University of Swaziland

Final Examination – December 2016

BSc I, BEng I, BEd I

Title of Paper : Algebra, Trigonometry & Analytic Geometry
Course Number : MAT111
Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY 3 (out of 5) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

A.1 a. On the same axes, make a sketch of the graphs of

i. $y = e^{-x}$ [2 marks]

ii. $y = \ln x$ [2 marks]

iii. $x = -y^2$ [2 marks]

b. Evaluate and leave your answer in the form $a + ib$.

i. $(4 - 3i^7)(3 + 4i^9)$ [4 marks]

ii. $\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6}) + e^{-i\pi/6}$ [3 marks]

c. Given the vectors $\boldsymbol{A} = 9\hat{\imath} - 12\hat{k}$ and $\boldsymbol{B} = -8\hat{\imath} + 20\hat{\jmath} + 3\hat{k}$, find

i. $|\boldsymbol{A}|$ [2 marks]

ii. $\boldsymbol{A} \cdot \boldsymbol{B}$ [3 marks]

d. Find the value of

i. $\sum_{n=-10}^{60} (7 - 5n)$ [3 marks]

ii. $\sum_{n=0}^{\infty} \left(\frac{10}{9}\right)^n$ (correct to 2 d.p.) [3 marks]

e. Simplify

$$2\log_2\left(2x^2\right) - \log_2\sqrt{8x^8}.$$ [3 marks]

f. Evaluate and simplify

$$\begin{vmatrix} \sin\theta & e^{-\theta} & \cos\theta \\ \cos\theta & \ln\theta & -\sin\theta \\ 0 & 1 & 0 \end{vmatrix}.$$ [4 marks]

g.

i. State the *Remainder Theorem.* [2 marks]

ii. Find the quotient and remainder of

$$\frac{x^4 - 1}{x^2 + 1}.$$ [4 marks]

h. Find the 17th term of the binomial expansion of

$$\left(\frac{2x^2}{\sqrt{y}} + \frac{\sqrt{y}}{x}\right)^{20}.$$ [3 marks]

Section B

Answer ANY 3 Questions in this section

B.1 a. Use de Moivre's theorem to expand

$$\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{100}$$

and express in the form $x + iy$. [4 marks]

b. Solve and express your answer in the form $z = x + iy$.

i. $2iz + 4 = 3 - 4\overline{z}$ [6 marks]

ii. $z^4 + 8z^2 - 9 = 0$ [4 marks]

c. Given the complex number $z = x + iy$, prove that

$$\overline{(z_1 \cdot z_2)} = \overline{z}_1 \cdot \overline{z}_2.$$ [6 marks]

B.2 a. A curve is defined by the parametric equations

$$\begin{aligned} x &= \sin\theta + \cos\theta \\ y &= \sin\theta - \cos\theta. \end{aligned}$$

i. By eliminating θ, derive the equation of the curve in terms of x and y only. [5 marks]

ii. State the name of the curve and make a sketch of it. [3 marks]

b. Consider the trigonometric identity

$$1 + \cos 2\theta + \cos 4\theta + \cos 6\theta = 4\cos\theta\cos 2\theta\cos 3\theta.$$

i. Prove the identity. [6 marks]

ii. Hence, or otherwise, find all values of θ (in radians) satisfying

$$1 + \cos 2\theta + \cos 4\theta + \cos 6\theta = 0$$

in the interval $0 \leqslant \theta \leqslant \pi$. [6 marks]

B.3 a. Solve for x given

$$\log_2 x + \log_2(8x + 15) = 1.$$ [4 marks]

b. Consider the number sequence

$$\ln x,\ \ln\left(xr^2\right),\ \ln\left(xr^4\right),\ \ln\left(xr^6\right),\cdots,$$

where $x > 0$ and r are real numbers.

i. Show that the sequence is an *arithmetic progression* and hence find the value of the *common difference.* [3 marks]

ii. Find the formula for the n-th term T_n (expressing your answer as a single logarithm with coefficient 1) [3 marks]

iii. Find the formula for the sum od the first n terms S_n (expressing your answer as a single logarithm with coefficient 1) [4 marks]

iv. Given that $T_5 = \ln 1280$ while $S_3 = \ln 8000$, find the values of x and r. [6 marks]

B.4 a. Consider the polynomial

$$P(x) = Ax^3 + Bx^2 - 5x + 2,$$

where A and B are constants. You are given that $x + 1$ is a factor of $P(x)$ while dividing $P(x)$ by $x + 2$ leaves a remainder of -36.

i. Find the values of A and B. [8 marks]

ii. Hence, factorise $P(x)$ [4 marks]

b. Use mathematical induction to prove the formula

$$\sum_{i=1}^{n} i \cdot 2^{i-1} = 1 + (n-1)2^n, \quad n \in \mathbb{Z}^+.$$ [8 marks]

B.5 a. Use Cramer's rule to solve

$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 0 \\ 2x & + & y & & & = & -1 \\ 3x & + & 5y & - & 4z & = & -23 \end{array}$$

[14 marks]

b. Find the first 3 terms of the binomial expansion of

$$\left(\frac{1}{x^2} - 2x^2\right)^{-\frac{1}{2}}.$$

[6 marks]

END OF EXAMINATION