University of Swaziland

Final Examination December, 2016

B.A.S.S I

Title of Paper : Elementary Quantitative Techniques I

Course Number : MAT101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Show all your working.

3. Start each question on a fresh page.

4. Non programmable calculators may be used (unless otherwise stated).

5. A formula sheet is provided on the last page.

6. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A

Answer ALL questions from section A.

A1. (a) Simplify:

i. $\dfrac{3}{y+3}+\dfrac{2}{y+5}$, [5]

ii. $5-\dfrac{x-1}{7x}$, [5]

iii. $\dfrac{x^2+2x-3}{x^2+5x-6}$. [8]

(b) Consider the matrices $M=\begin{pmatrix}3 & -2\\ 2 & 4\end{pmatrix}$ and $N=\begin{pmatrix}4 & 6\\ -1 & 5\end{pmatrix}$. Find

i. $-3M^T$, [3]

ii. N^T-M. [3]

(c) i. Use a calculator to compute $20C_5$, [2]

ii. Factorize completely $2t^2-72$. [3]

(d) i. Solve the simultaneous equations

$$3x-2y=5,$$
$$4x+5y=-24.$$

[6]

ii. Use the quadratic formula to solve $5x^2=20x-4$. (Give your answer correct to 1 d.p.) [5]

SECTION B

Answer any THREE questions from section B.

B2. (a) Consider the AP

$$5, 2, -1, -4, \ldots,$$

i. Write down the next two terms. [2]

ii. Find a formula for the nth term. [3]

iii. Use the formula in ii. to find the 81^{st} term. [2]

iv. Find the sum of the first 40 terms. [4]

(b) Find the value of

i. $1 + 2 + 4 + 8 + \cdots + 16384.$ [5]

ii. $\sum_{n=1}^{40} 4n.$ [4]

B3. (a) Consider the matrices

$$A = \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix},\ B = \begin{pmatrix} 5 & -1 \\ 1 & 3 \\ 5 & -4 \end{pmatrix},\ C = \begin{pmatrix} 4 & 1 & 3 \\ -2 & -1 & 8 \end{pmatrix}.$$

Find (where possible);

i. $|A|$, [2]

ii. AB^T, [3]

iii. BC. [4]

(b) Use Cramer's rule to solve the following linear system of equations.

$$\begin{aligned} 4x + 3y - 2z &= 7, \\ x + y &= 5, \\ 3x + z &= 4. \end{aligned}$$

[11]

B4. (a) Expand and simplify term by term $\left(x - \frac{3}{x^2}\right)^5$. [8]

(b) Find the 11^{th} term in the binomial expansion of $\left(\frac{2}{x} + x\right)^{15}$. [6]

(c) Simplify and leave your answers in terms of positive indices.

i. $\left(\frac{C^{-3}}{3}\right)^2$. [2]

ii. $\frac{10m^4n^{-3}}{m^{-1}} \times \frac{2m^2n}{5n}$. [4]

B5. (a) Consider the straight line, H given by $18x + 3y = -10$.

i. State the y-intercept of H. [3]

ii. State the gradient (slope) of H. [3]

iii. Find the equation of a line parallel to H, passing through the point $(-2, 1)$. [4]

(b) Use synthetic division to work out $\frac{x^3 - x^2 - 5x + 1}{x + 2}$. [6]

(c) Given that $x - 2$ is a factor of $x^3 + Bx^2 - 5x + 4$, find the value of B. [4]

B6. (a) Express as a single logarithm $\log_3(x+4) - \log_3(2x)$. [2]

(b) Express in terms of logarithms $3^{-4} = \frac{1}{81}$. [3]

(c) Solve for x in each of the following.

i. $2^{x-5} = 512$. [3]

ii. $\log_2 x + \log_2(x-2) = 3$. [5]

(d) The population of a city grows according to the formula

$$p(t) = 60000e^{0.028t},$$

where t is the number of years from year 2000. Find

i. the population in 2012. [2]

ii. the year when the population will reach 100000. [5]

END OF EXAMINATION

Formula Sheet

Arithmetic Progressions:

$$T_n = T_1 + (n-1)d, \quad S_n = \frac{n}{2}[T_1 + T_n], \quad S_n = \frac{n}{2}[2T_1 + (n-1)d].$$

Geometric Progressions:

$$T_n = T_1 r^{n-1}, \quad S_n = \frac{T_1(1-r^n)}{1-r}.$$

Binomial Theorem:

$$(a+b)^n = a^n + nC_1 a^{n-1}b + nC_2 a^{n-2}b^2 + nC_3 a^{n-3}b^3 + \cdots + b^n.$$

rth term of $(a+b)^n = nC_{r-1} a^{n-r+1} b^{r-1}$.

Matrices:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

logarithms and Exponential Functions:

$$\log_b x = y \Leftrightarrow x = b^y.$$
$$\log_b(AB) = \log_b A + \log_b B.$$
$$\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B.$$
$$\log_b A^n = n \log_b A.$$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$