swaziland

Final Examination, December 2016

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211 / MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a). **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.
   b). **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Suppose that $f'(x) = 0$ for all $x \in (a, b)$. Prove that $f(x)$ is constant on (a, b). [8]

(b) Determine the exact length of

$$x = \frac{2}{3}(y-1)^{\frac{3}{2}}, \quad 1 \leq y \leq 4.$$

[5]

(c) State the **First Derivative Test** for determining local maximum and local minimum. [3]

(d) Determine the volume of the solid obtained by rotating the region bounded by

$$y = x^2 - 4x + 5,$$

$x = 1$, $x = 4$ and the x-axis about the x-axis. [6]

(e) Find the absolute maximum and absolute minimum values of

$$f(x) = 12 + 4x - x^2, \quad \text{in} \quad [0, 5].$$

[4]

(f) Determine if the sequence

$$\left\{ \frac{e^{3n}}{n^2+1} \right\}_{n=1}^{\infty}$$

converges or diverges. If it converges, find the limit. [4]

(g) Find the average value of $f(x) = \sin(2x)$ over the closed interval $[0, \pi/2]$. [3]

(h) Consider the series

$$\sum_{n=0}^{\infty} \frac{4n^2 - n^3}{10 + 2n^3}.$$

Determine if the following series is convergent or divergent. [4]

(i) Find the Maclaurin series of $f(x) = x^2 e^{3x}$. [3]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Suppose that $\phi(x)$ and $\gamma(x)$ are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $\phi(a) = \gamma(a)$ and $\phi'(x) < \gamma'(x)$ for $a < x < b$. Prove that $\phi(b) < \gamma(b)$. [6]

(b) Consider the function $f(x) = 3x^5 - 5x^3 + 3$.

  (i) Find the intervals of increase or decrease. [7]

  (ii) Find the intervals where the function is concave up and concave down. [7]

Question 3

(a) Use the method of slicing to find the volume of the solid obtained by rotating the region bounded by

$$y = x^3, \quad y = x, \quad x \geq 0,$$

about the x-axis. Sketch the region, the solid and a typical disk or washer. [7]

(b) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by

$$y = 4x - x^2, \quad y = 3,$$

about $x = 1$. [8]

(c) Find the horizontal asymptotes of the curve $y = x^{\frac{1}{x}}$. [5]

Question 4

(a) The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}\left(R^2 - r^2\right)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity over the interval $0 \leq r \leq R$. [7]

(b) Find the arc length of the function

$$y = \ln(\sec(x)), \quad 0 \leq x \leq \frac{\pi}{4}.$$

[6]

(c) Determine the surface area of the solid obtained by rotating,

$$y = \sqrt{9 - x^2}, \quad -2 \leq x \leq 2$$

about the x-axis. [7]

Question 5

(a) Determine the area of the region bounded by

$$y = 2x^2 + 10, \quad y = 4x + 16$$

. [6]

(b) Find the Taylor Series for

$$f(x) = \sin(x),$$

about $x = 0$. [8]

(c) Use Maclaurin series to evaluate

$$\int x^2 \sin(x^4)\, dx$$

as an infinite series. [6]

Question 6

(a) Determine if the sequence

$$\left\{ \frac{4n^3 + 1}{10n - 8n^3} \right\}_{n=2}^{\infty}$$

converges or diverges. [4]

(b) Consider the series

$$\sum_{n=1}^{\infty} 4^{2-n} 2^{n+1}$$

(i) Express the series in the form $\sum_{n=1}^{\infty} ar^{n-1}$ and determine the values of a and r. [4]

(ii) Determine if the series converges or diverges. If it is convergent, determine the value of the series. [3]

(c) Consider the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x+3)^n}{4^n}.$$

Determine the radius of convergence and interval of convergence of the power series. [9]