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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2005

TITLE OF PAPER: OPERATIONS RESEARCH I

COURSE CODE : ST 307

TIME ALLOWED : TWO (2) HOURS

INSTRUCTIONS : ANSWER ANY THREE (3) QUESTIONS.

REQUIREMENTS: CALCULATOR

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Question 1.

Consider the following linear programming problem

Maximize $z = 2x_1 + 3x_2$
subject to

$$\begin{aligned} x_1 + 2x_2 &\le 10 \\ 3x_1 + x_2 &\le 15 \\ x_2 &\le 4 \\ x_1, x_2 &\ge 0 \end{aligned}$$

(a) Solve this problem using the graphical method.
(b) Find the range of values for the objective function coefficients for which the current optimal solution will remain optimal.
(c) Which resource is to be given top priority when the allocation of resources is made and why?

Question 2

(a) Briefly define the following terms as used in linear programming

(i) Infeasible solution
(ii) Degeneracy
(iii) Alternative optimal solution

(b) (i) Solve the following linear program using the simplex method

Maximize $z = 3x_1 + 2x_2 + 5x_3$
subject to

$$\begin{aligned} x_1 + 2x_2 + x_3 &\le 430 \\ 3x_1 + 2x_3 &\le 460 \\ x_1 + 4x_2 &\le 420 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

(ii) Identify shadow prices for the resources and explain their significance.

Question 3

Given the following primal problem

$$\begin{aligned} &\textit{Minimize } z = 10x_1 + 5x_2 + 4x_3 \\ &\textit{subject to} \\ &\quad 3x_1 + 2x_2 - 3x_3 \geq 3 \\ &\quad 4x_1 \qquad\;\; + 2x_3 \geq 10 \\ &\quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(a) Obtain the dual for this problem.
(b) Solve the dual problem using the simplex method.
(c) Use the dual solution to identify the optimal solution to the original primal problem.
(d) Verify that the optimal objective values for the primal and the dual are equal.

Question 4.

A product is produced at three plants and shipped to three warehouses. The transportation costs per unit are shown in the following table

	Ware house			
Plant	W1	W2	W3	**Plant Capacity/Supply**
P1	20	16	24	300
P2	10	10	8	500
P3	12	18	10	100
Warehouse demand	200	400	300	

(a) Use the Least Cost Method to find the initial basic feasible solution.
(b) Find the optimal solution to this problem.
(c) Express the transportation problem as a linear programming problem.