University of Swaziland

Final Examination, May 2017

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Code : MAT216

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) By eliminating the constant, find the differential equation satisfied by the equation $y = c_1 e^{-2x} + c_2 e^{2x}$. [5]

(ii) Solve

$$(1 + y^2)dx + (1 + x^2)dy = 0.$$

[5]

(iii) Solve

$$(x + 2y)dx + (2x + y)dy = 0.$$

[5]

(iv) Solve

$$(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0.$$

[5]

(b) Determine the value of a for which the following differential equation is exact: Hence, solve the differential equation.

$$xy^3dx + ax^2y^2dy = 0.$$

[5]

(c) Solve the Bernoulli equation

$$3y' + xy = xy^{-2}.$$

[5]

(d) Solve

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0.$$

[5]

(e) Using Laplace transform method solve

$$y'' - 2y' - 8y = 0, \quad y(0) = 3, \; y'(0) = 6.$$

[5]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Use the method of variation of parameters to solve

$$y'' + 3y' + 2y = 2e^{x}.$$

[10]

(b) Solve the system of equations

$$\frac{dx}{dt} - 7x + y = 0$$
$$\frac{dy}{dt} - 2x - 5y = 0.$$

[10]

Question 3

Find the series solution, about $x = 0$, of the equation

$$xy'' + y' - xy = 0,$$

by the Frobenious method. [20]

Question 4

(a) Use the method of undetermined coefficients to find the solution of

$$y'' + 9y = \cos 3x.$$

[10]

(b) Using Laplace transform method, solve

$$y'' + 2y' + 5y = e^{-t}\sin t, \quad y(0) = 0, \quad y'(0) = 1.$$

[10]

Question 5

(a) Solve

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2.$$

[10]

(b) It is given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are two linearly independent solutions of the associated homogeneous equation of

$$x^2y'' + xy' - y = x, \qquad x \neq 0.$$

Find a particular solution and the general solution of the equation. [10]

Question 6

(a) Solve

$$x^2y'' - 3xy' + 3y = 0, \quad y(1) = 0, \quad y'(1) = -2.$$

[10]

(b) Find the general solution of the differential equation

$$y' = y^2 + (2x-1)y + x^2 - x + 1$$

if $y = x$ is a solution of the differential equation. [10]

Table 1: Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
t^n	$\dfrac{n!}{s^{n+1}}$
$\dfrac{1}{\sqrt{t}}$	$\sqrt{\dfrac{\pi}{s}}$
e^{at}	$\dfrac{1}{s-a}$
$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
$\dfrac{1}{a-b}\left(e^{at} - e^{bt}\right)$	$\dfrac{1}{(s-a)(s-b)}$
$\dfrac{1}{a-b}\left(ae^{at} - be^{bt}\right)$	$\dfrac{s}{(s-a)(s-b)}$
$\sin(at)$	$\dfrac{a}{s^2+a^2}$
$\cos(at)$	$\dfrac{s}{s^2+a^2}$
$\sin(at) - at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\dfrac{a}{s^2-a^2}$
$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\dfrac{2a^2}{s^4+4a^4}$
$\dfrac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$g(t) = \begin{cases} 0, & 0 \le t \le a; \\ f(t-a), & a < t. \end{cases}$	$e^{-as}F(s)$