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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2005

TITLE OF PAPER:	PROBABILITY THEORY
COURSE CODE :	ST 201
TIME ALLOWED :	THREE (3) HOURS
INSTRUCTIONS :	THIS PAPER HAS SEVEN QUESTIONS. ANSWER ANY SIX (6) QUESTIONS. EACH QUESTION CARRIES 10 MARKS.
REQUIREMENTS:	Scientific Calculator

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QUESTION ONE

(a) *For any value of $k \geq 1$, define a discrete random variables X to have the probability function*

$$P_{x(x)} = (k^2-1)/k \ , \quad x=0$$
$$= \tfrac{1}{2}\{k^2\} \ , \quad x=-k\,,k$$
$$= 0, \text{ otherwise.}$$

Compute the mean and variance of this random variable.

(b) The joint probability density function of the random variables X and Y is given as :

x, y	1,1	1,2	1,3	2,1	2,2	2,3
f(x, y)	2/15	4/15	3/15	1/15	1/15	4/15

Find σ_{xy} and ρ_{xy} when f(x,y)=0 elsewhere.

QUESTION TWO

(a) *Obtain the characteristic function of the distribution of a random variable X with density function* e^{-x}, *for* $x > 0$.

(b) The distribution function of X is

$G_x(x) = 0$, $x<1$
$= \log x$, $1 \le x \le e$
$= 1$, $x>e$.

Obtain the interquartile range of this random variable.

QUESTION THREE

A man is allowed to flip a fair coin until the first head appears and will win 2^x Dollar at the occurrence of the first head, there X is the total number of flips required.

(a) What is the expected value of his winning the game?

(b) If the amount he pays to play a game is equal to the amount he expects to win, the game is fair. How much should he pay to play the game to make it fair?

(c) If $Y \sim b(n,p)$,find the smallest value of n for which $P(|Y-P|<\xi) \ge 0.9 \ \forall \ \xi > 0$.

QUESTION FOUR

(a) *Let X be a uniform random variable on the interval (1,2). A square is constructed with sides of length X. Derive the probability density function of $Y=X^2$ and compute $P(Y>2)$.*

(b) *The probability set function of the random variable X is given as $P(A)=\int_A e^{-x}dx$, where:*

$A=\{x:0<x<\infty\}$, define
$A_k=\{ x: 2-1/k <x<3\}$, for k=1,2,3...

Obtain $\lim_{k\to\infty}(A_k)$ and $P(\lim_{k\to\infty}(A_k))$.

QUESTION FIVE

Every Saturday a fisherman goes to one of three locations: the river, the sea and a lake to catch fishes with probabities ¼,1/2 and ¼ respectively. If he goes to the sea, there is an 80% chance of catching fish, the corresponding figures for the river and the lake are 40% and 60% respectively.

(a) *Find the probability that he catches fish on a given Saturday.*

(b) *What is the probability that he catches fish on at least three of the five consecutives Saturdays?*

(c) *If on a particular Saturday, he comes home without catching anything, what is the probability of being at each of the locations without catching fish?*

QUESTION SIX

(a) *Suppose a random variable Y is defined as*

$Y= a+bx$, when $E(X)= \mu$, $Var(X)= \sigma^2$.

Show that :

(i) If $Z= (x-\mu)/\sigma$ then $E(Z)=0$ and $Var(Z)=1$.

(ii) $E(Y)=a+b\mu$ and $Var(Y)=\sigma^2 b^2$

(b) *Show that the moment generating function of the random variable* $X \sim N(\mu,\sigma^2)$ is $Exp(t\mu+\sigma^2 t^2/2)$

QUESTION SEVEN

(a) *State clearly without proof, the Bayes theorem.*

(b) *Assume that the probability is 0.95 that the jury selected to try a criminal case will arrive at the appropriate verdict. Suppose that the local police force is quite diligent in its duties and that 99% of* *the people brought before the court are actually guilty. Compute the probability that the defendant is innocent, given that the jury finds him innocent.*