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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2005

TITLE OF PAPER	:	OPERATIONS RESEARCH I
COURSE CODE	:	ST307
TIME ALLOWED	:	2 (TWO) HOURS
REQUIRMENTS	:	CALCULATORS
INSTRUCTIONS	:	ANSWER QUESTION ONE AND ANY TWO QUESTIONS. ALL QUESTIONS CARRY MARKS AS GIVEN WITHIN THE PARENTHESIS.

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QUESTION ONE: [10 marks]

Consider the following linear programming problem:

Maximize $z = 120x + 100y$
Subject to

$$2x + 2y \leq 8$$
$$5x + 3y \leq 15$$

$$x \geq 0,\ y \geq 0$$

Solve this problem using the graphical method.

QUESTION TWO: [25 marks]

Solve the following linear programming problem using the simplex method:

Maximize $z = 8x_1 + 9x_2 + 5x_3$
Subject to

$$x_1 + x_2 + 2x_3 \leq 2$$
$$2x_1 + 3x_2 + 4x_3 \leq 3$$
$$6x_1 + 6x_2 + 2x_3 \leq 8$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

(a) Set-up the initial tableau. Write the initial solution. Is it also an optimal solution? Why or why not?

(b) Find the next tableau by using one iteration. Write the solution. Is it an optimal solution? Why or why not?

(c) Suppose you are given the following tableau, find the optimal solution.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$-\frac{1}{3}$	0	1
x_2	$\frac{2}{3}$	1	$\frac{4}{3}$	0	$\frac{1}{3}$	0	1
x_6	2	0	−6	0	−2	1	2
	−2	0	7	0	3	0	9

QUESTION THREE: [25 marks]

Given the following primal problem

Maximize $z = 3x_1 + 4x_2$

Subject to

$$x_1 + x_2 \leq 10$$
$$2x_1 + 3x_2 \leq 18$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$

$$x_1 \geq 0, \; x_2 \geq 0$$

(a) Obtain the dual for this problem.
(b) Solve the dual problem using the simplex method.
(c) Use the dual solution to identify the optimal solution to the original primal problem.
(d) Verify that the optimal objective values for the primal and the dual are equal.

QUEASTION FOUR: [25 marks]

A retail-store chain purchased a product at three wholesale markets and distributed to its three retail stores. The transportation costs per unit are shown in the following table:

Wholesale Markets	Retail Stores S_1	S_2	S_3	Purchase Capacity
M_1	20	16	24	300
M_2	10	12	8	500
M_3	12	18	10	100
Store Demand	200	400	300	

(a) Use the Least Cost Method to find the initial basic feasible solution.
(b) Find the optimal solution to this problem.
(c) Express the transportation problem as a linear programming problem.