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UNIVERSITY OF SWAZILAND

EXAMINATION PAPER 2005

TITLE OF PAPER	: SAMPLE SURVEY THEORY
COURSE CODE	: ST 306 *(OLD PROGRAMME)*
TIME ALLOWED	: THREE (3) HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES FORMULA SHEET ATTACHED
INSTRUCTIONS	: ANSWER QUESTION ONE AND ANY OTHER THREE QUESTIONS (ALL QUESTION CARRY EQUAL MARKS)

Question 1

Swaziland's National Income for 2004 is to be estimated from a sample of 10 industries that report their income earlier than the remaining 35. There are 45 industries used to determine the total national income. Income data for 2003 is available for all 45 industries and totals 2174.2 (in millions). The data are given in the following table;

Industry	2003	2004
Textile mill products	13.6	14.5
Chemicals and allied	37.7	42.7
Lumber and wood	15.2	15.1
Electrical and electronic equipment	48.4	53.6
Motor vehicles and equipment	19.6	25.4
Trucking and warehousing	33.58	35.9
Banking	44.4	48.5
Real estate	198.3	221.2
Health services	99.2	114.0
Educational services	15.4	17.0

a) Find the SRS estimate of the 2004 total income along with the estimated error bound. (10 Marks)

b) Find a minimum variance unbiased estimate of the 2004 total income along with the estimated 95% confidence interval. (15 Marks)

c) Which is a more efficient estimate , between a) and b), to estimate total income? (10 Marks)

Question 2

Suppose the circulation manager of The Times of Swaziland wishes to estimate the average number of newspapers purchased per household and the total number of newspapers purchased in Mbabane. Travel costs from household to household are substantial. Therefore, the 3000 households in this city are listed in 30 geographical clusters of equal size each, and a simple random sample of 4 clusters is selected. Interviews are conducted, with the results as shown in the accompanying table.

Clusters	Number of newspapers	Total
1	1 2 1 3 3 2 1 4 1 1	19
2	1 3 2 2 3 1 4 1 1 2	20
3	2 1 1 1 1 3 2 1 3 1	16
4	1 1 3 2 1 5 1 2 3 1	20

a) Estimate the average number of newspapers purchased per household and the total number of newspapers purchased in Mbabane, and place a bound on the error of estimation of each estimate. (25 Marks)

b) If household 1 in cluster 3 and household 5 in cluster 2 do not respond to the interviews, estimate the average number of newspapers purchased per household, and the error bound. (10 Marks)

Question 3

The NIC (Pty) Ltd Corporation wishes to obtain information on the effectiveness of a business machine. A number of division heads will be interviewed by telephone and asked to rate the equipment on a numerical scale. The divisions are located in Swaziland, South Africa and Botswana. The costs are larger for interviewing division heads located outside Swaziland. The accompanying table gives the costs per interview, approximate variances of the ratings, and N_i's that have been established. The corporation wants to estimate the average rating with $V\left(\bar{y}_{st}\right)=0.1$. Choose the sample size n that achieves this bound, and find the appropriate allocation.

Swaziland	**South Africa**	**Botswana**
c_1=E9	c_2=E25	c_3=E36
$\sigma_1^2=2.25$	$\sigma_2^2=3.24$	$\sigma_3^2=3.24$
$N_1=112$	$N_2=68$	$N_3=39$

(25 Marks)

Suppose data on costs and variances were not available, choose the sample size n that achieves this bound, and find the appropriate allocation. (10 Marks)

Question 4

Wage earners in Taung Textiles are stratified into management and clerical classes, the first having 300 and second having 500 employees. To assess attitude on sick-leave policy, independent random samples of 100 workers each were selected, one sample from each of the classes. After the sample data were collected, the responses were divided according to gender. In the table of results,

a= Number who **like** the policy
b= Number who **dislike** the policy
c= Number who have **no opinion** on the policy

	Management	**Clerical**
Male	a=60	a=24
	b=15	b=4
	c= 5	c=2
Female	a=10	a=42
	b=7	b=20
	c=3	c=8

a) Estimate the proportion of wage earners who like the policy, and place bound on the error of estimation. (10 Marks)
b) Estimate the proportion of female wage earners who like the policy, and place bound on the error of estimation. (15 Marks)
c) Estimate the total number of female wage earners who like the policy, and place bound on the error of estimation. (10 Marks)

Question 5

A market research firm constructed a sampling plan to estimate the weekly sales of brand A cereal in the Gauteng Province. The firm decided to sample cities within the areas and then to sample supermarkets within cities The number of boxes of brand A cereal sold in a specified week is the measurement of interest. Five cities were sampled from the 20 in the area.

City	No. of supermarkets	No. of supermarkets sampled	Mean	Variance
1	45	9	102	20
2	36	7	90	16
3	20	4	76	22
4	18	4	94	26
5	28	6	120	12

Using the data given in table above;

a) Estimate the average sales for the week for all supermarkets in the area with a 95% confidence interval. (10 Marks)

b) Estimate the total sales for the week for all supermarkets in the area and margin of error of the estimate. (10 Marks)

c) If 6 supermarkets are sampled from each of the 5 cities, estimate the average sales for the week for all supermarkets in the area and margin of error of the estimate. (15 Marks)

ST306 II

$s^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n-1}$ $\sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}$

$\hat{\mu}_{srs} = \bar{y}$ $\hat{V}(\hat{\mu})_{srs} = \frac{s^2}{n}(\frac{N-n}{N})$

$\hat{\tau}_{srs} = N\hat{\mu}_{srs}$ $\hat{V}(\hat{\tau})_{srs} = N^2\hat{V}(\hat{\mu})_{srs}$

$\hat{p}_{srs} = \sum_{i=1}^{n} \frac{y_i}{n}$ $\hat{V}(\hat{p})_{srs} = \frac{\hat{p}(1-\hat{p})}{(n-1)}(\frac{N-n}{N})$

$\hat{\tau}_{pps} = \frac{1}{n}\sum_{i=1}^{n}(\frac{y_i}{\pi_i})$ $\hat{V}(\hat{\tau})_{pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\frac{y_i}{\pi_i} - \hat{\tau}_{pps})^2$

$\hat{\mu}_{pps} = \frac{1}{N}\hat{\tau}_{pps}$ $\hat{V}(\hat{\mu})_{pps} = \frac{1}{N^2}\hat{V}(\hat{\tau})_{pps}$

$\hat{\mu}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n}$ $\hat{V}(\hat{\mu})_{sys} = \frac{s^2}{n}(\frac{N-n}{N})$

$\hat{\tau}_{sys} = N\hat{\mu}_{sys}$ $\hat{V}(\hat{\tau})_{sys} = N^2\hat{V}(\hat{\mu})_{sys}$

$\hat{p}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n}$ $\hat{V}(\hat{p})_{sys} = \frac{\hat{p}(1-\hat{p})}{(n-1)}(\frac{N-n}{N})$

$\hat{\mu}_{rsys} = \sum_{i=1}^{ns} \frac{\hat{\mu}_i}{ns}$ $\hat{V}(\hat{\mu})_{rsys} = (\frac{N-n}{N})\sum_{i=1}^{ns}\frac{(\hat{\mu}_i - \hat{\mu}_{rsys})^2}{ns(ns-1)}$

$\hat{\tau}_{rsys} = N\hat{\mu}_{rsys}$ $\hat{V}(\hat{\tau})_{rsys} = N^2\hat{V}(\hat{\mu})_{rsys}$

$\hat{\mu}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\bar{y}_i$ $\hat{V}(\hat{\mu})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2(\frac{N_i - n_i}{N_i})\frac{s_i^2}{n_i}$

$\hat{\tau}_{str} = N\hat{\mu}_{str}$ $\hat{V}(\hat{\tau})_{str} = N^2\hat{V}(\hat{\mu})_{str}$

$\hat{p}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p}_i$ $\hat{V}(\hat{p})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}_i)}{n_i - 1}\right)$

$\hat{\mu}_{pstr} = \sum_{i=1}^{L} w_i\bar{y}_i$ $\hat{V}(\hat{\mu})_{pstr} = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{i=1}^{L} w_i s_i^2 + \frac{1}{n^2}\sum_{i=1}^{L}(1 - w_i)s_i^2$

$$r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{V}(r) = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{(n-1)}$$

$$\hat{\rho} = \frac{\hat{cov}(x,y)}{s_x s_y} \qquad \hat{V}(r) = \frac{1-(n/N)}{n}\left(\frac{1}{\mu_x^2}\right)(s_y^2 + r^2 s_x^2 - 2r\hat{\rho}s_x s_y)$$

$$\hat{\tau}_{ratio} = r\tau_x \qquad \hat{V}(\hat{\tau})_{ratio} = \tau_x^2 \hat{V}(r)$$

$$\hat{\mu}_{ratio} = r\mu_x \qquad \hat{V}(\hat{\mu})_{ratio} = \mu_x^2 \hat{V}(r)$$

$$Y_i = \beta_0 + \beta_1(X_i) + \varepsilon_i \qquad \sum_{i=1}^{n}(y_i - rx_i)^2 = \sum_{i=1}^{n} y_i^2 + r^2 \sum_{i=1}^{n} x_i^2 - 2r\sum_{i=1}^{n} y_i x_i$$

$$b_0 = \bar{y} - b_1\bar{x} \qquad b_1 = \hat{\rho}(s_y/s_x)$$

$$b_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \qquad \hat{\rho} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y}$$

$$\hat{\mu}_{reg} = \bar{y} + b_1(\mu_x - \bar{x}) \qquad \hat{V}(\hat{\mu})_{reg} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n(n-1)}$$

$$\hat{y}_i = b_0 + b_1(x_i) \qquad \hat{V}(\hat{\mu})_{reg} \approx \left(\frac{N-n}{N}\right)\frac{MSE}{n}$$

$$\hat{\mu}_{diff} = \bar{y} + (\mu_x - \bar{x}) \qquad \hat{V}(\hat{\mu})_{diff} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(d_i-\bar{d})^2}{n(n-1)}$$

$$\sum_{i=1}^{n}(d_i - \bar{d})^2 = \sum_{i=1}^{n} d_i^2 - n\bar{d}^2 \qquad \hat{RE}\left(\frac{E1}{E2}\right) = \frac{\hat{V}(E2)}{\hat{V}(E1)}$$

$$\hat{\mu}_{cts1} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i} \qquad \hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i - \bar{y}m_i)^2}{n\bar{M}^2(n-1)}$$

$$\hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)(s_y^2 + \hat{\mu}_{cts1}^2 s_m^2 - 2\hat{\mu}_{cts1}\hat{\rho}s_y s_m)$$

$$\hat{\tau}_{cts1(1)} = M\hat{\mu}_{cts1} \qquad \hat{V}(\hat{\tau})_{cts1(1)} = M^2\hat{V}(\hat{\mu})_{cts1}$$

$$\hat{\tau}_{cts1(2)} = N\bar{y}_t = N\left(\frac{\sum_{i=1}^{n} y_i}{n}\right)$$

$$\hat{V}(\hat{\tau})_{cts1(2)} = \left(\frac{N-n}{N}\right)\left(\frac{N^2}{n}\right)\frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)}$$

$$\bar{m} = \frac{\sum_{i=1}^{n} m_i}{n}$$

$$\sum_{i=1}^{n}(y_i - \bar{y}m_i)^2 = \sum_{i=1}^{n} y_i^2 + \bar{y}^2\sum_{i=1}^{n} m_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i m_i$$

$$\hat{p}_{cts1} = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i}$$

$$\hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{nM^2}\right)\frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

$$\Pi_i = \frac{m_i}{M}$$

$$\hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{nM^2}\right)(s_a^2 + \hat{p}^2 s_m^2 - 2\hat{p}\hat{\rho}s_a s_m)$$

$$\hat{\tau}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\Pi_i}$$

$$\sum_{i=1}^{n}(a_i - \hat{p}m_i)^2 = \sum_{i=1}^{n} a_i^2 + \hat{p}^2\sum_{i=1}^{n} m_i^2 - 2\hat{p}\sum_{i=1}^{n} a_i m_i$$

$$\hat{\tau}_{cts1,pps} = \frac{M}{n}\sum_{i=1}^{n}\bar{y}_i$$

$$\hat{V}(\hat{\tau})_{cts1,pps} = \frac{M^2}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\tau})^2$$

$$\hat{\mu}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i$$

$$\hat{V}(\hat{\mu})_{cts1,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu})^2$$

$$\hat{\mu}_{cts2} = \left(\frac{N}{M}\right)\frac{\sum_{i=1}^{n} M_i\bar{y}_i}{n}$$

$$\hat{V}(\hat{\mu})_{cts2} = \left(\frac{N-n}{N}\right)\left(\frac{1}{nM^2}\right)s_b^2 + \frac{1}{nNM^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_b^2 = \frac{\sum_{i=1}^{n}(M_i\bar{y}_i - \bar{M}\hat{\mu})^2}{n-1}$$

$$s_i^2 = \frac{\sum_{i=1}^{m_i}(y_{ij}-\bar{y}_i)^2}{m_i-1}$$

$$\hat{\tau}_{cts2} = M\hat{\mu}_{cts2}$$

$$\hat{V}(\hat{\tau})_{cts2} = M^2\hat{V}(\hat{\mu})_{cts2}$$

$$\hat{\mu}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i}$$

$$\hat{V}(\hat{\mu})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{nM^2}\right)s_r^2 + \frac{1}{nNM^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\bar{y}_i - \hat{\mu}_{cts2,r})^2}{n-1}$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\hat{p}_i - \hat{p}_{cts2,r})^2}{n-1}$$

$$\hat{p}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i}$$

$$\hat{V}(\hat{p})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{nM^2}\right)s_r^2 + \frac{1}{nNM^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}_i)}{m_i-1}\right)$$

$\hat{\mu}_{cts2,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i$ $\qquad$ $\hat{V}(\hat{\mu})_{cts2,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu}_{cts2,pps})^2$

$\hat{\tau}_{cts2,pps} = M\hat{\mu}_{cts2,pps}$ $\qquad$ $\hat{V}(\hat{\tau}) = M^2\hat{V}(\hat{\mu})_{cts2,pps}$

n for μ (SRS): $\qquad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for τ (SRS): $\qquad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4N^2)+\sigma^2}$

n for p (SRS): $\qquad$ $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

n for μ (SYS): $\qquad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for p (SYS): $\qquad$ $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

$k \leq \frac{N}{n}$ $\qquad$ $k' = k(ns)$

n for μ (STR): $\qquad$ $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

n for τ (STR): $\qquad$ $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4N^2)+\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR μ:

$n_i = n\left(\frac{N_i\sigma_i/\sqrt{c_i}}{\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}}\right)$ $\qquad$ $n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{i=1}^{L} N_i\sigma_i\sqrt{c_i}\right)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_i}\right)$ $\qquad$ $n = \frac{\left(\sum_{i=1}^{L} N_i\sigma_i\right)^2}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i}{N}\right)$ $\qquad$ $n = \frac{\sum_{i=1}^{L} N_i\sigma_i^2}{N^2(B^2/4)+(1/N)\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR τ: change $N^2(B^2/4)$ to $N^2(B^2/4N^2)$

Allocations for STR p:

$$n_i = n\left(\frac{N_i\sqrt{p_i(1-p_i)/c_i}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{i=1}^{L} N_i^2 p_i(1-p_i)/w_i}{N^2(B^2/4)+\sum_{i=1}^{L} N_i p_i(1-p_i)}$$

n for μ (ratio): $n = \frac{N\sigma^2}{N(B^2/4)+\sigma^2}$

n for τ (ratio): $n = \frac{N\sigma^2}{N(B^2/4N^2)+\sigma^2}$

n for μ (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

n for τ (CTS1(1)): $n = \frac{N\sigma_c^2}{N(B^2/4N^2)+\sigma_c^2}$

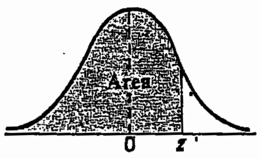
n for τ (CTS1(2)): $n = \frac{N\sigma_t^2}{N(B^2/4N^2)+\sigma_t^2}$

$$s_c^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}m_i)^2}{(n-1)} \text{ with } \bar{y} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i} \qquad s_t^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)} \text{ with } \bar{y}_t = \frac{\sum_{i=1}^{n} y_i}{n}$$

n for p (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

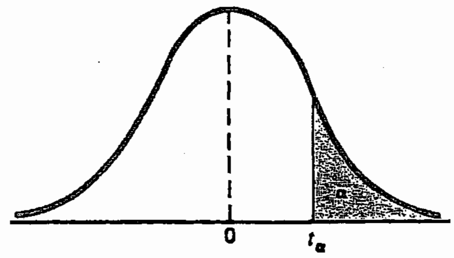
$$s_c^2 = \frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

TABLE A.4
Areas Under the Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE A.5*
Critical Values of the t Distribution



ν	α = 0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

*Table A.5 is taken from Table IV of R. A. Fisher, *Statistical Methods for Research Workers*, Oliver & Boyd Ltd., Edinburgh, by permission of the author and publishers.