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**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2005**

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| **TITLE OF PAPER:** | **INTRODUCTION TO DISTRIBUTION THEORY** |
| **COURSE CODE :** | **ST 301** |
| **TIME ALLOWED :** | **TWO(2) HOURS** |
| **INSTRUCTIONS :** | **THIS PAPER HAS FIVE QUESTIONS.<br>ANSWER ANY FOUR(4) QUESTIONS.<br>EACH QUESTION CARRIES 15 MARKS.** |
| **REQUIREMENTS:** | **Scientific Calculator** |

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## QUESTION ONE

*Let $Y_1=(x_1-x_2)/2$ and $Y_2=X_2$, where $X_1$ and $X_2$ are stochastically independent random variables with joint probability density function given as*

$f(x_1,x_2)=1/4 \exp(-x_1+x_2)/2$ ; $0<x_1<\infty$ and $0<x_2<\infty$.

*Find the joint probability density function of $Y_1$ and $Y_2$, hence obtain the marginal probability density function of $Y_1$.*

## QUESTION TWO

*(a) The cumulative distribution function of a random variable X is*

$$
\begin{aligned}
F_X(x) &= 0, x<-10 \\
&= 1/4, -10 \le x < 0 \\
&= 3/4, 0 \le x < 10 \\
&= 1, x \ge 10
\end{aligned}
$$

*Find the distribution function for a random variable*

$Z=7x-50$

(b) If Y is a function of a random variable X, such

that Y= $f(\mu_x)+f^{(1)}(\mu_x)(x-\mu_x)$.

Show that $F_y(y)=F_x\left(\left\{\langle y-f(y)\rangle / f^{(1)}(\mu_x)\right\}+\mu_x\right)$,

For $f^{(1)}(\mu_x)>0$ and $\mu_x \, \mathrm{A} f(\mu_x)$

## QUESTION THREE

(a) If Y~b(n,1/3). Find the smallest value of n that yields P(Y $\geq 1$) $\geq 0.80$

(b) If x~b(n,p), show that

E(x/n)=P and E{(x/n-P)$^2$}=P(1-P)/n

## QUESTION FOUR

(a) What are the roles of Normal Distribution in Sampling distribution theory?

(b) Suppose a random variable Y is defined as

Y= a+bx, when E(X)= $\mu$, Var(X)= $\sigma^2$.

*Show that* 150

(i) *If $Z=(x-\mu)/\sigma$ then $E(Z)=0$ and $Var(Z)=1$.*

(ii) *$E(Y)=a+b\mu$ and $Var(Y)=\sigma^2b^2$*

( C) *Show that the moment generating function of the random variable $X \sim N(\mu,\sigma^2)$ is $Exp(t\mu+\sigma^2t^2/2)$*

## QUESTION FIVE

(a) *Suppose that the length of a time a transistor radio will work is a random variable Z, with density fucnction*

$f_Z(z)=1000e^{-1000z}, z>0$ *or zero otherwise.*

*Obtain the moment generating function*

(b) *Find the k-th moment of random variable X with probability density function given as*

$f_X(x)=1/10, 20<x<30$ *or zero otherwise.*