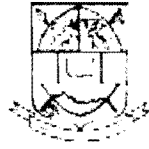

University of Swaziland



Final Examination – May 2017

BSc I, BEd I, BEng I, BASS I

Title of Paper : Introduction to Calculus

Course Number : MAT112

Time Allowed : Three (3) hours

Instructions:

1. This paper consists of 2 sections.
2. Answer ALL questions in Section A.
3. Answer ANY THREE (3) questions in Section B.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN

Section A
Answer ALL Questions in this section

A.1 a. $\lim_{x \rightarrow \frac{1}{2}} \left(\frac{2x^2 + x - 1}{2x^2 - 5x + 2} \right)$ [4 marks]

b. $\lim_{\theta \rightarrow 0} \left(\frac{6\theta + \sin 4\theta}{\theta} \right)$ [3 marks]

A.2 Find $\frac{dy}{dx}$ if

a. $y = (1 + 4\sqrt{x})^{20}$ [3 marks]

b. $y = \sin^{-1}(\cos x)$ [4 marks]

c. $x^2y - y^4 = 5$, where $y = y(x)$ [4 marks]

A.3 Use the *limit definition* to find $\frac{df}{dx}$ given

$f(x) = 7 - 3x^2$. [7 marks]

A.4 Integrate

a. $\int_0^1 4xe^{-2x^2} dx$ [3 marks]

b. $\int_0^1 4x^2e^{-2x} dx$ [4 marks]

c. $\int_0^{\frac{\pi}{4}} 8 \cos^3 \theta \sin \theta d\theta$ [4 marks]

d. $\int \frac{dx}{1-x^2}$ [4 marks]

Section B

Answer ANY THREE (3) Questions in this section

B.1 a. Make a sketch of the graph of

$$y = 2H(x + 2) + H(x - 2),$$

where $H(x)$ is the Heaviside function of x .

[4 marks]

b. Evaluate

$$\lim_{x \rightarrow \infty} \left(x - \sqrt{x^2 - 8x} \right)$$

[6 marks]

c. Use the *limit definition* to find y' given

$$y = 7 + \frac{8}{\sqrt{x}}.$$

[10 marks]

B.2 a. Find $\frac{dy}{dx}$ and simplify given

$$y = \tan^{-1} \left(\frac{x}{3} \right) + \frac{3x}{x^2 + 9}.$$

[6 marks]

b. Consider the function

$$y = 10 + x^2 - 10 \ln(1 + x^2).$$

i. Find the stationary points of y .

[6 marks]

ii. Determine the nature of each stationary point.

[4 marks]

iii. Make a sketch of the graph of y .

[4 marks]

B.3 a. Differentiate the function

$$y = \frac{\sin \theta}{\sin \theta - \cos \theta}$$

and show that

$$\frac{dy}{d\theta} = \frac{1}{\sin 2\theta - 1}.$$

[8 marks]

b. Find the dimensions of the *largest* rectangle that can be inscribed inside 1 ellipse

$$\frac{x^2}{36} + \frac{y^2}{9} = 1.$$

[12 marks]

B.4 a. Integrate

i. $\int (4x + 1) \ln x \, dx$

[4 marks]

ii. $\int_0^4 \frac{x^2 \, dx}{16 + x^2}$

[7 marks]

b. Derive the formula

$$A = \pi ab$$

for the area of an ellipse with major axis a and minor axis b .

[9 marks]

B.5 Integrate

a. $\int \frac{x^4 - x^3 + x^2}{x^3 - x^2 - x + 1} \, dx$

[15 marks]

b. $\int \frac{d\theta}{1 + e^\theta}$

[5 marks]

END OF EXAMINATION
