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UNIVERSITY OF SWAZILAND

EXAMINATION PAPER 2005

TITLE OF PAPER	: SAMPLE SURVEY THEORY
COURSE CODE	: ST 306 *(NEW PROGRAMME)*
TIME ALLOWED	: TWO (2) HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES FORMULA SHEET ATTACHED
INSTRUCTIONS	: ANSWER QUESTION ONE AND ANY OTHER FOUR QUESTIONS

(EACH QUESTION CARRIES 20 MARKS)

Question 1

Swaziland's National Income for 2004 is to be estimated from a sample of 10 industries that report their income earlier than the remaining 35. There are 45 industries used to determine the total national income. Income data for 2003 is available for all 45 industries and totals 2174.2 (in millions). The data are given in the following table;

Industry	2003	2004
Textile mill products	13.6	14.5
Chemicals and allied	37.7	42.7
Lumber and wood	15.2	15.1
Electrical and electronic equipment	48.4	53.6
Motor vehicles and equipment	19.6	25.4
Trucking and warehousing	33.6	35.9
Banking	44.4	48.5
Real estate	198.3	221.2
Health services	99.2	114.0
Educational services	15.4	17.0

a) Find the SRS estimate of the 2004 total income along with the estimated error bound. (10 Marks)

b) Find a minimum variance unbiased estimate of the 2004 total income along with the estimated 95% confidence interval. (15 Marks)

Question 2

Suppose the circulation manager of The Times of Swaziland wishes to estimate the average number of newspapers purchased per household and the total number of newspapers purchased in Mbabane. Travel costs from household to household are substantial. Therefore, the 3000 households in this city are listed in 30 geographical clusters of equal size each, and a simple random sample of 4 clusters is selected. Interviews are conducted, with the results as shown in the accompanying table.

Clusters	Number of newspapers										Total
1	1	2	1	3	3	2	1	4	1	1	19
2	1	3	2	2	3	1	4	1	1	2	20
3	2	1	1	1	1	3	2	1	3	1	16
4	1	1	3	2	1	5	1	2	3	1	20

Estimate the average number of newspapers purchased per household and the total number of newspapers purchased in Mbabane, and place a bound on the error of estimation on each estimate.

(25 Marks)

Question 3

The NIC (Pty) Ltd Corporation wishes to obtain information on the effectiveness of a business machine. A number of division heads will be interviewed by telephone and asked to rate the equipment on a numerical scale. The divisions are located in Swaziland, South Africa and Botswana. The costs are larger for interviewing division heads located outside Swaziland. The accompanying table gives the costs per interview, approximate variances of the ratings, and N_i's that have been established. The corporation wants to estimate the average rating with $V(\overline{y}_{st})=0.1$. Choose the sample size n that achieves this bound, and find the appropriate allocation.

Swaziland	**South Africa**	**Botswana**
c_1=E9	c_2=E25	c_3=E36
$\sigma_1^2=2.25$	$\sigma_2^2=3.24$	$\sigma_3^2=3.24$
$N_1=112$	$N_2=68$	$N_3=39$

(25 Marks)

Question 4

Wage earners in Taung Textiles are stratified into management and clerical classes, the first having 300 and second having 500 employees. To assess attitude on sick-leave policy, independent random samples of 100 workers each were selected, one sample from each of the classes. After the sample data were collected, the responses were divided according to gender. In the table of results,

a= Number who **like** the policy
b= Number who **dislike** the policy
c= Number who have **no opinion** on the policy

	Management	**Clerical**
Male	a=60	a=24
	b=15	b=4
	c= 5	c=2
Female	a=10	a=42
	b=7	b=20
	c=3	c=8

a) Estimate the proportion of wage earners who like the policy, and place bound on the error of estimation. (10 Marks)
b) Estimate the proportion of female wage earners who like the policy, and place bound on the error of estimation. (15 Marks)

Question 5

A market research firm constructed a sampling plan to estimate the weekly sales of brand A cereal in the Gauteng Province. The firm decided to sample cities within the area and then to sample supermarkets within cities The number of boxes of brand A cereal sold in a specified week is the measurement of interest. Five cities were sampled from the 20 in the area.

City	No. of supermarkets	No. of supermarkets sampled	Mean	Variance
1	45	9	102	20
2	36	7	90	16
3	20	4	76	22
4	18	4	94	26
5	28	6	120	12

Using the data given in table above;

a) Estimate the average sales for the week for all supermarkets in the area with a 95% confidence interval. (15 marks)

b) Estimate the total sales for the week for all supermarkets in the area and margin of error of the estimate. (10 marks)

$$s^2 = \sum_{i=1}^{n} \frac{(y_i-\bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i-\bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{V}(\hat{\mu})_{srs} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{V}(\hat{\tau})_{srs} = N^2\hat{V}(\hat{\mu})_{srs}$$

$$\hat{p}_{srs} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{srs} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{pps} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i}\right) \qquad \hat{V}(\hat{\tau})_{pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i} - \hat{\tau}_{pps}\right)^2$$

$$\hat{\mu}_{pps} = \frac{1}{N}\hat{\tau}_{pps} \qquad \hat{V}(\hat{\mu})_{pps} = \frac{1}{N^2}\hat{V}(\hat{\tau})_{pps}$$

$$\hat{\mu}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{\mu})_{sys} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{sys} = N\hat{\mu}_{sys} \qquad \hat{V}(\hat{\tau})_{sys} = N^2\hat{V}(\hat{\mu})_{sys}$$

$$\hat{p}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{sys} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\mu}_{rsys} = \sum_{i=1}^{ns} \frac{\hat{\mu}_i}{ns} \qquad \hat{V}(\hat{\mu})_{rsys} = \left(\frac{N-n}{N}\right)\sum_{i=1}^{ns}\frac{(\hat{\mu}_i-\hat{\mu}_{rsys})^2}{ns(ns-1)}$$

$$\hat{\tau}_{rsys} = N\hat{\mu}_{rsys} \qquad \hat{V}(\hat{\tau})_{rsys} = N^2\hat{V}(\hat{\mu})_{rsys}$$

$$\hat{\mu}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i-n_i}{N_i}\right)\frac{s_i^2}{n_i}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{V}(\hat{\tau})_{str} = N^2\hat{V}(\hat{\mu})_{str}$$

$$\hat{p}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p}_i \qquad \hat{V}(\hat{p})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i-n_i}{N_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}_i)}{n_i-1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{i=1}^{L} w_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{pstr} = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{i=1}^{L} w_i s_i^2 + \frac{1}{n^2}\sum_{i=1}^{L}(1-w_i)s_i^2$$

$r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$ $\hat{V}(r) = (\frac{N-n}{N})(\frac{1}{n\mu_x^2})\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{(n-1)}$

$\hat{\rho} = \frac{c\hat{o}v(x,y)}{s_x s_y}$ $\hat{V}(r) = \frac{1-(n/N)}{n}(\frac{1}{\mu_x^2})(s_y^2 + r^2 s_x^2 - 2r\hat{\rho}s_x s_y)$

$\hat{\tau}_{ratio} = r\tau_x$ $\hat{V}(\hat{\tau})_{ratio} = \tau_x^2 \hat{V}(r)$

$\hat{\mu}_{ratio} = r\mu_x$ $\hat{V}(\hat{\mu})_{ratio} = \mu_x^2 \hat{V}(r)$

$Y_i = \beta_0 + \beta_1(X_i) + \varepsilon_i$ $\sum_{i=1}^{n}(y_i - rx_i)^2 = \sum_{i=1}^{n} y_i^2 + r^2 \sum_{i=1}^{n} x_i^2 - 2r \sum_{i=1}^{n} y_i x_i$

$b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \hat{\rho}(s_y/s_x)$

$b_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$ $\hat{\rho} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$

$\hat{\mu}_{reg} = \bar{y} + b_1(\mu_x - \bar{x})$ $\hat{V}(\hat{\mu})_{reg} = (\frac{N-n}{N})\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n(n-1)}$

$\hat{y}_i = b_0 + b_1(x_i)$ $\hat{V}(\hat{\mu})_{reg} \approx (\frac{N-n}{N})\frac{MSE}{n}$

$\hat{\mu}_{diff} = \bar{y} + (\mu_x - \bar{x})$ $\hat{V}(\hat{\mu})_{diff} = (\frac{N-n}{N})\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n(n-1)}$

$\sum_{i=1}^{n}(d_i - \bar{d})^2 = \sum_{i=1}^{n} d_i^2 - n\bar{d}^2$ $\hat{RE}(\frac{E1}{E2}) = \frac{\hat{V}(E2)}{\hat{V}(E1)}$

$\hat{\mu}_{cts1} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}$ $\hat{V}(\hat{\mu})_{cts1} = (\frac{N-n}{N})\frac{\sum_{i=1}^{n}(y_i - \bar{y}m_i)^2}{n\bar{M}^2(n-1)}$

$\hat{V}(\hat{\mu})_{cts1} = (\frac{N-n}{N})(\frac{1}{n\bar{M}^2})(s_y^2 + \hat{\mu}_{cts1}^2 s_m^2 - 2\hat{\mu}_{cts1}\hat{\rho}s_y s_m)$

$\hat{\tau}_{cts1(1)} = M\hat{\mu}_{cts1}$ $\hat{V}(\hat{\tau})_{cts1(1)} = M^2 \hat{V}(\hat{\mu})_{cts1}$

$$\hat{\tau}_{cts1(2)} = N\bar{y}_t = N\left(\frac{\sum_{i=1}^{n} y_i}{n}\right) \qquad \hat{V}(\hat{\tau})_{cts1(2)} = (\tfrac{N-n}{N})(\tfrac{N^2}{n})\frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)}$$

$$\bar{m} = \frac{\sum_{i=1}^{n} m_i}{n} \qquad \sum_{i=1}^{n}(y_i - \bar{y}m_i)^2 = \sum_{i=1}^{n} y_i^2 + \bar{y}^2 \sum_{i=1}^{n} m_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i m_i$$

$$\hat{p}_{cts1} = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i} \qquad \hat{V}(\hat{p})_{cts1} = (\tfrac{N-n}{N})(\tfrac{1}{n\bar{M}^2})\frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

$$\Pi_i = \frac{m_i}{M} \qquad \hat{V}(\hat{p})_{cts1} = (\tfrac{N-n}{N})(\tfrac{1}{n\bar{M}^2})(s_a^2 + \hat{p}^2 s_m^2 - 2\hat{p}\hat{\rho}s_a s_m)$$

$$\hat{\tau}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\Pi_i} \qquad \sum_{i=1}^{n}(a_i - \hat{p}m_i)^2 = \sum_{i=1}^{n} a_i^2 + \hat{p}^2\sum_{i=1}^{n} m_i^2 - 2\hat{p}\sum_{i=1}^{n} a_i m_i$$

$$\hat{\tau}_{cts1,pps} = \frac{M}{n}\sum_{i=1}^{n}\bar{y}_i \qquad \hat{V}(\hat{\tau})_{cts1,pps} = \frac{M^2}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\tau})^2$$

$$\hat{\mu}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i \qquad \hat{V}(\hat{\mu})_{cts1,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu})^2$$

$$\hat{\mu}_{cts2} = (\tfrac{N}{M})\frac{\sum_{i=1}^{n} M_i\bar{y}_i}{n} \qquad \hat{V}(\hat{\mu})_{cts2} = (\tfrac{N-n}{N})(\tfrac{1}{n\bar{M}^2})s_b^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2(\tfrac{M_i-m_i}{M_i})(\tfrac{s_i^2}{m_i})$$

$$s_b^2 = \frac{\sum_{i=1}^{n}(M_i\bar{y}_i - \bar{M}\hat{\mu})^2}{n-1} \qquad s_i^2 = \frac{\sum_{i=1}^{m_i}(y_{ij}-\bar{y}_i)^2}{m_i-1}$$

$$\hat{\tau}_{cts2} = M\hat{\mu}_{cts2} \qquad \hat{V}(\hat{\tau})_{cts2} = M^2\hat{V}(\hat{\mu})_{cts2}$$

$$\hat{\mu}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{\mu})_{cts2,ratio} = (\tfrac{N-n}{N})(\tfrac{1}{n\bar{M}^2})s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2(\tfrac{M_i-m_i}{M_i})(\tfrac{s_i^2}{m_i})$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\bar{y}_i - \hat{\mu}_{cts2,r})^2}{n-1} \qquad s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\hat{p}_i - \hat{p}_{cts2,r})^2}{n-1}$$

$$\hat{p}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p})_{cts2,ratio} = (\tfrac{N-n}{N})(\tfrac{1}{n\bar{M}^2})s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2(\tfrac{M_i-m_i}{M_i})(\tfrac{\hat{p}_i(1-\hat{p}_i)}{m_i-1})$$

$\hat{\mu}_{cts2,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i$ $\hat{V}(\hat{\mu})_{cts2,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu}_{cts2,pps})^2$

$\hat{\tau}_{cts2,pps} = M\hat{\mu}_{cts2,pps}$ $\hat{V}(\hat{\tau}) = M^2\hat{V}(\hat{\mu})_{cts2,pps}$

n for μ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for τ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4N^2)+\sigma^2}$

n for p (SRS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

n for μ (SYS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for p (SYS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

$k \leq \frac{N}{n}$ $k' = k(ns)$

n for μ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

n for τ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4N^2)+\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR μ:

$n_i = n\left(\frac{N_i\sigma_i/\sqrt{c_i}}{\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}}\right)$ $n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{i=1}^{L} N_i\sigma_i\sqrt{c_i}\right)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_i}\right)$ $n = \frac{\left(\sum_{i=1}^{L} N_i\sigma_i\right)^2}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i}{N}\right)$ $n = \frac{\sum_{i=1}^{L} N_i\sigma_i^2}{N^2(B^2/4)+(1/N)\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR τ: change $N^2(B^2/4)$ to $N^2(B^2/4N^2)$

Allocations for STR p:

$$n_i = n\left(\frac{N_i\sqrt{p_i(1-p_i)/c_i}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{i=1}^{L} N_i^2 p_i(1-p_i)/w_i}{N^2(B^2/4)+\sum_{i=1}^{L} N_i p_i(1-p_i)}$$

n for μ (ratio): $n = \frac{N\sigma^2}{N(B^2/4)+\sigma^2}$

n for τ (ratio): $n = \frac{N\sigma^2}{N(B^2/4N^2)+\sigma^2}$

n for μ (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

n for τ (CTS1(1)): $n = \frac{N\sigma_c^2}{N(B^2/4N^2)+\sigma_c^2}$

n for τ (CTS1(2)): $n = \frac{N\sigma_t^2}{N(B^2/4N^2)+\sigma_t^2}$

$$s_c^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}m_i)^2}{(n-1)} \text{ with } \bar{y} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i} \qquad s_t^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)} \text{ with } \bar{y}_t = \frac{\sum_{i=1}^{n} y_i}{n}$$

n for p (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

$$s_c^2 = \frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

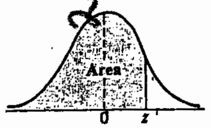
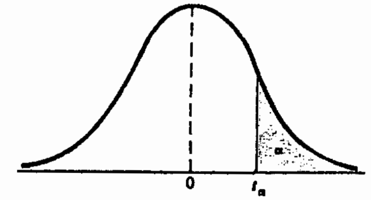


TABLE A.4
Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE A.5*
Critical Values of the t Distribution



	α				
v	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

*Table A.5 is taken from Table IV of R. A. Fisher, *Statistical Methods for Research Workers*, Oliver & Boyd Ltd., Edinburgh, by permission of the author and publishers.

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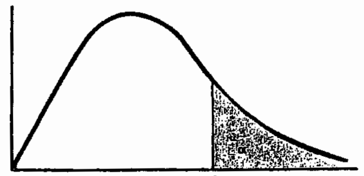
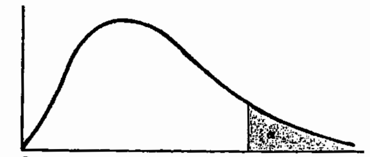
TABLE A.6*
Critical Values of the Chi-Square Distribution



	α							
ν	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

*Abridged from Table 8 of *Biometrika Tables for Statisticians*, Vol. I, by permission of E. S. Pearson and the Biometrika Trustees.

TABLE A.7*
Critical Values of the *F* Distribution



$f_{0.05}(\nu_1, \nu_2)$

	ν_1								
ν_2	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

*Reproduced from Table 18 of *Biometrika Tables for Statisticians*, Vol. I, by permission of E. S. Pearson and the Biometrika Trustees.

TABLE A.7 (*continued*)
Critical Values of the *F* Distribution
$f_{0.05}(\nu_1, \nu_2)$

ν_2	ν_1 = 10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

TABLE A.7 (*continued*)
Critical Values of the *F* Distribution
$f_{0.01}(\nu_1, \nu_2)$

ν_2	ν_1 = 1	2	3	4	5	6	7	8	9
1	4052	4999.5	5403	5625	5764	5859	5928	5981	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

TABLE A.7 (*continued*)
Critical Values of the *F* Distribution

$f_{0.01}(\nu_1, \nu_2)$

	ν_1									
ν_2	10	12	15	20	24	30	40	60	120	∞
1	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00