

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2005

**TITLE OF PAPER** : DESIGN AND ANALYSIS OF EXPERIMENTS

**COURSE CODE** : ST 404

**TIME ALLOWED** : TWO (2) HOURS

**REQUIREMENTS** : CALCULATOR AND STATISTICAL TABLES

**INSTRUCTIONS** : ANSWER QUESTION ONE AND ANY OTHER  
TWO QUESTIONS

**Question 1**

The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valves shutoff mechanism. The results are shown in the following table: (SSE = 202.8)

Circuit type	Response Time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

- a) Test the hypothesis that the three circuit types have same response time. Use 1% level of significance.  
(10 marks)
- b) Assuming all assumptions are satisfied, conduct pair wise comparisons and state the conclusions using 5% error rate. What error rate is the procedure controlling? Why do you choose this procedure?  
(15 Marks)

**Question 2**

The following data are from a two-factor experiment on sugar beet. The two factors are nitrogen (0 kg, 50 kg and 100 kg sulphate of ammonia per acre) and depth of winter ploughing (8 cm and 12 cm).

Yield of sugar (kg per acre) per treatment combination

<i>Nitrogen</i>	0 kg	50 kg	100 kg	0 kg	50 kg	100 kg
<i>Depth of ploughing</i>	8 cm	8 cm	8 cm	12 cm	12 cm	12 cm
	1054	1218	1406	1177	1374	1554
	1099	1258	1423	1160	1350	1572
	1080	1279	1468	1151	1351	1536
	1093	1273	1430	1145	1362	1561
Means	1081.50	1257.00	1431.75	1158.25	1359.25	1555.75

The analysis of variance (ANOVA) table is

Source	DF	SS	MS
Nitrogen	2	****	****
Depth	****	****	****
Nitrogen x Depth	****	2237.3	****
Error	****	****	397.9
Total	23	629744.5	

Complete the ANOVA table and use it to assess what evidence the experiment provides regarding the six treatment combinations. Summarise your conclusions in non-technical language that the experimenter would understand.

(25 Marks)

**Question 3**

- a) The yields of two aubergine varieties,  $V_1$  and  $V_2$ , were examined in two soil types,  $S_1$  and  $S_2$ , and at two levels of moisture,  $M$  (low,  $M_1$ , and high,  $M_2$ ). Four complete replicates of a  $2^3$  factorial design were run in 4 greenhouses. The yields  $y$  (coded in suitable units) and treatment combinations are given below.

In the coding of the treatment combinations,  
 presence of  $v$  indicates  $V_2$  was used, otherwise  $V_1$  was used;  
 presence of  $s$  indicates  $S_2$  was used, otherwise  $S_1$  was used;  
 presence of  $m$  indicates  $M_2$  was used, otherwise  $M_1$  was used.

	Greenhouse				Total
	I	II	III	IV	
(1)	7	19	13	11	50
$v$	30	33	28	31	122
$s$	24	30	19	25	98
$vs$	39	36	35	43	153
$m$	21	30	24	21	96
$vm$	31	36	31	33	131
$sm$	27	31	26	29	113
$vsm$	39	41	36	66	182
Total	218	256	212	259	

Effect estimates	
$V$	14.44
$S$	9.19
$M$	6.19
$VS$	1.06
$VM$	-1.44
$SM$	-0.69
$VSM$	3.19

$$\Sigma y = 945 \quad \Sigma y^2 = 31513$$

- i) Explain briefly why the above is called a factorial layout. What are the advantages of this type of design?
  - ii) Show how the value 3.19 for the  $VSM$  interaction estimate was obtained.
  - iii) Construct the analysis of variance. Carry out any further significance tests that you consider appropriate, and report on the results.
- b) Suppose now that only four aubergine plants can be grown in each greenhouse, and that the greenhouse used may influence the yield. However, 8 greenhouses of this size are available.
- i) Explain briefly the importance of *confounding* in  $2^k$  factorial experiments.
  - ii) Write down an appropriate design for an experiment using the same treatment combinations as in part (a), in which each of these 8 greenhouses is treated as a block.

(25 Marks)

**Question 4**

A field trial was conducted to compare four seed treatments with treatment - control on Soya beans. The data are the number of plants which failed to emerge out of 100 planted on each plot.

Treatment	Block				
	I	II	III	IV	V
Control	8	10	12	13	11
Arason	2	6	7	11	5
Spargon	4	10	9	8	10
Samesan	3	5	9	10	6
Fermate	9	7	5	5	3

Create an ANOVA table and carryout an appropriate hypotheses testing, multiple comparisons and conclude about this study. Use 5% level of significance. (25 Marks)

I. Cumulative Standard Normal Distribution<sup>a</sup>

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	.00	.01	.02	.03	.04	z
0	.50000	.50399	.50798	.51197	.51595	0
1	.53983	.54379	.54776	.55172	.55567	1
2	.57926	.58317	.58706	.59095	.59483	2
3	.61791	.62172	.62551	.62930	.63307	3
4	.65542	.65910	.66276	.66640	.67003	4
5	.69146	.69497	.69847	.70194	.70540	5
6	.72575	.72907	.73237	.73565	.73891	6
7	.75803	.76115	.76424	.76730	.77035	7
8	.78814	.79103	.79389	.79673	.79954	8
9	.81594	.81859	.82121	.82381	.82639	9
10	.84134	.84375	.84613	.84849	.85083	10
11	.86433	.86650	.86864	.87076	.87285	11
12	.88493	.88686	.88877	.89065	.89251	12
13	.90320	.90490	.90658	.90824	.90988	13
14	.91924	.92073	.92219	.92364	.92506	14
15	.93319	.93448	.93574	.93699	.93822	15
16	.94520	.94630	.94738	.94845	.94950	16
17	.95543	.95637	.95728	.95818	.95907	17
18	.96407	.96485	.96562	.96637	.96711	18
19	.97128	.97193	.97257	.97320	.97381	19
20	.97725	.97778	.97831	.97882	.97932	20
21	.98214	.98257	.98300	.98341	.98382	21
22	.98610	.98645	.98679	.98713	.98746	22
23	.98928	.98956	.98983	.99010	.99036	23
24	.99180	.99202	.99224	.99245	.99266	24
25	.99379	.99396	.99413	.99430	.99446	25
26	.99534	.99547	.99560	.99573	.99585	26
27	.99653	.99664	.99674	.99683	.99693	27
28	.99744	.99752	.99760	.99767	.99774	28
29	.99813	.99819	.99825	.99831	.99836	29
30	.99865	.99869	.99874	.99878	.99882	30
31	.99903	.99906	.99910	.99913	.99916	31
32	.99931	.99934	.99936	.99938	.99940	32
33	.99952	.99953	.99955	.99957	.99958	33
34	.99966	.99968	.99969	.99970	.99971	34
35	.99977	.99978	.99978	.99979	.99980	35
36	.99984	.99985	.99985	.99986	.99986	36
37	.99989	.99989	.99990	.99990	.99991	37
38	.99993	.99993	.99993	.99994	.99994	38
39	.99995	.99995	.99996	.99996	.99996	39

I. Cumulative Standard Normal Distribution (continued)

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	.05	.06	.07	.08	.09	z
0	.51994	.52392	.52790	.53188	.53586	0
1	.55962	.56356	.56749	.57142	.57534	1
2	.59871	.60257	.60642	.61026	.61409	2
3	.63683	.64058	.64431	.64803	.65173	3
4	.67364	.67724	.68082	.68438	.68793	4
5	.70884	.71226	.71566	.71904	.72240	5
6	.74215	.74537	.74857	.75175	.75490	6
7	.77337	.77637	.77935	.78230	.78523	7
8	.80234	.80510	.80785	.81057	.81327	8
9	.82894	.83147	.83397	.83646	.83891	9
10	.85314	.85543	.85769	.85993	.86214	10
11	.87493	.87697	.87900	.88100	.88297	11
12	.89435	.89616	.89796	.89973	.90147	12
13	.91499	.91658	.91815	.91971	.92125	13
14	.92647	.92785	.92922	.93056	.93189	14
15	.93943	.94062	.94179	.94295	.94408	15
16	.95053	.95154	.95254	.95352	.95448	16
17	.95994	.96080	.96164	.96246	.96327	17
18	.96784	.96856	.96926	.96995	.97062	18
19	.97441	.97500	.97558	.97615	.97670	19
20	.97982	.98030	.98077	.98124	.98169	20
21	.98422	.98461	.98500	.98537	.98574	21
22	.98778	.98809	.98840	.98870	.98899	22
23	.99061	.99086	.99111	.99134	.99158	23
24	.99286	.99305	.99324	.99343	.99361	24
25	.99461	.99477	.99492	.99506	.99520	25
26	.99598	.99609	.99621	.99632	.99643	26
27	.99702	.99711	.99720	.99728	.99736	27
28	.99781	.99788	.99795	.99801	.99807	28
29	.99841	.99846	.99851	.99856	.99861	29
30	.99886	.99889	.99893	.99897	.99900	30
31	.99918	.99921	.99924	.99926	.99929	31
32	.99942	.99944	.99946	.99948	.99950	32
33	.99960	.99961	.99962	.99964	.99965	33
34	.99972	.99973	.99974	.99975	.99976	34
35	.99981	.99981	.99982	.99983	.99983	35
36	.99987	.99987	.99988	.99988	.99989	36
37	.99991	.99992	.99992	.99992	.99992	37
38	.99994	.99994	.99995	.99995	.99995	38
39	.99996	.99996	.99996	.99997	.99997	39

<sup>a</sup> Reproduced with permission from *Probability and Statistics in Engineering and Management Science*, 3rd edition, by W. W. Hines and D. C. Montgomery, Wiley, New York, 1990.

II. Percentage Points of the *t* Distribution<sup>a</sup>

<i>v</i>	$\alpha$	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	6.869
6	.265	.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.019	4.785	5.408	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	3.373
$\infty$	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	3.291

*v* = degrees of freedom.

<sup>a</sup> Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

III. Percentage Points of the  $\chi^2$  Distribution<sup>a</sup>

<i>v</i>	$\alpha$	.995	.990	.975	.950	.500	.050	.025	.010	.005
1	0.00 +	0.00 +	0.00 +	0.00 +	0.00 +	0.45	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.10	1.39	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.35	2.37	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	0.71	3.36	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.15	4.35	11.07	12.38	15.09	16.75
6	0.68	0.87	1.24	1.64	1.64	5.35	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.17	6.35	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	2.73	7.34	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	3.33	8.34	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	3.94	9.34	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	4.57	10.34	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	5.23	11.34	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	5.89	12.34	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	6.57	13.34	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	7.26	14.34	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	7.96	15.34	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	8.67	16.34	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	9.39	17.34	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	10.12	18.34	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	10.85	19.34	31.41	34.17	37.57	40.00
25	10.52	11.52	13.12	14.61	14.61	24.34	37.65	40.65	44.31	46.93
30	13.79	14.95	16.79	18.49	18.49	29.34	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	26.51	39.34	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	34.76	49.33	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	43.19	59.33	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	51.74	69.33	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	60.39	79.33	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	69.13	89.33	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	77.93	99.33	124.34	129.56	135.81	140.17

*v* = degrees of freedom.

<sup>a</sup> Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

$\nu_2$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49	9.58	9.63	9.67	9.71	9.76	9.80	9.85
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41	3.43	3.43	3.44	3.45	3.46	3.47	3.48
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.88	1.88	1.88	1.88	1.87	1.87	1.87
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.75	1.75	1.75	1.74	1.74	1.74
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.70	1.69	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.65	1.65
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58	1.58
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.54	1.53	1.53
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52	1.52	1.51	1.51	1.50	1.49	1.48
11	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47	1.46	1.45
12	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42
13	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.40
14	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.43	1.41	1.41	1.40	1.39	1.38	1.37	1.36
16	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
17	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
18	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32
19	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.30
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.29
21	1.40	1.48	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28
22	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28
23	1.39	1.47	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.29	1.28
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.27
25	1.39	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.27	1.25
26	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.26	1.25
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.33	1.32	1.31	1.30	1.28	1.27	1.26	1.24
28	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.25	1.24
29	1.38	1.45	1.45	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.27	1.26	1.25	1.23
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.24	1.23
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.22	1.21	1.19
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.27	1.25	1.24	1.22	1.21	1.19	1.17	1.15
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.13	1.10
$\infty$	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.22	1.19	1.18	1.16	1.14	1.12	1.08	1.00

$\nu$  = degrees of freedom  
 \* Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

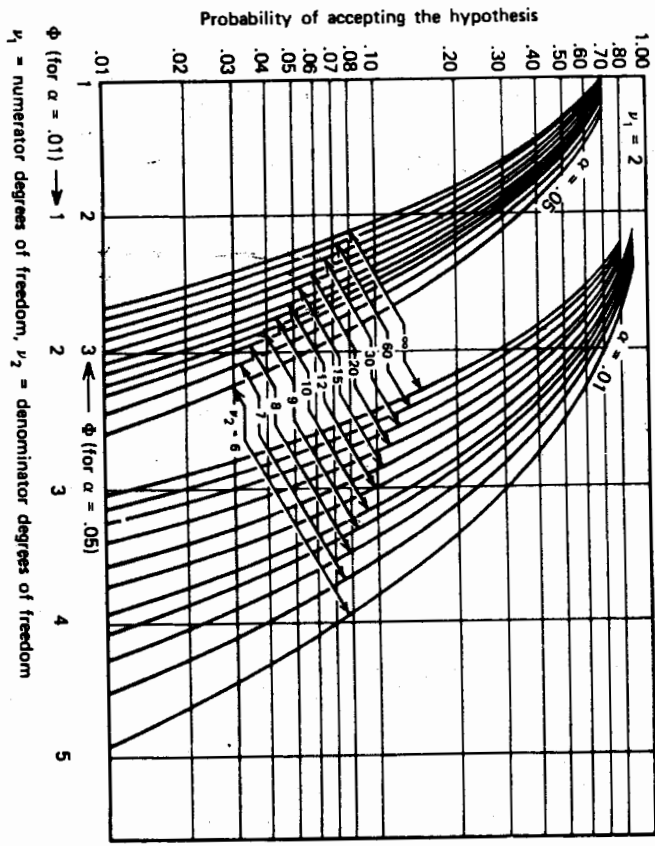
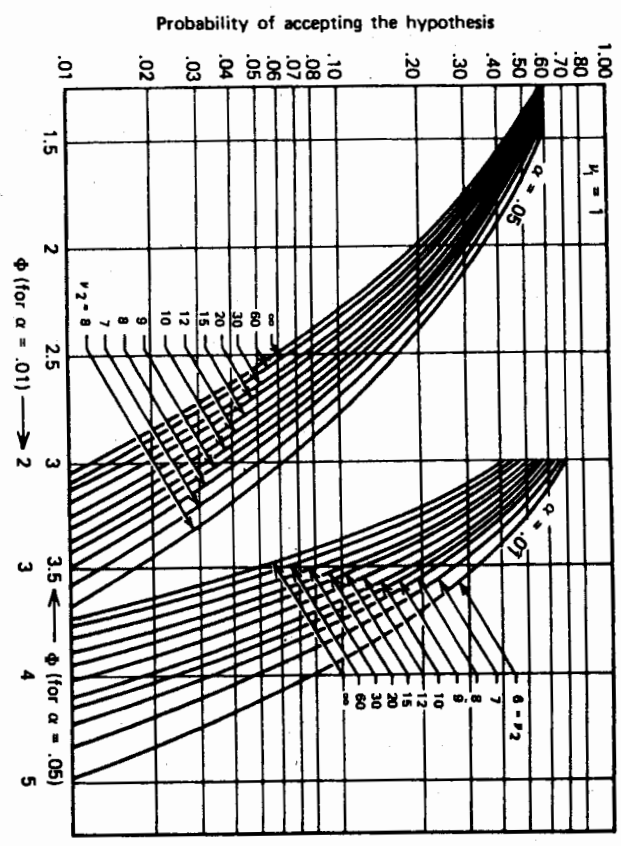
$\nu_2$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23																



$\nu_2 \backslash \nu_1$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.49	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

$\nu_2 \backslash \nu_1$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
12	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
15	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
20	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
24	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
30	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
40	6.12	4.69	4.06	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
60	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
120	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
$\infty$	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
2	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
3	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
4	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
5	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
6	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
7	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
8	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
9	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
10	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
12	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
15	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
20	5.42	4.05	3.46	3.13	2.90														

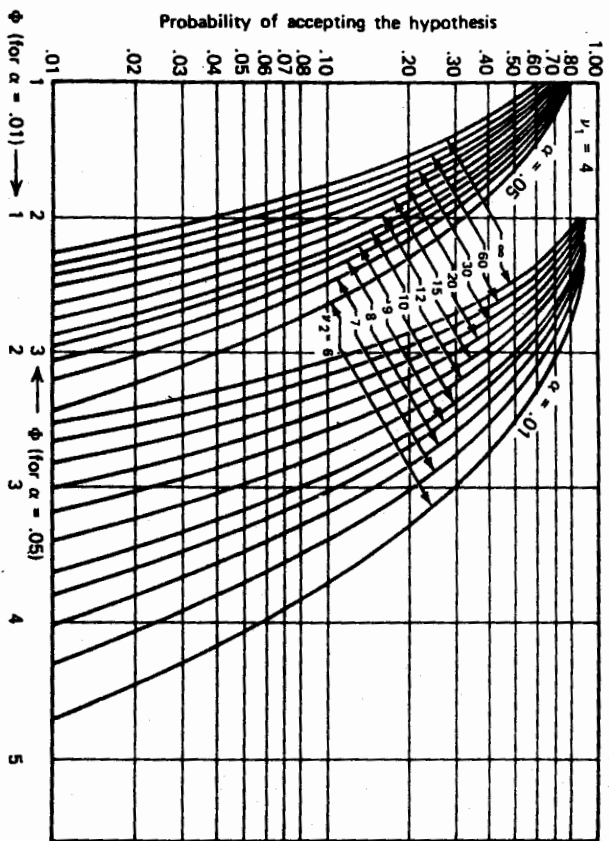
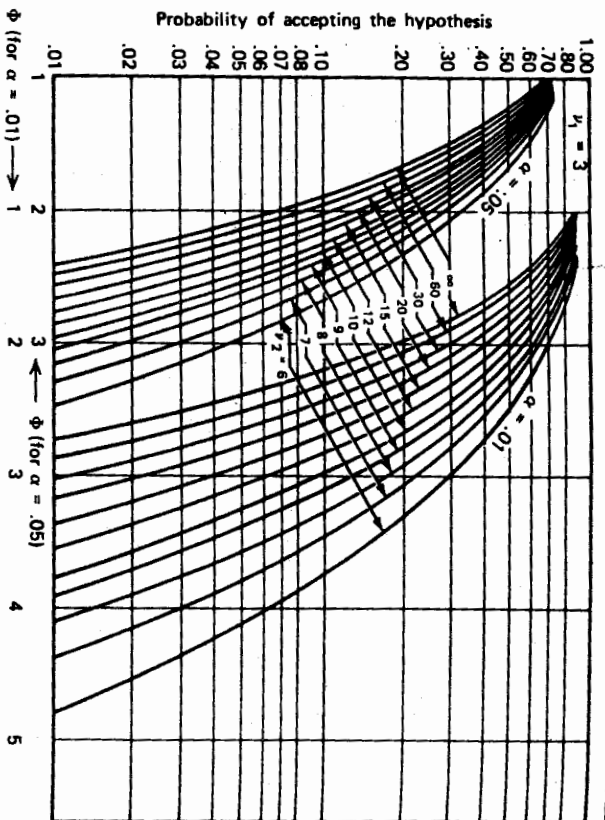
$\nu_1$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.00	26.50	26.41	26.32	26.22	26
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.1
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.1
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.1
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.0
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.0
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.0
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.8
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.6
80	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.55	1.4
100	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.35	1.2



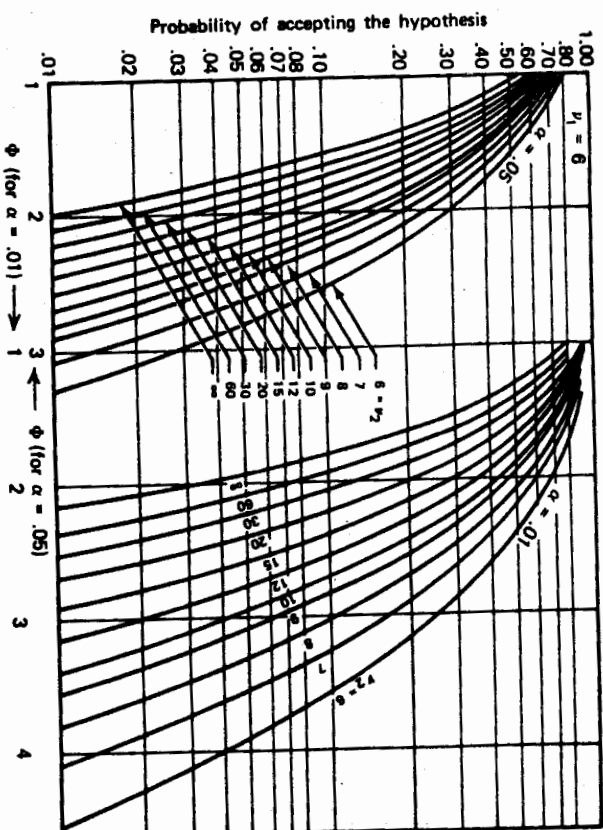
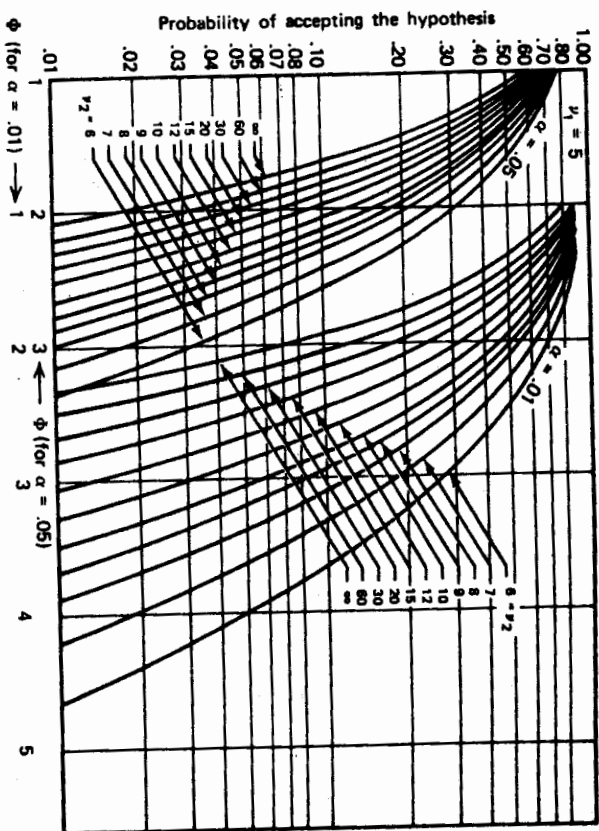
V. Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance\*

\* Adapted with permission from Biometrika Tables for Statisticians, Vol. 2, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1972.

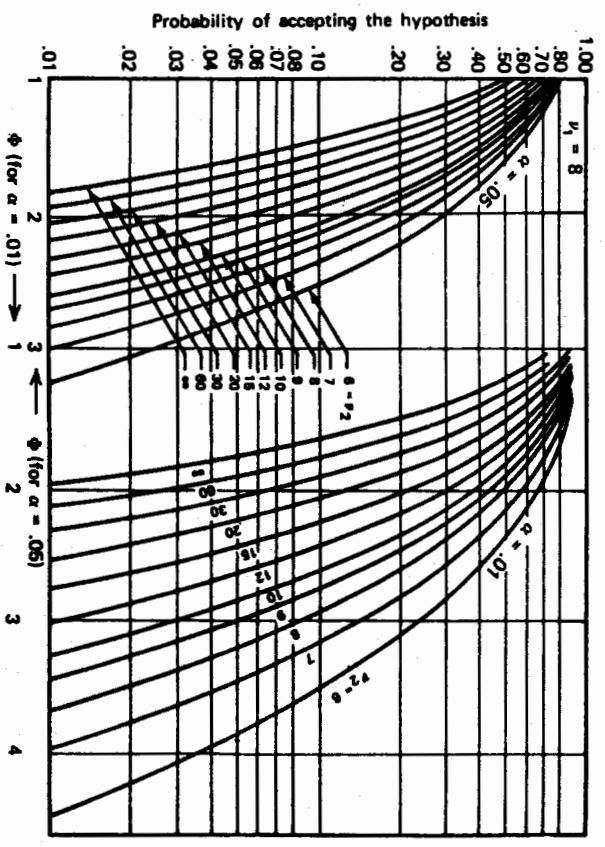
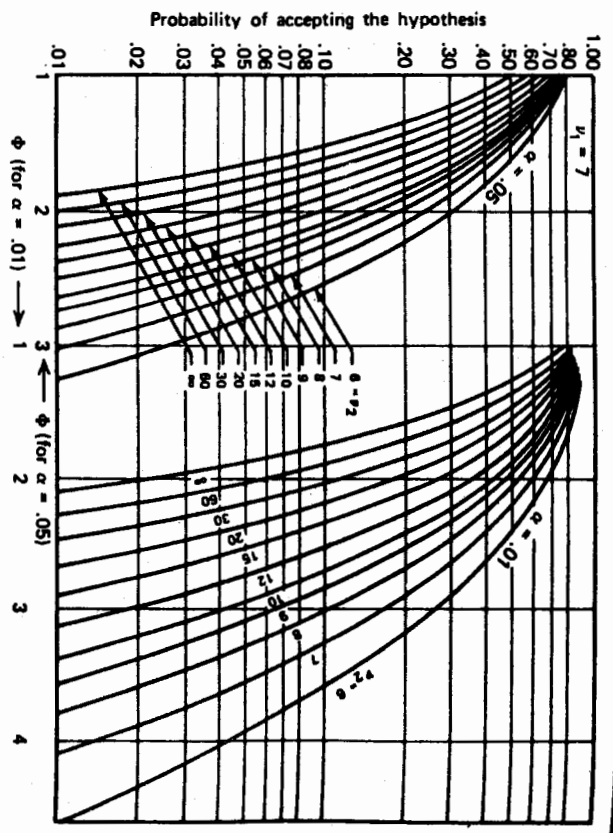
V. Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (continued)



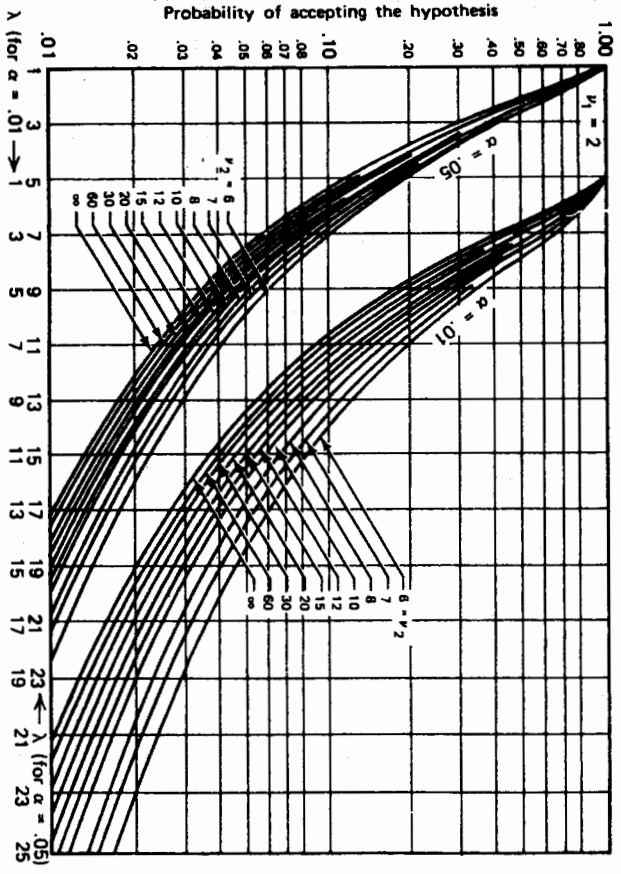
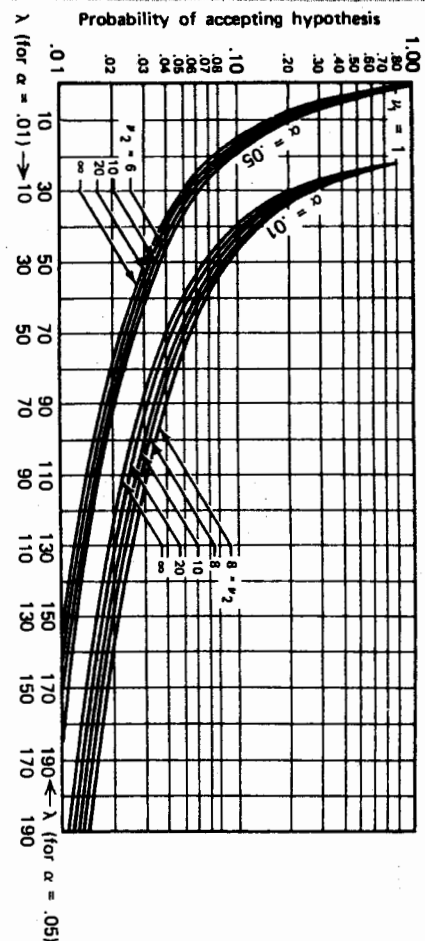
V. Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (continued)



V. Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (continued)

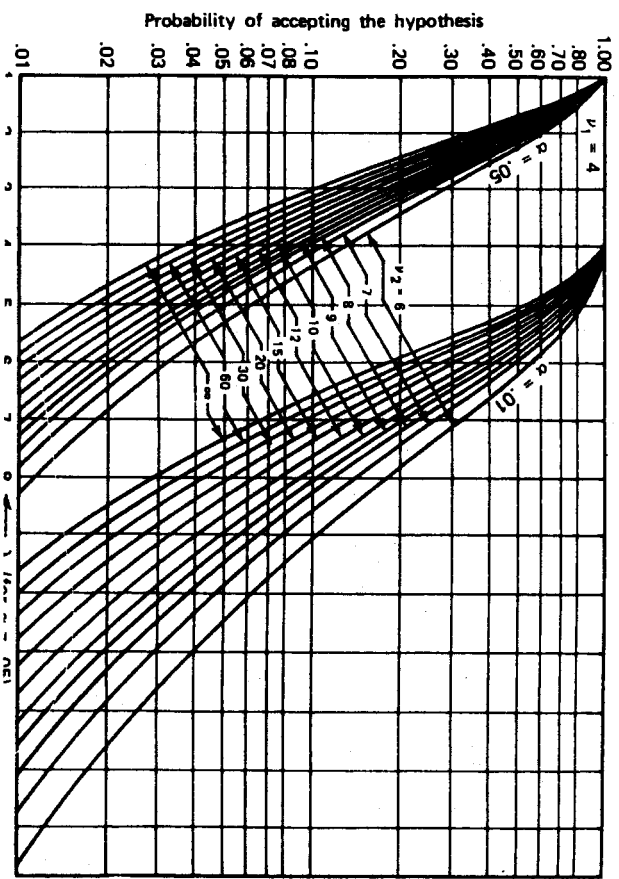
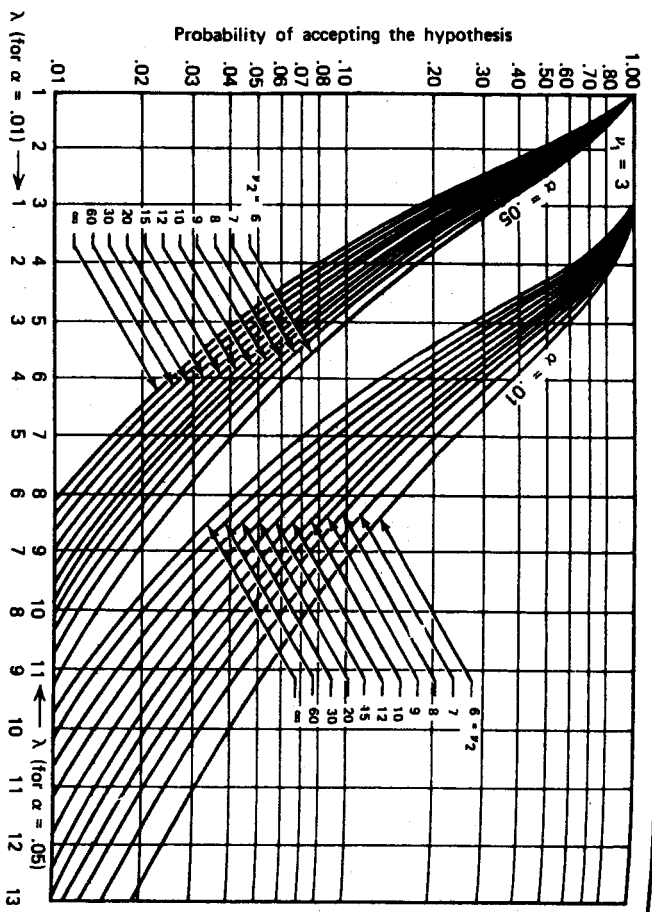


VI. Operating Characteristic Curves for the Random Effects Model Analysis of Variance\*

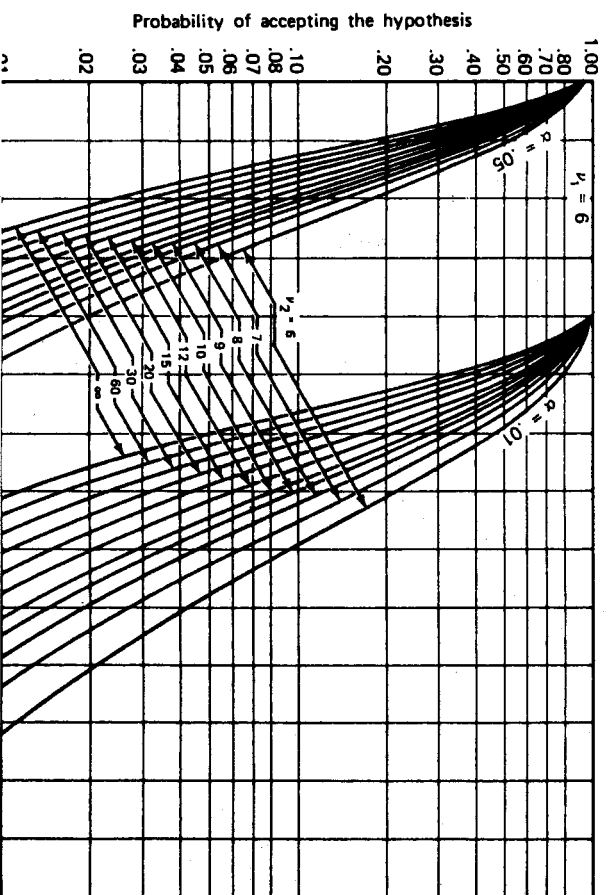
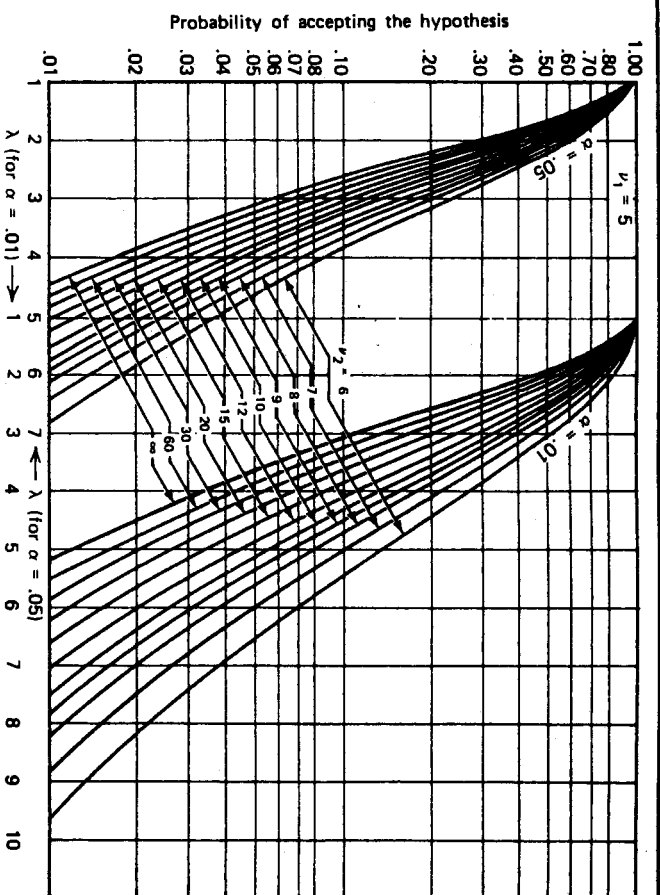


\* Reproduced with permission from Engineering Statistics, 2nd edition, by A. H. Bowker and G. J. Lieberman, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1972.

VI. Operating Characteristic Curves for the Random Effects Model Analysis of Variance (continued)

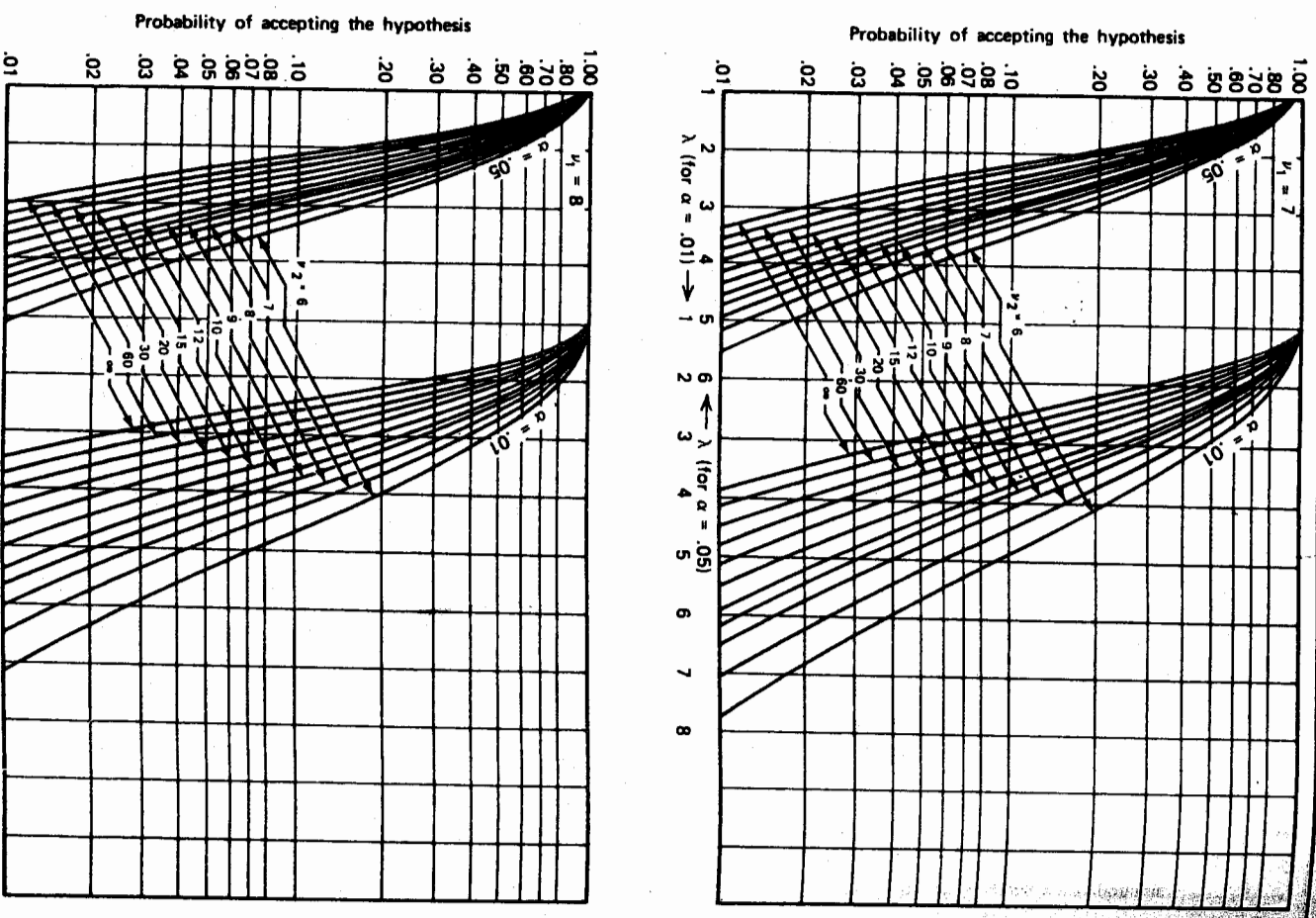


VI. Operating Characteristic Curves for the Random Effects Model Analysis of Variance (continued)





VI. Operating Characteristic Curves for the Random Effects Model Analysis of Variance (continued)



VII. Significant Ranges for Duncan's Multiple Range Test\*

f	p											
	2	3	4	5	6	7	8	9	10	20	50	100
1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
2	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
3	8.26	8.5	8.6	8.7	8.8	8.9	8.9	9.0	9.0	9.3	9.3	9.3
4	6.51	6.8	6.9	7.0	7.1	7.1	7.2	7.2	7.3	7.5	7.5	7.5
5	5.70	5.96	6.11	6.18	6.26	6.33	6.40	6.44	6.5	6.8	6.8	6.8
6	5.24	5.51	5.65	5.73	5.81	5.88	5.95	6.00	6.0	6.3	6.3	6.3
7	4.95	5.22	5.37	5.45	5.53	5.61	5.69	5.73	5.8	6.0	6.0	6.0
8	4.74	5.00	5.14	5.23	5.32	5.40	5.47	5.51	5.5	5.8	5.8	5.8
9	4.60	4.86	4.99	5.08	5.17	5.25	5.32	5.36	5.4	5.7	5.7	5.7
10	4.48	4.73	4.88	4.96	5.06	5.13	5.20	5.24	5.28	5.55	5.55	5.55
11	4.39	4.63	4.77	4.86	4.94	5.01	5.06	5.12	5.15	5.39	5.39	5.39
12	4.32	4.55	4.68	4.76	4.84	4.92	4.96	5.02	5.07	5.26	5.26	5.26
13	4.26	4.48	4.62	4.69	4.74	4.84	4.88	4.94	4.98	5.15	5.15	5.15
14	4.21	4.42	4.55	4.63	4.70	4.78	4.83	4.87	4.91	5.07	5.07	5.07
15	4.17	4.37	4.50	4.58	4.64	4.72	4.77	4.81	4.84	5.00	5.00	5.00
16	4.13	4.34	4.45	4.54	4.60	4.67	4.72	4.76	4.79	4.94	4.94	4.94
17	4.10	4.30	4.41	4.50	4.56	4.63	4.68	4.73	4.75	4.89	4.89	4.89
18	4.07	4.27	4.38	4.46	4.53	4.59	4.64	4.68	4.71	4.85	4.85	4.85
19	4.05	4.24	4.35	4.43	4.50	4.56	4.61	4.64	4.67	4.82	4.82	4.82
20	4.02	4.22	4.33	4.40	4.47	4.53	4.58	4.61	4.65	4.79	4.79	4.79
30	3.89	4.06	4.16	4.22	4.32	4.36	4.41	4.45	4.48	4.65	4.71	4.71
40	3.82	3.99	4.10	4.17	4.24	4.30	4.34	4.37	4.41	4.59	4.69	4.69
60	3.76	3.92	4.03	4.12	4.17	4.23	4.27	4.31	4.34	4.53	4.66	4.66
100	3.71	3.86	3.98	4.06	4.11	4.17	4.21	4.25	4.29	4.48	4.64	4.65
∞	3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17	4.20	4.41	4.60	4.68

\* = degrees of freedom.  
 Reproduced with permission from "Multiple Range and Multiple F Tests," by D. B. Duncan, Biometrics, Vol. 1, No. 1, pp. 1-42, 1955.

VII. Significant Ranges for Duncan's Multiple Range Test (continued)

f	p																							
	2	3	4	5	6	7	8	9	10	20	50	100	2	3	4	5	6	7	8	9	10	20	50	100
1	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
2	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
3	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
4	3.93	4.01	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02
5	3.64	3.74	3.79	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83
6	3.46	3.58	3.64	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68
7	3.35	3.47	3.54	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58
8	3.26	3.39	3.47	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52
9	3.20	3.34	3.41	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47
10	3.15	3.30	3.37	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43
11	3.11	3.27	3.35	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39
12	3.08	3.23	3.33	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36	3.36
13	3.06	3.21	3.30	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35
14	3.03	3.18	3.27	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33
15	3.01	3.16	3.25	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31	3.31
16	3.00	3.15	3.23	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30	3.30
17	2.98	3.13	3.22	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.28
18	2.97	3.12	3.21	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27
19	2.96	3.11	3.19	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26	3.26
20	2.95	3.10	3.18	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25
30	2.89	3.04	3.12	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20
40	2.86	3.01	3.10	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.17
60	2.83	2.98	3.08	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14
100	2.80	2.95	3.05	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12
∞	2.77	2.92	3.02	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09

f = degrees of freedom.

VIII. Percentage Points of the Studentized Range Statistic<sup>a</sup>

q <sub>0.01</sub> (p, f)																			
p																			
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
90.0	135	164	186	202	216	227	237	246	253	260	266	272	277	282	286	290	294	198	
14.0	19.0	22.3	24.7	26.6	28.2	29.5	30.7	31.7	32.6	33.4	34.1	34.8	35.4	36.0	36.5	37.0	37.5	37.9	
8.26	10.6	12.2	13.3	14.2	15.0	15.6	16.2	16.7	17.1	17.5	17.9	18.2	18.5	18.8	19.1	19.3	19.5	19.8	
6.51	8.12	9.17	9.96	10.6	11.1	11.5	11.9	12.3	12.6	12.8	13.1	13.3	13.5	13.7	13.9	14.1	14.2	14.4	
5.70	6.97	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48	10.70	10.89	11.08	11.24	11.40	11.55	11.68	11.81	11.93	
5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.49	9.65	9.81	9.95	10.08	10.21	10.32	10.43	10.54	
4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.61	
4.74	5.63	6.20	6.63	6.96	7.24	7.47	7.68	7.87	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.00	
4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.32	7.49	7.65	7.78	7.91	8.03	8.13	8.23	8.32	8.41	8.49	8.55	
4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.48	7.60	7.71	7.81	7.91	7.99	8.07	8.15	8.22	
4.39	5.14	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95	
4.32	5.04	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73	
4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19	7.27	7.34	7.42	7.48	7.55	
4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05	7.12	7.20	7.27	7.33	7.39	
4.17	4.83	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26	
4.13	4.78	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15	
4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48	6.57	6.66	6.73	6.80	6.87	6.94	7.00	7.05	
4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65	6.72	6.79	6.85	6.91	6.96	
4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89	
4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.29	6.37	6.45	6.52	6.59	6.65	6.71	6.76	6.82	
3.96	4.54	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61	
3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41	
3.82	4.37	4.70	4.93	5.11	5.27	5.39	5.50	5.60	5.69	5.77	5.84	5.90	5.96	6.02	6.07	6.12	6.17	6.21	
3.76	4.28	4.60	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.79	5.84	5.89	5.93	5.98	6.03	
3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.38	5.44	5.51	5.56	5.61	5.66	5.71	5.75	5.79	5.83	
3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65	

degrees of freedom.

from J. M. May, "Extended and Corrected Tables of the Upper Percentage Points of the Studentized Range," *Biometrika*, Vol. 39, pp. 192-193, 1952. Reproduced by permission of the publishers, of *Biometrika*.

VIII. Percentage Points of the Studentized Range Statistic (continued)

$q_{\alpha}(p, f)$																			
$p$																			
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
18.1	26.7	32.8	37.2	40.5	43.1	45.4	47.3	49.1	50.6	51.9	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6	
6.09	8.28	9.80	10.89	11.73	12.43	13.03	13.54	13.99	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.36	16.57	16.77	
4.50	5.88	6.83	7.51	8.04	8.47	8.85	9.18	9.46	9.72	9.95	10.16	10.35	10.52	10.69	10.84	10.98	11.12	11.24	
3.93	5.00	5.76	6.31	6.73	7.06	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.67	8.80	8.92	9.03	9.14	9.24	
3.61	4.54	5.18	5.64	5.99	6.28	6.52	6.74	6.93	7.10	7.25	7.39	7.52	7.64	7.75	7.86	7.95	8.04	8.13	
3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.04	7.14	7.24	7.34	7.43	7.51	7.59	
3.34	4.16	4.68	5.06	5.35	5.59	5.80	5.99	6.15	6.29	6.42	6.54	6.65	6.75	6.84	6.93	7.01	7.08	7.16	
3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87	
3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.65	
3.15	3.88	4.33	4.66	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.12	6.20	6.27	6.34	6.41	6.47	
3.11	3.82	4.26	4.58	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.14	6.20	6.27	6.33	
3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21	
3.06	3.73	4.15	4.46	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	6.00	6.06	6.11	
3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.56	5.64	5.72	5.79	5.86	5.92	5.98	6.03	
3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.79	5.85	5.91	5.96	
3.00	3.65	4.05	4.34	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90	
2.98	3.62	4.02	4.31	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.55	5.61	5.68	5.74	5.79	5.84	
2.97	3.61	4.00	4.28	4.49	4.67	4.83	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79	
2.96	3.59	3.98	4.26	4.47	4.64	4.79	4.92	5.04	5.14	5.23	5.32	5.39	5.46	5.53	5.59	5.65	5.70	5.75	
2.95	3.58	3.96	4.24	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.50	5.56	5.61	5.66	5.71	
2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.50	5.55	5.59	
2.89	3.48	3.84	4.11	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.48	
2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.90	4.98	5.05	5.11	5.17	5.22	5.27	5.32	5.36	
2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24	
2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13	
2.77	3.32	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.84	4.89	4.93	4.97	5.01	

IX. Critical Values for Dunnett's Test for Comparing Treatments with a Control<sup>a</sup>

$d_{\alpha}(a-1, f)$   
Two-Sided Comparisons

$f$	$a-1 = \text{Number of treatment means (excluding control)}$								
	1	2	3	4	5	6	7	8	9
5	2.57	3.03	3.29	3.48	3.62	3.73	3.82	3.90	3.97
6	2.45	2.86	3.10	3.26	3.39	3.49	3.57	3.64	3.71
7	2.36	2.75	2.97	3.12	3.24	3.33	3.41	3.47	3.53
8	2.31	2.67	2.88	3.02	3.13	3.22	3.29	3.35	3.41
9	2.26	2.61	2.81	2.95	3.05	3.14	3.20	3.26	3.32
10	2.23	2.57	2.76	2.89	2.99	3.07	3.14	3.19	3.24
11	2.20	2.53	2.72	2.84	2.94	3.02	3.08	3.14	3.19
12	2.18	2.50	2.68	2.81	2.90	2.98	3.04	3.09	3.14
13	2.16	2.48	2.65	2.78	2.87	2.94	3.00	3.06	3.10
14	2.14	2.46	2.63	2.75	2.84	2.91	2.97	3.02	3.07
15	2.13	2.44	2.61	2.73	2.82	2.89	2.95	3.00	3.04
16	2.12	2.42	2.59	2.71	2.80	2.87	2.92	2.97	3.02
17	2.11	2.41	2.58	2.69	2.78	2.85	2.90	2.95	3.00
18	2.10	2.40	2.56	2.68	2.76	2.83	2.89	2.94	2.98
19	2.09	2.39	2.55	2.66	2.75	2.81	2.87	2.92	2.96
20	2.09	2.38	2.54	2.65	2.73	2.80	2.86	2.90	2.95
24	2.06	2.35	2.51	2.61	2.70	2.76	2.81	2.86	2.90
30	2.04	2.32	2.47	2.58	2.66	2.72	2.77	2.82	2.86
40	2.02	2.29	2.44	2.54	2.62	2.68	2.73	2.77	2.81
60	2.00	2.27	2.41	2.51	2.58	2.64	2.69	2.73	2.77
120	1.98	2.24	2.38	2.47	2.55	2.60	2.65	2.69	2.73
$\infty$	1.96	2.21	2.35	2.44	2.51	2.57	2.61	2.65	2.69

$f = \text{degrees of freedom.}$   
<sup>a</sup> Reproduced with permission from C. W. Dunnett, "New Tables for Multiple Comparison with a Control," *Biometrics*, Vol. 20, No. 3, 1964, and from C. W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," *Journal of the American Statistical Association*, Vol. 50, 1955.



**IX. Critical Values for Dunnett's Test for Comparing Treatments with a Control**

$$d_{05}(a-1, f)$$

Two-Sided Comparisons

(continued)

f	a - 1 = number of treatment means (excluding control)								
	1	2	3	4	5	6	7	8	9
5	4.03	4.63	4.98	5.22	5.41	5.56	5.69	5.80	5.89
6	3.71	4.21	4.51	4.71	4.87	5.00	5.10	5.20	5.28
7	3.50	3.95	4.21	4.39	4.53	4.64	4.74	4.82	4.89
8	3.36	3.77	4.00	4.17	4.29	4.40	4.48	4.56	4.62
9	3.25	3.63	3.85	4.01	4.12	4.22	4.30	4.37	4.43
10	3.17	3.53	3.74	3.88	3.99	4.08	4.16	4.22	4.28
11	3.11	3.45	3.65	3.79	3.89	3.98	4.05	4.11	4.16
12	3.05	3.39	3.58	3.71	3.81	3.89	3.96	4.02	4.07
13	3.01	3.33	3.52	3.65	3.74	3.82	3.89	3.94	3.99
14	2.98	3.29	3.47	3.59	3.69	3.76	3.83	3.88	3.93
15	2.95	3.25	3.43	3.55	3.64	3.71	3.78	3.83	3.88
16	2.92	3.22	3.39	3.51	3.60	3.67	3.73	3.78	3.83
17	2.90	3.19	3.36	3.47	3.56	3.63	3.69	3.74	3.79
18	2.88	3.17	3.33	3.44	3.53	3.60	3.66	3.71	3.75
19	2.86	3.15	3.31	3.42	3.50	3.57	3.63	3.68	3.72
20	2.85	3.13	3.29	3.40	3.48	3.55	3.60	3.65	3.69
24	2.80	3.07	3.22	3.32	3.40	3.47	3.52	3.57	3.61
30	2.75	3.01	3.15	3.25	3.33	3.39	3.44	3.49	3.52
40	2.70	2.95	3.09	3.19	3.26	3.32	3.37	3.41	3.44
60	2.66	2.90	3.03	3.12	3.19	3.25	3.29	3.33	3.37
120	2.62	2.85	2.97	3.06	3.12	3.18	3.22	3.26	3.29
∞	2.58	2.79	2.92	3.00	3.06	3.11	3.15	3.19	3.22

**IX. Critical Values for Dunnett's Test for Comparing Treatments with a Control**

$$d_{05}(a-1, f)$$

One-Sided Comparisons

(continued)

f	a - 1 = number of treatment means (excluding control)								
	1	2	3	4	5	6	7	8	9
5	2.02	2.44	2.68	2.85	2.98	3.08	3.16	3.24	3.30
6	1.94	2.34	2.56	2.71	2.83	2.92	3.00	3.07	3.12
7	1.89	2.27	2.48	2.62	2.73	2.82	2.89	2.95	3.01
8	1.86	2.22	2.42	2.55	2.66	2.74	2.81	2.87	2.92
9	1.83	2.18	2.37	2.50	2.60	2.68	2.75	2.81	2.86
10	1.81	2.15	2.34	2.47	2.56	2.64	2.70	2.76	2.81
11	1.80	2.13	2.31	2.44	2.53	2.60	2.67	2.72	2.77
12	1.78	2.11	2.29	2.41	2.50	2.58	2.64	2.69	2.74
13	1.77	2.09	2.27	2.39	2.48	2.55	2.61	2.66	2.71
14	1.76	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.69
15	1.75	2.07	2.24	2.36	2.44	2.51	2.57	2.62	2.67
16	1.75	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65
17	1.74	2.05	2.22	2.33	2.42	2.49	2.54	2.59	2.64
18	1.73	2.04	2.21	2.32	2.41	2.48	2.53	2.58	2.62
19	1.73	2.03	2.20	2.31	2.40	2.47	2.52	2.57	2.61
20	1.72	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60
24	1.71	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57
30	1.70	1.99	2.15	2.25	2.33	2.40	2.45	2.50	2.54
40	1.68	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51
60	1.67	1.95	2.10	2.21	2.28	2.35	2.39	2.44	2.48
120	1.66	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45
∞	1.64	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42

IX. Critical Values for Dunnett's Test for Comparing Treatments with a Control  
 $d_{\alpha}(a-1, f)$   
 One-Sided Comparisons  
 (continued)

f	a - 1 = number of treatment means (excluding control)								
	1	2	3	4	5	6	7	8	9
5	3.37	3.90	4.21	4.43	4.60	4.73	4.85	4.94	5.03
6	3.14	3.61	3.88	4.07	4.21	4.33	4.43	4.51	4.59
7	3.00	3.42	3.66	3.83	3.96	4.07	4.15	4.23	4.30
8	2.90	3.29	3.51	3.67	3.79	3.88	3.96	4.03	4.09
9	2.82	3.19	3.40	3.55	3.66	3.75	3.82	3.89	3.94
10	2.76	3.11	3.31	3.45	3.56	3.64	3.71	3.78	3.83
11	2.72	3.06	3.25	3.38	3.48	3.56	3.63	3.69	3.74
12	2.68	3.01	3.19	3.32	3.42	3.50	3.56	3.62	3.67
13	2.65	2.97	3.15	3.27	3.37	3.44	3.51	3.56	3.61
14	2.62	2.94	3.11	3.23	3.32	3.40	3.46	3.51	3.56
15	2.60	2.91	3.08	3.20	3.29	3.36	3.42	3.47	3.52
16	2.58	2.88	3.05	3.17	3.26	3.33	3.39	3.44	3.48
17	2.57	2.86	3.03	3.14	3.23	3.30	3.36	3.41	3.45
18	2.55	2.84	3.01	3.12	3.21	3.27	3.33	3.38	3.42
19	2.54	2.83	2.99	3.10	3.18	3.25	3.31	3.36	3.40
20	2.53	2.81	2.97	3.08	3.17	3.23	3.29	3.34	3.38
24	2.49	2.77	2.92	3.03	3.11	3.17	3.22	3.27	3.31
30	2.46	2.72	2.87	2.97	3.05	3.11	3.16	3.21	3.24
40	2.42	2.68	2.82	2.92	2.99	3.05	3.10	3.14	3.18
60	2.39	2.64	2.78	2.87	2.94	3.00	3.04	3.08	3.12
120	2.36	2.60	2.73	2.82	2.89	2.94	2.99	3.03	3.06
$\infty$	2.33	2.56	2.68	2.77	2.84	2.89	2.93	2.97	3.00

X. Coefficients of Orthogonal Polynomials<sup>a</sup>

$x_j$	n = 3		n = 4			n = 5				n = 6					n = 7					
	$P_1$	$P_2$	$P_1$	$P_2$	$P_3$	$P_1$	$P_2$	$P_3$	$P_4$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
1	-1	1	-3	1	-1	-2	2	-1	1	-5	5	-5	1	-1	-3	5	-1	3	-1	.1
2	0	-2	-1	-1	3	-1	-1	2	-4	-3	-1	7	-3	5	-2	0	1	-7	4	-6
3	1	1	1	-1	-3	0	-2	0	6	-1	-4	4	2	-10	-1	-3	1	1	-5	15
4			3	1	1	1	-1	-4	-4	1	-4	-4	2	10	0	-4	0	6	0	-20
5						2	2	1	1	3	-1	-7	-3	-5	1	-3	-1	1	5	15
6										5	5	5	1	1	2	0	-1	-7	-4	-6
7															3	5	1	3	1	1
$\sum_{j=1}^n (P_j(x_j))^2$	2	6	20	4	20	10	14	10	70	70	84	180	28	252	28	84	6	154	84	924
$\lambda$	1	3	2	1	$\frac{2}{3}$	1	1	$\frac{2}{3}$	$\frac{11}{12}$	2	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{18}$	1	1	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{36}$

$x_j$	n = 8						n = 9						n = 10					
	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
1	-7	7	-7	7	-7	1	-4	28	-14	14	-4	4	-9	6	-42	18	-6	3
2	-5	1	5	-13	23	-5	-3	7	7	-21	11	-17	-7	2	14	-22	14	-11
3	-3	-3	7	-3	-17	9	-2	-8	13	-11	-4	22	-5	-1	35	-17	-1	10
4	-1	-5	3	9	-15	-5	-1	-17	9	9	-9	1	-3	-3	31	3	-11	6
5	1	-5	-3	9	15	-5	0	-20	0	18	0	-20	-1	-4	12	18	-6	-8
6	3	-3	-7	-3	17	9	1	-17	-9	9	9	1	1	-4	-12	18	6	-8
7	5	1	-5	-13	-23	-5	2	-8	-13	-11	4	22	3	-3	-31	3	11	6
8	7	7	7	7	7	1	3	7	-7	-21	-11	-17	5	-1	-35	-17	1	10
9							4	28	14	14	4	4	7	2	-14	-22	-14	-11
10													9	6	42	18	6	3
$\sum_{j=1}^n (P_j(x_j))^2$	168	168	264	616	2184	264	60	2772	990	2002	468	1980	330	132	8580	2860	780	660
$\lambda$	2	1	$\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{24}$	1	3	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{36}$	2	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{24}$

<sup>a</sup> Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.