UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

B.A.S.S. II

TITLE OF PAPER   :   MATHEMATICS FOR STATISTICIANS

COURSE NUMBER   :   ST 202

TIME ALLOWED   :   TWO (2) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF
                                                 FIVE QUESTIONS.
                                          2.   ANSWER ANY THREE QUESTIONS

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find the limit of the function $f(x) = \dfrac{x^2 + 3x + 2}{x^2 + 5x + 4}$ as

    (i) $x \to 0$ [2]

    (ii) $x \to -1$ [3]

    (iii) $x \to \infty$ [3]

    (b) Use the limit definition of the derivative (i.e from first principles) to find $f'(x)$ if $f(x) = \sqrt{2x + 1}$ [6]

    (c) Find the derivative $\dfrac{dy}{dx}$ if
    $y = \sin^{-1} x^2$ (N.B: $\sin^{-1} = \arcsin$)   [6]

QUESTION 2

2. (a) Use Newton's method with four (4) iterations and $x_0 = 1$ to calculate the square root of 2.   [6]

    (b) Given that $f(x, y) = xye^{xy}$, find

    (i) f_y [3]

    (ii) f_{yx} [3]

    (c) Find the derivative of $y = x^{\ln x}$ [4]

    (d) Find the slope of the curve $x^2 + xy + y^2 = 2$ at the point (1,2).   [4]

QUESTION 3

3. (a) Evaluate the following integrals

   (i) $\int \sin^7 x \, dx$ [4]

   (ii) $\int \dfrac{4x}{\sqrt{1 - 4x^4}} dx$ [4]

   (iii) $\int_0^2 \int_{-1}^1 (1 - 6x^2 y) \, dx \, dy$ [4]

   (b) Given that $\mathbf{A} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 3\mathbf{i} + \mathbf{j}$, find

   (i) $\mathbf{A} \cdot \mathbf{B}$ [4]

   (ii) $\mathbf{A} \times \mathbf{B}$ [4]

QUESTION 4

4. (a) Use Gaussian elimination method with back substitution
   to solve the system

$$
\begin{aligned}
2x &+ 4y &+ 6z &= 2 \\
4x &+ 5y &+ 6z &= 1 \\
3x &+ y &- 2z &= 1
\end{aligned}
$$
                                                                                [8]

   (b) Use elementary row operations to find the inverse of the matrix

$$
A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}
$$
                                                                                [6]

   (c) Reduce the following matrix to row echelon form

$$
A = \begin{pmatrix} 1 & 1 & 4 & 4 \\ 0 & -1 & -2 & -3 \\ 1 & 3 & 7 & 9 \\ -1 & -2 & -4 & -6 \end{pmatrix}
$$
                                                                                [6]

QUESTION 5

5. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{pmatrix}$$

[10]

(b) Solve the following homogeneous system of equations.

$$
\begin{aligned}
x + 2y - 3z &= 0 \\
2x - y + 4z &= 0 \\
4x + 3y - 2z &= 0
\end{aligned}
$$

[5]

(c) Find the adjoint of the following matrix

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

[5]