UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2017

BA in Social Sciences I

Title of Paper : Elementary Quantitative Techniques II

Course Number : MAT102

Time Allowed : Two (2) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.

2. Section A is **COMPULSORY** and is worth 50%. Answer ALL questions in this section.

3. Section B consists of FOUR questions, each worth 25%. Answer ANY TWO (2) questions in this section.

4. Show all your working.

5. Start each new major question (A1, B2 – B7) on a new page and clearly indicate the question number at the top of the page.

6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

QUESTION A1

a . Evaluate

    i. $\lim_{x \to -2}(3x^2 + 6x - 7)$ [2]

    ii. $\lim_{x \to 1}\left(\dfrac{1 + x}{(2x - 5)^2}\right)$ [2]

    iii. $\lim_{x \to -1}\left(\dfrac{x^2 - 1}{x + 1}\right)$ [3]

    iv. $\lim_{x \to -\infty}\left(\dfrac{2x^2 - 3}{4 - 5x^2}\right)$ [4]

QUESTION A2

a. State the *limit definition* of the derivative of the function $f(x)$.   [2]

b. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = 3x^2 - 2x$$

       [7]

c. Find y' if

    i. $y = \dfrac{2}{3}x^6 - 7x^4 - 8$ [2]

    ii. $y = 2X^{\frac{1}{5}} + X^{-\frac{6}{5}}$ [3]

    iii. $y = 5t^2 - \dfrac{4}{\sqrt{t}}$ [3]

    iv. $y = e^{3x+2}$ [2]

    v. $y = \ln(3 - 5x)$ [3]

QUESTION A3

a. State the *Fundamental Theorem of Calculus*.   [3]

b. Integrate

    i. $\displaystyle\int_{-1}^{2}\left(4x - 8x^2\right)dx$ [5]

    ii. $\displaystyle\int\left(2X^{\frac{1}{2}} + \frac{3}{X}\right)dX$ [3]

    iii. $\displaystyle\int\left(\frac{12}{x^3} - \frac{20}{x^7}\right)dx$ [3]

    iv. $\displaystyle\int e^{-\frac{1}{3}x}\,dx$ [3]

Section B

Answer and 2 Questions in this section

QUESTION B4

a. Find the value of the limit of

$$\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2}$$

[7]

b. Find the derivative

    i. y' for $y = (x - x^3)^5$ [3]

    ii. y' for $y = (5x + 7)e^{-3x}$ [4]

    iii. y' for $y = \dfrac{1 - t^2}{5 + t^3}$ [6]

    iv. y''' for $y = 4x + \dfrac{3}{x}$ [5]

QUESTION B5

a. Consider the function

$$y = x^2 - 4x + 3$$

    i.   Find the *stationary point(s)* and classify them as relative maxima or minima. [10]

    ii. Find the y-intercept. [2]

    iii. Make a neat sketch of the graph of y. [4]

b. The profit P of a company is given by

$$P(x) = 75x - 0.03x^2 - 5400,$$

    i. find the profit if the monthly sales stand at 200 units. [3]

    ii. find the number of units that maximises the profit. Hence find the maximum profit. [6]

QUESTION B6

a. By first making the substitution $u = 3 - x^2$, evaluate the integral

$$\int 8x\left(3 - x^2\right)^{-2} dx.$$

[10]

b. Use the method of partial fractions to integrate

$$\int \frac{3x - 2}{x(x - 2)} dx.$$

[15]

QUESTION B7

a. Use the method of tabular integration to evaluate

$$\int 12x^2 e^{2x} dx.$$

[12]

b. i. Evaluate

$$\int_0^1 \left(\frac{2}{3\sqrt{x}} + x^{3/2} + 6\right) dx$$

[7]

ii. Find the area of the shaded region in the figure below   [6]

$y = 36 - x^2$

_____END OF EXAMINATION PAPER_____