University of Swaziland

Supplementary Examination July, 2017

B.A.S.S. I, B.Comm I, D.Comm I (IDE), B. Ed I

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Number : MAT107/MAT121/MS101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Show all your working.

3. Start each question on a fresh page.

4. Non programmable calculators may be used (unless otherwise stated).

5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A

Answer ALL questions from section A.

A1. (a) State the remainder theorem and use it to find the remainder when $P(x) = 5x^3 - 6x^2 - 28x - 2$ is divided by $x - 3$. [5]

(b) Use the long division method to find the quotient and remainder when the following polynomial $P(x) = x^3 + 5x^2 + 2x - 8$ is divided by $D(x) = x + 2$. [6]

(c) Solve the following equations.

    i. $4^{3x} = 16^{x+3}$. [3]

    ii. $12^x = 3^{4x+1}$. [3]

    iii. $\log(x^2 + 1) - \log(x - 2) = 1$. [4]

(d) Calculate $A^T B$ if the matrices A and B be given by

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -6 \\ 7 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -9 \\ 2 & 7 \\ -3 & 3 \end{pmatrix}.$$

[4]

(e) Write an equation of a line passing through $(-1, 3)$ and perpendicular to the line with equation $2y - x = 1$. [5]

(f) Given that $z_1 = 2 + 3i$ and $z_2 = 1 - i$, find $\dfrac{z_1 + z_2}{z_1}$. [5]

(g) Find the 9th term of the geometric progression

$$4, 12, 36, 108, \ldots$$

[5]

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SECTION B

Answer any THREE questions from section B.

B2. Given the following polynomial

$$P(x) = x^4 - x^3 - 19x^2 + 49x - 30.$$

(a) List all the possible roots of $P(x)$. [3]

(b) Find the number of positive real zeros (roots) of $P(x)$. [3]

(c) Find the number of negative real zeros (roots) of $P(x)$. [3]

(d) Use the factor theorem and synthetic division (**ONLY**) to find the factors of $P(x)$. [11]

B3. (a) Using mathematical induction, prove that

$$1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}.$$ [10]

(b) Find the annual interest rate required to treble a certain amount if the interest is compounded monthly for 10 years. [5]

(c) A student wants to buy a new computer after 4 years that will cost $E8000$. If she only has $E4000$ available to deposit now, what interest rate is required for it to increase to $E8000$ in 4 years if the interest is compounded continuously. [5]

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B4. (a) Consider the binomial expansion of $\left(x + \dfrac{1}{2x}\right)^8$.

      i. Find the constant term, and   [5]

      ii. the term involving x^6.   [5]

  (b) i. Write down the first 4 terms of the binomial expansion of $\dfrac{1}{\sqrt{1-x}}$.   [4]

      ii. Use your result in part B3(b)i above to estimate $\dfrac{1}{\sqrt{0.99}}$.   [6]

B5. (a) Prove the trigonometric identity

$$(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x.$$ [4]

  (b) If $x = 2\cos x$ and $y = 3\sin x$ prove that $9x^2 + 4y^2 = 36$.   [4]

  (c) Convert the decimal $1.3712712712\ldots$ into a common fraction.   [6]

  (d) Find the sum of the following progression

$$2 + 5 + 8 + 11 + \cdots + 1001.$$ [6]

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B6. (a) Find the equation of the straight line which passes through the point $(2, 4)$ and through the intersection of the lines $3x+4y-5 = 0$ and $2x - y + 2 = 0$. [5]

(b) Find the center and radius of a circle given by
$7x^2 + 7y^2 + 14x - 56y - 25 = 0.$ [5]

(c) Given that $z_1 = 2 + 3i$ and $z_2 = 1 - i$, find

   i. $z_1 z_2$, [5]

   ii. $\dfrac{z_1}{z_2}$. [5]

END OF EXAMINATION