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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2005

TITLE OF PAPER:	**INTRODUCTION TO REGRESSION ANALYSIS**
COURSE CODE :	**ST 304**
TIME ALLOWED :	**TWO(2) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS FIVE QUESTIONS. ANSWER ANY FOUR(4) QUESTIONS. EACH QUESTION CARRIES 15 MARKS.**
REQUIREMENTS:	**Scientific Calculator**

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QUESTION ONE

Given the linear multiple regression model

$Y=X\beta+U$, where $U \sim NID(0,\sigma^2 I)$. *Show that the residual sum of squares is quadratic form in **U**, assuming the least squares estimate of **β** to be $\hat{\beta}$. Hence or otherwise, show that $s^2 = e'e/n-k$, is unbiased for **σ^2** where n is the number of observations and k is the number of explanatory variables.*

QUESTION TWO

For a linear regression model ***$Y=X\beta+U$, where $U \sim NID(0,\sigma^2 I)$.***

(i) *Show that the least square estimate $\hat{\beta}$ is distributed independently of e'e where $e=Y-x\hat{\beta}$.*

(ii)Show that the expected value of Y_{n+1} is $C'\hat{\beta}$, where $C=[1\ X_{2,n+1}\ X_{3,n+1}\ldots\ldots\ldots]$

QUESTION THREE

In a multiple linear regression model $Y=X\beta+U$, *if all the assumptions necessary for the least squares method hold except that* $E(UU') \neq \sigma^2 I$.

(a) *What happens to the estimates of the parameters by the Ordinary least squares method?*

(b) *Suggest an alternative estimating procedure and find (i) the estimates of the parameters.*

(ii) *the var-covariance matrix of the estimates.*

QUESTION FOUR

In the analysis of variance table given below for a regression data set of twenty cases.

(i) *Find the values of the asterisked cells.*

(ii) *Test for* $\beta_2=0$ *, given that* $t_{n-1,0.25}=2.093$

(iii) *Compute F-value for the regression coefficients.*

ANALYSIS OF VARIANCE TABLE

(a) REGRESSION COEFFICIENTS

Regression coefficient	*Estimated regression coefficient*	*Estimated standard deviation*	*t-value*
B_0	33.87407	***	18.68
B_1	***	0.00889	-11.44
B_2	8.05547	1.45911	***

(b) ANOVA RESULTS

Source of variation	***Df***	***SS***	***MS***
Regression	***	**1504.41**	***
Error	***	***	**10.38**
Total	***	**1680.80**	

QUESTION FIVE

In a study of factors thought to be related to admission patterns of a large general hospital. The administrator obtained these data on ten communities in the hospital's catchment's area

Persons per 1000 popultion admitted during study period.(Y)	*Index of availability of other health services.(x_1)*	*Index of indigency.(x_2)*
61.6	6.0	6.3
53.2	4.4	5.5
65.5	9.1	3.6
64.9	8.1	5.8
72.7	9.7	6.8
52.5	4.8	7.9
50.2	7.6	4.2
44.0	4.4	6.0
53.8	9.1	2.8
53.5	6.7	6.7

Given that

$\sum X_1^2 = 525.73, \sum X_1 X_2 = 374.31, \sum Y^2 = 33349.92, \sum X_2^2 = 331.56, \sum X_1 Y = 4104.32, \sum X_2 Y = 3138$

(i) Obtain the regression equation of Y on X_1 and X_2

(ii) Predict the admission population when X_1=11.5 and X_2=5, using the fitted regression model.