University of Swaziland

Supplementary Examination, July 2017

B.A.S.S., B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : M211 / MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a). **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.
   b). **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) Expand $f(x) = x^2e^{3x}$ as a Taylor series about $x = 0$. [3]

(b) Determine the exact length of

$$y = \frac{2}{3}(x+4)^{\frac{3}{2}}, \qquad 1 \leq x \leq 4.$$

[4]

(c) Consider the function

$$f(x) = (x+1)^5 - 5x - 2.$$

Find the local maximum and minimum values. [6]

(d) Determine the area of the region bounded by

$$y = 2x^2 + 10, \qquad y = 4x + 16$$

. [5]

(e) Find the absolute maximum and absolute minimum values of

$$f(x) = 12 + 4x - x^2, \quad \text{in} \quad [0, 5].$$

[4]

(f) Consider the sequence

$$\left\{ \frac{e^{2n}}{n^2 - 1} \right\}_{n=2}^{\infty}$$

(i) Write down the first three terms of the sequence. [3]

(ii) Determine if the sequence converges or diverges. If it converges, find the limit. [4]

(g) Find the average value of $f(x) = e^x$ over $[0, \pi]$. [3]

(h) Consider the series

$$\sum_{n=0}^{\infty} \frac{4n^3 - n^2}{10 + 2n^3}.$$

Determine if the following series is convergent or divergent. [4]

(i) Suppose that $\rho(x)$ is an even function that is differentiable everywhere. Prove that for every positive number κ, there exist a number α in $(-\kappa, \kappa)$ such that $\rho'(\alpha) = 0$. [4]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) If $f(x)$ is continuous on a closed interval $[a, b]$, show that $f(x)$ attains both an absolute maximum value β and an absolute minimum value α in $[a, b]$. That is, show that there are two numbers x_1 and x_2 in $[a, b]$ with $f(x_1) = \alpha$ and $f(x_2) = \beta$ and $\alpha \leq f(x) \leq \beta$ for every other $x \in [a, b]$. [6]

(b) Consider the function $f(x) = 3x^5 - 5x^3 + 3$.

- (i) Find the intervals of increase or decrease. [7]
- (ii) Find the intervals where the function is concave up and concave down. [7]

Question 3

(a) Determine the volume of the solid obtained by rotating the region bounded by

$$y = (x-1)(x-3)^2$$

and the x-axis about the y-axis. [7]

(b) Determine the volume of the solid obtained by rotating the region bounded by

$$y = 2\sqrt{x-1}, \quad \text{and} \quad y = x - 1$$

about the line $x = -1$. [8]

(c) Find the horizontal asymptotes of the curve $y = \frac{\ln(x)}{x}$. [5]

Question 4

(a) In a certain city, the Temperature (in °F) t hours after 9 am was modeled by the function

$$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$$

Find the average temperature during the period from 9:00 am to 9:00 pm. [7]

(b) Set up the integral that could be used to find the arc length of the function

$$x = \frac{y^2}{2}, \quad 0 \leq x \leq \frac{1}{2}.$$

[6]

(c) Determine the surface area of the solid obtained by rotating,

$$y = \sqrt{9 - x^2}, \quad -2 \leq x \leq 2$$

about the x-axis. [7]

Question 5

(a) Determine the area of the region bounded by

$$y = xe^{-x^2}, \quad y = x + 1, \quad x = 0, \quad x = 2.$$

[6]

(b) Find the Taylor Series for

$$f(x) = e^x,$$

about $x = 0$. [8]

(c) Use Maclaurin series to evaluate

$$\int t^3 e^{t^2}\, dt$$

as an infinite series. [6]

Question 6

(a) Determine if the sequence

$$\left\{ \frac{n^3+1}{5n-7n^3} \right\}_{n=2}^{\infty}$$

converges or diverges. [4]

(b) (i) Determine if the series $\sum_{n=0}^{\infty} ne^{-n^2}$ converges or diverges. [5]

(ii) Determine if the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges. [3]

(c) Consider the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x+4)^n}{6^n}.$$

Determine the radius of convergence and interval of convergence of the power series. [8]