

UNIVERSITY OF SWAZILAND**FINAL EXAMINATION PAPER 2005**

TITLE OF PAPER : **TOPICS IN STATISTICS
(MULTIVARIATE DATA ANALYSIS)**

COURSE CODE : **ST405**

TIME ALLOWED : **3 (THREE) HOURS**

REQUIRMENTS : **STATISTICAL TABLES
AND CALCULATOR**

INSTRUCTIONS : **ANSWER ANY 4 (FOUR) QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[7 + 8 + 10 marks]

- 1.1 Define Principal component analysis and discuss its all important properties.
- 1.2 State and discuss the four steps of the procedure for a principal component analysis.
- 1.3 Consider the following table:

| Component | Eigenvalue | Eigenvectors | | | | |
|-----------|------------|----------------|----------------|----------------|----------------|----------------|
| | | X ₁ | X ₂ | X ₃ | X ₄ | X ₅ |
| 1 | 2.616 | 0.452 | 0.462 | 0.451 | 0.471 | 0.398 |
| 2 | 1.532 | -0.051 | 0.300 | 0.325 | 0.185 | -0.377 |
| 3 | 0.386 | 0.691 | 0.341 | -0.455 | -0.411 | -0.179 |
| 4 | 0.302 | -0.420 | 0.548 | -0.606 | 0.388 | 0.069 |
| 5 | 0.165 | 0.374 | -0.530 | -0.343 | 0.652 | -0.192 |

- a. How many components will you choose? Explain why.
- b. List those selected components and interpret those in terms of original variables, X_i'^s.

QUESTION TWO.

[3 + 3 + 3 + 4 + 12 marks]

- 2.1 Discuss the purpose of the discriminant function analysis.
- 2.2 State the criterion for selecting canonical discriminant functions and also state the number of functions you can select.
- 2.3 The following table shows the eigenvalues and corresponding eigenvectors of $\mathbf{W}^{-1}\mathbf{B}$:

| Component | Eigenvalue | Eigenvectors | | | |
|-----------|------------|----------------|----------------|----------------|----------------|
| | | X ₁ | X ₂ | X ₃ | X ₄ |
| 1 | 0.437 | -0.0107 | 0.0040 | 0.0119 | -0.0068 |
| 2 | 0.035 | 0.0031 | 0.0168 | -0.0046 | -0.0022 |
| 3 | 0.015 | -0.0068 | 0.0010 | 0.0000 | 0.0247 |
| 4 | 0.002 | 0.0126 | -0.0001 | 0.0112 | 0.0054 |

- a. How many groups and variables were considered in this problem?
- b. List all the canonical discriminant functions.
- c. Assuming that the ith sample size, n_i = 30 for all i = 1, 2, 3, 4, 5; test whether each of these functions varies significantly from group to group.

QUESTION THREE.

[8 + 10 + 5 + 2 marks]

- 3.1 Define factor analysis and compare its properties with those of principal component analysis.
- 3.2 Discuss the procedures of principal component factor analysis to determine the final factors.
- 3.3 Using the table given in Question 1.3, write the unrotated factor model.
- 3.4 Compute the communalities.

QUESTION FOUR.

[15 + 10 marks]

- 4.1 Suppose we have two groups, with 10 subjects in each group. The means for the two variables (X_1 and X_2) measured in group A are 10 and 7.5, while the means in group B are 9 and 9.5. The respective pooled sample variances are 9 and 4 for variables X_1 and X_2 , while the pooled correlation is 0.7. Perform Hotellings' T^2 test and also perform univariate t tests for each of these two variables at 5% level of significance. Compare the results.
- 4.2 Suppose we have three variables in each of the 3 groups, with 10 subjects per group. Let the sum of squares matrices are as follows:

$$B = \begin{bmatrix} 1.68 & 1.38 & -1.26 \\ 1.38 & 1.14 & -1.08 \\ -1.26 & -1.08 & 1.26 \end{bmatrix}, \quad W = \begin{bmatrix} 1.24 & 0.06 & 0.56 \\ 0.06 & 1.08 & 0.18 \\ 0.56 & 0.18 & 2.74 \end{bmatrix}, \quad \& \quad T = \begin{bmatrix} 2.92 & 1.44 & -0.70 \\ 1.44 & 2.22 & -0.90 \\ -0.70 & -0.90 & 4.00 \end{bmatrix}$$

Compute Wilk's Λ statistic and ϕ . Use χ^2 approximation to test the equality of population mean vectors.

QUESTION FIVE.

[3 + 3 + 4 + 4 + 3 + 8 marks]

The following tables are part of the complete output running SPSS for a set of multivariate variables; not necessarily from the same set of variables. Tables 1-6 are obtained running Factor Analysis and Tables 7-9 are from Discriminant Function Analysis:

Table 1:

| Correlation Matrix | | | | | | | |
|--------------------|-------|-------|-------|-------|-------|-------|-------|
| | X1 | X2 | X3 | X4 | X5 | X6 | X7 |
| Correlatio X1 | 1.000 | .036 | -.671 | -.400 | -.538 | -.645 | -.764 |
| X2 | .036 | 1.000 | .445 | .405 | -.026 | -.495 | -.221 |
| X3 | -.671 | .445 | 1.000 | .385 | .494 | .080 | .200 |
| X4 | -.400 | .405 | .385 | 1.000 | .060 | .199 | .185 |
| X5 | -.538 | -.026 | .494 | .060 | 1.000 | .271 | .210 |
| X6 | -.645 | -.495 | .080 | .199 | .271 | 1.000 | .424 |
| X7 | -.764 | -.221 | .200 | .185 | .210 | .424 | 1.000 |

Table 2:

| Component | Eigenvalues |
|-----------|-------------|
| 1 | 2.965 |
| 2 | 1.850 |
| 3 | 0.971 |
| 4 | 0.643 |
| 5 | 0.359 |
| 6 | 0.211 |
| 7 | 0.000 |

Table 3:

Component Matrix^a

| | Component | | | | | |
|----|-----------|-----------|-----------|-----------|-----------|-----------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| X1 | -.986 | 5.070E-02 | -2.47E-02 | .113 | .102 | -2.99E-02 |
| X2 | -1.24E-02 | .936 | 7.822E-02 | -.109 | 3.541E-04 | .325 |
| X3 | .683 | .566 | -.258 | -3.93E-02 | -.333 | -.186 |
| X4 | .483 | .481 | .590 | .327 | .241 | -.153 |
| X5 | .621 | 4.503E-02 | -.677 | .186 | .344 | 4.014E-02 |
| X6 | .641 | -.562 | .179 | .402 | -.186 | .212 |
| X7 | .705 | -.319 | .246 | -.560 | .166 | 2.041E-02 |

Extraction Method: Principal Component Analysis.

a. 6 components extracted.

Table 4:

Component Matrix^a

| | Component | | |
|----|-----------|-----------|-----------|
| | 1 | 2 | 3 |
| X1 | -.986 | 5.070E-02 | -2.47E-02 |
| X2 | -1.24E-02 | .936 | 7.822E-02 |
| X3 | .683 | .566 | -.258 |
| X4 | .483 | .481 | .590 |
| X5 | .621 | 4.503E-02 | -.677 |
| X6 | .641 | -.562 | .179 |
| X7 | .705 | -.319 | .246 |

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

Table 5:

Rotated Component Matrix^a

| | Component | | |
|----|-----------|-----------|-------|
| | 1 | 2 | 3 |
| X1 | -.764 | .550 | -.301 |
| X2 | -.515 | .195 | .761 |
| X3 | 8.552E-02 | .756 | .523 |
| X4 | .295 | -2.72E-02 | .851 |
| X5 | .185 | .892 | -.127 |
| X6 | .858 | 8.626E-02 | -.124 |
| X7 | .790 | .141 | .120 |

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 6 iterations.

Table 6:

Component Score Coefficient Matrix

| | Component | | |
|----|-----------|-------|-------|
| | 1 | 2 | 3 |
| X1 | -.270 | -.170 | -.103 |
| X2 | -.263 | .078 | .433 |
| X3 | -.102 | .422 | .169 |
| X4 | .192 | -.291 | .584 |
| X5 | -.113 | .658 | -.290 |
| X6 | .400 | -.097 | -.063 |
| X7 | .365 | -.099 | .083 |

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

Table 7:

Wilks' Lambda

| Test of Function(s) | Wilks' Lambda | Chi-square | df | Sig. |
|---------------------|---------------|------------|----|------|
| 1 through 2 | .147 | 39.244 | 12 | .000 |
| 2 | .656 | 8.655 | 5 | .124 |

Table 8:

Standardized Canonical Discriminant Function Coefficients

| | Function | |
|----|----------|--------|
| | 1 | 2 |
| X1 | .268 | 1.193 |
| X2 | .202 | -1.065 |
| X3 | .412 | .938 |
| X4 | .573 | .478 |
| X5 | .129 | .097 |
| X6 | -1.040 | -.249 |

Table 9:

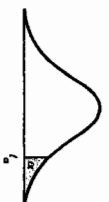
Canonical Discriminant Function Coefficients

| | Function | |
|------------|----------|--------|
| | 1 | 2 |
| X1 | .019 | .083 |
| X2 | .305 | -1.603 |
| X3 | .059 | .135 |
| X4 | 1.512 | 1.263 |
| X5 | .076 | .057 |
| X6 | -.268 | -.064 |
| (Constant) | -.217 | -3.732 |

Unstandardized coefficients

- a. Examine Table 1 and discuss the suitability of principal component analysis. How many principal components will you have?
- b. How many factors will you choose if you wish to use factor analysis method? Explain your answer.
- c. How many factors will you get in your factor model from Table 3? List the last two equations of your model and compute their communalities.
- d. Suppose the same data were analyzed using with a restriction on the number of factors. How many factors were chosen in Table 4? List the first two equations of your model and compute their communalities.
- e. List all equations needed to compute factor scores.
- f. Write all the discriminant functions and test whether each of those are significant at 5% level of significance.

Table 5
Percentage points of the *t* distributions

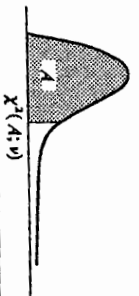


| <i>t</i> ₁₀₀ | <i>t</i> ₉₅₀ | <i>t</i> ₉₀₀ | <i>t</i> ₈₀₀ | <i>t</i> ₇₀₀ | <i>t</i> ₅₀₀ | df |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|------|
| 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 63.657 | 1 |
| 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 9.925 | 2 |
| 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 5.841 | 3 |
| 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4.604 | 4 |
| 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.032 | 5 |
| 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 3.707 | 6 |
| 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 3.499 | 7 |
| 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.355 | 8 |
| 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.250 | 9 |
| 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.169 | 10 |
| 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.106 | 11 |
| 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.055 | 12 |
| 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.012 | 13 |
| 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 2.977 | 14 |
| 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 2.947 | 15 |
| 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 2.921 | 16 |
| 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 2.898 | 17 |
| 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 2.878 | 18 |
| 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 2.861 | 19 |
| 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 2.845 | 20 |
| 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 2.831 | 21 |
| 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 2.819 | 22 |
| 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 2.807 | 23 |
| 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 2.797 | 24 |
| 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 2.787 | 25 |
| 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 2.779 | 26 |
| 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 2.771 | 27 |
| 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 2.763 | 28 |
| 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 2.756 | 29 |
| 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.576 | inf. |

From "Table of Percentage Points of the *t*-Distribution,"
Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p.
300. Reproduced by permission of Professor E. S. Pearson.

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



| ν | .005 | .010 | .025 | .050 | .100 | .900 | .950 | .975 | .990 | .995 |
|-------|---------|---------|---------|---------|---------|-------|-------|-------|-------|-------|
| 1 | 0.00393 | 0.00786 | 0.01572 | 0.03144 | 0.06288 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | 0.01000 | 0.02000 | 0.05000 | 0.10000 | 0.211 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 |
| 6 | 0.676 | 0.872 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 7 | 0.989 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

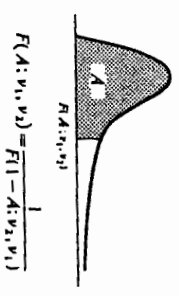


TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | | |
|---------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| 1 | .50 | 1.00 | 1.50 | 1.71 | 1.82 | 1.89 | 1.94 | 1.98 | 2.00 | 2.03 |
| .90 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 | 60.3 |
| .95 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 241 |
| .975 | 648 | 800 | 864 | 900 | 922 | 937 | 948 | 958 | 963 | 966 |
| .99 | 4,032 | 5,000 | 5,403 | 5,625 | 5,764 | 5,859 | 5,928 | 5,981 | 6,022 | 6,052 |
| .995 | 16,211 | 20,000 | 21,615 | 22,500 | 23,056 | 23,437 | 23,715 | 23,935 | 24,091 | 24,191 |
| .999 | 405,280 | 500,000 | 540,380 | 562,500 | 576,400 | 585,940 | 592,870 | 598,140 | 602,280 | 605,280 |
| 2 | .50 | 0.667 | 1.00 | 1.13 | 1.21 | 1.25 | 1.28 | 1.30 | 1.32 | 1.33 |
| .90 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 |
| .95 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 |
| .975 | 38.5 | 39.0 | 39.2 | 39.2 | 39.3 | 39.3 | 39.4 | 39.4 | 39.4 | 39.4 |
| .99 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 |
| .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 |
| .999 | 998.5 | 999.0 | 999.2 | 999.2 | 999.3 | 999.3 | 999.4 | 999.4 | 999.4 | 999.4 |
| 3 | .50 | 0.585 | 0.881 | 1.00 | 1.06 | 1.10 | 1.13 | 1.15 | 1.16 | 1.17 |
| .90 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.24 |
| .95 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.81 |
| .975 | 17.4 | 16.0 | 15.4 | 15.1 | 14.9 | 14.7 | 14.6 | 14.5 | 14.5 | 14.5 |
| .99 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.3 |
| .995 | 55.6 | 49.8 | 47.5 | 46.2 | 45.4 | 44.8 | 44.4 | 44.1 | 43.9 | 43.9 |
| .999 | 167.0 | 148.5 | 141.1 | 137.1 | 134.6 | 132.8 | 131.6 | 130.6 | 129.9 | 129.9 |
| 4 | .50 | 0.549 | 0.828 | 0.941 | 1.00 | 1.04 | 1.06 | 1.08 | 1.09 | 1.10 |
| .90 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.98 | 3.94 | 3.94 |
| .95 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 6.00 |
| .975 | 12.2 | 10.6 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 | 8.90 |
| .99 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.7 |
| .995 | 31.3 | 26.3 | 24.3 | 23.2 | 22.5 | 22.0 | 21.6 | 21.4 | 21.1 | 21.1 |
| .999 | 74.1 | 61.2 | 56.2 | 53.4 | 51.7 | 50.5 | 49.7 | 49.0 | 48.5 | 48.5 |
| 5 | .50 | 0.528 | 0.799 | 0.907 | 0.965 | 1.00 | 1.02 | 1.04 | 1.05 | 1.06 |
| .90 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.32 |
| .95 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.77 |
| .975 | 10.0 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 | 6.68 |
| .99 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.2 |
| .995 | 22.8 | 18.3 | 16.5 | 15.6 | 14.9 | 14.5 | 14.2 | 14.0 | 13.8 | 13.8 |
| .999 | 47.2 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 27.2 | 27.2 |
| 6 | .50 | 0.515 | 0.780 | 0.886 | 0.942 | 0.977 | 1.00 | 1.02 | 1.03 | 1.04 |
| .90 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.96 |
| .95 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.10 |
| .975 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 | 5.52 |
| .99 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.98 |
| .995 | 18.6 | 14.5 | 12.9 | 12.0 | 11.5 | 11.1 | 10.6 | 10.4 | 10.4 | 10.4 |
| .999 | 35.5 | 27.0 | 23.7 | 21.9 | 20.8 | 20.0 | 19.5 | 19.0 | 18.7 | 18.7 |
| 7 | .50 | 0.506 | 0.767 | 0.871 | 0.926 | 0.960 | 0.983 | 1.00 | 1.01 | 1.02 |
| .90 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.72 |
| .95 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.68 |
| .975 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 | 4.82 |
| .99 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.72 |
| .995 | 16.2 | 12.4 | 10.9 | 10.1 | 9.52 | 9.16 | 8.99 | 8.89 | 8.51 | 8.51 |
| .999 | 29.2 | 21.7 | 18.8 | 17.2 | 16.2 | 15.5 | 15.0 | 14.6 | 14.3 | 14.3 |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | | | | | |
|---------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--|--|--|
| | 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ | | | | |
| 1 | .50 | 2.04 | 2.07 | 2.09 | 2.12 | 2.13 | 2.15 | 2.17 | 2.18 | 2.20 | | | |
| .90 | 60.2 | 60.7 | 61.2 | 61.7 | 62.0 | 62.3 | 62.8 | 63.1 | 63.3 | 63.3 | | | |
| .95 | 242 | 244 | 246 | 248 | 249 | 250 | 252 | 253 | 254 | 254 | | | |
| .975 | 605.6 | 610.6 | 615.7 | 620.9 | 623.5 | 626.1 | 631.3 | 633.9 | 636.6 | 636.6 | | | |
| .99 | 24,224 | 24,426 | 24,630 | 24,836 | 24,940 | 25,044 | 25,253 | 25,359 | 25,464 | 25,464 | | | |
| .995 | 605,620 | 610,670 | 615,760 | 620,910 | 623,500 | 626,100 | 631,340 | 633,970 | 636,620 | 636,620 | | | |
| 2 | .50 | 1.34 | 1.36 | 1.38 | 1.39 | 1.40 | 1.41 | 1.43 | 1.43 | 1.44 | | | |
| .90 | 9.39 | 9.41 | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.48 | 9.49 | 9.49 | | | |
| .95 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | | | |
| .975 | 39.4 | 39.4 | 39.4 | 39.4 | 39.5 | 39.5 | 39.5 | 39.5 | 39.5 | 39.5 | | | |
| .99 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | | | |
| .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 200 | | | |
| .999 | 999.4 | 999.4 | 999.4 | 999.4 | 999.5 | 999.5 | 999.5 | 999.5 | 999.5 | 999.5 | | | |
| 3 | .50 | 1.18 | 1.20 | 1.21 | 1.23 | 1.23 | 1.24 | 1.25 | 1.26 | 1.27 | | | |
| .90 | 5.73 | 5.72 | 5.70 | 5.68 | 5.66 | 5.64 | 5.62 | 5.61 | 5.60 | 5.60 | | | |
| .95 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.57 | 8.55 | 8.53 | 8.53 | | | |
| .975 | 14.4 | 14.3 | 14.3 | 14.2 | 14.1 | 14.1 | 14.0 | 13.9 | 13.9 | 13.9 | | | |
| .99 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.3 | 26.2 | 26.1 | 26.1 | | | |
| .995 | 43.7 | 43.4 | 43.1 | 42.8 | 42.6 | 42.5 | 42.1 | 42.0 | 41.8 | 41.8 | | | |
| .999 | 129.2 | 128.3 | 127.4 | 126.4 | 125.9 | 125.4 | 124.5 | 124.0 | 123.5 | 123.5 | | | |
| 4 | .50 | 1.11 | 1.13 | 1.14 | 1.15 | 1.16 | 1.18 | 1.18 | 1.19 | 1.19 | | | |
| .90 | 3.92 | 3.90 | 3.87 | 3.84 | 3.83 | 3.82 | 3.79 | 3.78 | 3.76 | 3.76 | | | |
| .95 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.69 | 5.66 | 5.63 | 5.63 | | | |
| .975 | 8.84 | 8.75 | 8.66 | 8.56 | 8.51 | 8.46 | 8.36 | 8.31 | 8.26 | 8.26 | | | |
| .99 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.6 | 13.5 | 13.5 | | | |
| .995 | 21.0 | 20.7 | 20.4 | 20.2 | 20.0 | 19.9 | 19.6 | 19.5 | 19.3 | 19.3 | | | |
| .999 | 48.1 | 47.4 | 46.8 | 46.1 | 45.8 | 45.4 | 44.7 | 44.4 | 44.1 | 44.1 | | | |
| 5 | .50 | 1.07 | 1.09 | 1.10 | 1.11 | 1.12 | 1.14 | 1.14 | 1.14 | 1.15 | | | |
| .90 | 3.30 | 3.27 | 3.24 | 3.21 | 3.19 | 3.17 | 3.14 | 3.12 | 3.11 | 3.11 | | | |
| .95 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.43 | 4.40 | 4.37 | 4.37 | | | |
| .975 | 6.62 | 6.52 | 6.43 | 6.33 | 6.28 | 6.23 | 6.12 | 6.07 | 6.02 | 6.02 | | | |
| .99 | 10.1 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.20 | 9.11 | 9.02 | 9.02 | | | |
| .995 | 13.6 | 13.4 | 13.1 | 12.9 | 12.8 | 12.7 | 12.4 | 12.3 | 12.1 | 12.1 | | | |
| .999 | 26.9 | 26.4 | 25.9 | 25.4 | 25.1 | 24.9 | 24.3 | 24.1 | 23.8 | 23.8 | | | |
| 6 | .50 | 1.05 | 1.06 | 1.07 | 1.08 | 1.09 | 1.10 | 1.11 | 1.12 | 1.12 | | | |
| .90 | 2.94 | 2.90 | 2.87 | 2.84 | 2.82 | 2.80 | 2.76 | 2.74 | 2.72 | 2.72 | | | |
| .95 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.74 | 3.70 | 3.67 | 3.67 | | | |
| .975 | 5.46 | 5.37 | 5.27 | 5.17 | 5.12 | 5.07 | 4.96 | 4.90 | 4.85 | 4.85 | | | |
| .99 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.06 | 6.97 | 6.88 | 6.88 | | | |
| .995 | 10.2 | 10.0 | 9.81 | 9.59 | 9.47 | 9.36 | 9.12 | 9.00 | 8.88 | 8.88 | | | |
| .999 | 18.4 | 18.0 | 17.6 | 17.1 | 16.9 | 16.7 | 16.2 | 16.0 | 15.7 | 15.7 | | | |
| 7 | .50 | 1.03 | 1.04 | 1.05 | 1.07 | 1.07 | 1.08 | 1.09 | 1.10 | 1.10 | | | |
| .90 | 2.70 | 2.67 | 2.63 | 2.59 | 2.58 | 2.56 | 2.51 | 2.49 | 2.47 | 2.47 | | | |
| .95 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.30 | 3.27 | 3.23 | 3.23 | | | |
| .975 | 4.76 | 4.67 | 4.57 | 4.47 | 4.42 | 4.36 | 4.25 | 4.20 | 4.14 | 4.14 | | | |
| .99 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.82 | 5.74 | 5.65 | 5.65 | | | |
| .995 | 8.38 | 8.18 | 7.97 | 7.75 | 7.65 | 7.53 | 7.31 | 7.19 | 7.08 | 7.08 | | | |
| .999 | 14.1 | 13.7 | 13.3 | 12.9 | 12.7 | 12.5 | 12.1 | 11.9 | 11.7 | 11.7 | | | |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | | |
|---------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| 8 | .50 | 0.499 | 0.757 | 0.860 | 0.915 | 0.948 | 0.971 | 0.988 | 1.00 | 1.01 |
| | .90 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 |
| | .95 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| | .975 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 |
| | .99 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| 9 | .50 | 0.494 | 0.749 | 0.852 | 0.906 | 0.939 | 0.962 | 0.978 | 0.990 | 1.00 |
| | .90 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 |
| | .95 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| | .975 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 |
| | .99 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| 10 | .50 | 0.490 | 0.743 | 0.845 | 0.899 | 0.932 | 0.954 | 0.971 | 0.983 | 0.992 |
| | .90 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 |
| | .95 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| | .975 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 |
| | .99 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| 12 | .50 | 0.484 | 0.735 | 0.835 | 0.888 | 0.921 | 0.943 | 0.959 | 0.972 | 0.981 |
| | .90 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 |
| | .95 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| | .975 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 |
| | .99 | 9.33 | 6.93 | 5.93 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| 15 | .50 | 0.478 | 0.726 | 0.826 | 0.878 | 0.911 | 0.933 | 0.949 | 0.960 | 0.970 |
| | .90 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
| | .95 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| | .975 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 |
| | .99 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| 20 | .50 | 0.472 | 0.718 | 0.816 | 0.868 | 0.900 | 0.922 | 0.938 | 0.950 | 0.959 |
| | .90 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
| | .95 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| | .975 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
| | .99 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| 24 | .50 | 0.469 | 0.714 | 0.812 | 0.863 | 0.895 | 0.917 | 0.932 | 0.944 | 0.953 |
| | .90 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |
| | .95 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| | .975 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |
| | .99 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| .999 | .999 | 9.55 | 6.66 | 5.52 | 4.89 | 4.49 | 4.20 | 3.99 | 3.83 | 3.69 |
| | .999 | 14.0 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | | | | | |
|---------|--------------|-------|-------|-------|-------|------|------|------|------|------|--|--|--|
| | 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ | | | | |
| 8 | .50 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.07 | 1.08 | 1.08 | 1.09 | | | |
| | .90 | 2.54 | 2.50 | 2.46 | 2.42 | 2.40 | 2.38 | 2.34 | 2.32 | 2.29 | | | |
| | .95 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.01 | 2.97 | 2.93 | | | |
| | .975 | 4.30 | 4.20 | 4.10 | 4.00 | 3.95 | 3.89 | 3.78 | 3.73 | 3.67 | | | |
| | .99 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.03 | 4.95 | 4.86 | | | |
| 9 | .50 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.05 | 1.07 | 1.07 | 1.08 | | | |
| | .90 | 2.42 | 2.38 | 2.34 | 2.30 | 2.28 | 2.25 | 2.21 | 2.18 | 2.16 | | | |
| | .95 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.79 | 2.75 | 2.71 | | | |
| | .975 | 3.96 | 3.87 | 3.77 | 3.67 | 3.61 | 3.56 | 3.45 | 3.39 | 3.33 | | | |
| | .99 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.48 | 4.40 | 4.31 | | | |
| 10 | .50 | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.06 | 1.07 | | | |
| | .90 | 2.32 | 2.28 | 2.24 | 2.20 | 2.18 | 2.16 | 2.11 | 2.08 | 2.06 | | | |
| | .95 | 2.98 | 2.91 | 2.84 | 2.77 | 2.74 | 2.70 | 2.62 | 2.58 | 2.54 | | | |
| | .975 | 3.72 | 3.62 | 3.52 | 3.42 | 3.37 | 3.31 | 3.20 | 3.14 | 3.08 | | | |
| | .99 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.08 | 4.00 | 3.91 | | | |
| 12 | .50 | 0.989 | 1.00 | 1.01 | 1.02 | 1.03 | 1.03 | 1.05 | 1.05 | 1.06 | | | |
| | .90 | 2.19 | 2.15 | 2.10 | 2.06 | 2.04 | 2.01 | 1.96 | 1.93 | 1.90 | | | |
| | .95 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.38 | 2.34 | 2.30 | | | |
| | .975 | 3.37 | 3.28 | 3.18 | 3.07 | 3.02 | 2.96 | 2.85 | 2.79 | 2.72 | | | |
| | .99 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.54 | 3.45 | 3.36 | | | |
| 15 | .50 | 0.977 | 0.989 | 1.00 | 1.01 | 1.02 | 1.03 | 1.03 | 1.05 | 1.05 | | | |
| | .90 | 2.06 | 2.02 | 1.97 | 1.92 | 1.90 | 1.87 | 1.82 | 1.79 | 1.76 | | | |
| | .95 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.16 | 2.11 | 2.07 | | | |
| | .975 | 3.06 | 2.96 | 2.86 | 2.76 | 2.70 | 2.64 | 2.52 | 2.46 | 2.40 | | | |
| | .99 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.05 | 2.96 | 2.87 | | | |
| 20 | .50 | 0.966 | 0.977 | 0.989 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 | 1.03 | | | |
| | .90 | 1.94 | 1.89 | 1.84 | 1.79 | 1.77 | 1.74 | 1.68 | 1.64 | 1.61 | | | |
| | .95 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.95 | 1.90 | 1.84 | | | |
| | .975 | 2.77 | 2.68 | 2.57 | 2.46 | 2.41 | 2.35 | 2.22 | 2.16 | 2.09 | | | |
| | .99 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.61 | 2.52 | 2.42 | | | |
| 24 | .50 | 0.961 | 0.972 | 0.983 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 | | | |
| | .90 | 1.88 | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.61 | 1.57 | 1.53 | | | |
| | .95 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.84 | 1.79 | 1.73 | | | |
| | .975 | 2.64 | 2.54 | 2.44 | 2.33 | 2.27 | 2.21 | 2.08 | 2.01 | 1.94 | | | |
| | .99 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.40 | 2.31 | 2.21 | | | |
| .999 | .999 | 3.59 | 3.42 | 3.25 | 3.06 | 2.97 | 2.87 | 2.66 | 2.55 | 2.43 | | | |
| | .999 | 4.64 | 4.39 | 4.14 | 3.87 | 3.74 | 3.59 | 3.29 | 3.14 | 2.97 | | | |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | |
|---------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 30 | 0.466 | 0.709 | 0.807 | 0.858 | 0.890 | 0.912 | 0.927 | 0.939 | 0.948 |
| .50 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
| .90 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| .95 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 |
| .975 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| .99 | 9.18 | 6.35 | 5.24 | 4.62 | 4.23 | 3.95 | 3.74 | 3.58 | 3.45 |
| .995 | 13.3 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
| .999 | | | | | | | | | |
| 60 | 0.461 | 0.701 | 0.798 | 0.849 | 0.880 | 0.901 | 0.917 | 0.928 | 0.937 |
| .50 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |
| .90 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| .95 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 |
| .975 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| .99 | 8.49 | 5.80 | 4.73 | 4.14 | 3.76 | 3.49 | 3.29 | 3.13 | 3.01 |
| .995 | 12.0 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |
| .999 | | | | | | | | | |
| 120 | 0.458 | 0.697 | 0.793 | 0.844 | 0.875 | 0.896 | 0.912 | 0.923 | 0.932 |
| .50 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 |
| .90 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 |
| .95 | 5.15 | 3.80 | 3.23 | 2.89 | 2.67 | 2.52 | 2.39 | 2.30 | 2.22 |
| .975 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 |
| .99 | 8.18 | 5.54 | 4.50 | 3.92 | 3.55 | 3.28 | 3.09 | 2.93 | 2.81 |
| .995 | 11.4 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 |
| .999 | | | | | | | | | |
| ∞ | 0.455 | 0.693 | 0.789 | 0.839 | 0.870 | 0.891 | 0.907 | 0.918 | 0.927 |
| .50 | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 |
| .90 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 |
| .95 | 5.02 | 3.69 | 3.12 | 2.79 | 2.57 | 2.41 | 2.29 | 2.19 | 2.11 |
| .975 | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 |
| .99 | 7.88 | 5.30 | 4.28 | 3.72 | 3.35 | 3.09 | 2.90 | 2.74 | 2.62 |
| .995 | 10.8 | 6.91 | 5.42 | 4.62 | 4.10 | 3.74 | 3.47 | 3.27 | 3.10 |
| .999 | | | | | | | | | |

TABLE A.4 (concluded) Percentiles of the F Distribution

| Den. df | Numerator df | | | | | | | | | | |
|---------|--------------|-------|-------|-------|-------|-------|-------|-------|------|------|--|
| | 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ | | |
| 30 | 0.955 | 0.966 | 0.978 | 0.989 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 | 1.02 | |
| .50 | 1.82 | 1.77 | 1.72 | 1.67 | 1.64 | 1.61 | 1.54 | 1.50 | 1.46 | 1.46 | |
| .90 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.74 | 1.68 | 1.62 | 1.62 | |
| .95 | 2.51 | 2.41 | 2.31 | 2.20 | 2.14 | 2.07 | 1.94 | 1.87 | 1.79 | 1.79 | |
| .975 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.21 | 2.11 | 2.01 | 2.01 | |
| .99 | 3.34 | 3.18 | 3.01 | 2.82 | 2.73 | 2.63 | 2.42 | 2.30 | 2.18 | 2.18 | |
| .995 | 4.24 | 4.00 | 3.75 | 3.49 | 3.36 | 3.22 | 2.92 | 2.76 | 2.59 | 2.59 | |
| .999 | | | | | | | | | | | |
| 60 | 0.945 | 0.956 | 0.967 | 0.978 | 0.983 | 0.989 | 1.00 | 1.01 | 1.01 | 1.01 | |
| .50 | 1.71 | 1.66 | 1.60 | 1.54 | 1.51 | 1.48 | 1.40 | 1.35 | 1.29 | 1.29 | |
| .90 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.53 | 1.47 | 1.39 | 1.39 | |
| .95 | 2.27 | 2.17 | 2.06 | 1.94 | 1.88 | 1.82 | 1.67 | 1.58 | 1.48 | 1.48 | |
| .975 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.84 | 1.73 | 1.60 | 1.60 | |
| .99 | 2.90 | 2.74 | 2.57 | 2.39 | 2.29 | 2.19 | 1.96 | 1.83 | 1.69 | 1.69 | |
| .995 | 3.54 | 3.32 | 3.08 | 2.83 | 2.69 | 2.55 | 2.25 | 2.08 | 1.89 | 1.89 | |
| .999 | | | | | | | | | | | |
| 120 | 0.939 | 0.950 | 0.961 | 0.972 | 0.978 | 0.983 | 0.994 | 1.00 | 1.01 | 1.01 | |
| .50 | 1.65 | 1.60 | 1.55 | 1.48 | 1.45 | 1.41 | 1.32 | 1.26 | 1.19 | 1.19 | |
| .90 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.43 | 1.35 | 1.25 | 1.25 | |
| .95 | 2.16 | 2.05 | 1.95 | 1.82 | 1.76 | 1.69 | 1.53 | 1.43 | 1.31 | 1.31 | |
| .975 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.66 | 1.53 | 1.38 | 1.38 | |
| .99 | 2.71 | 2.54 | 2.37 | 2.19 | 2.09 | 1.98 | 1.75 | 1.61 | 1.43 | 1.43 | |
| .995 | 3.24 | 3.02 | 2.78 | 2.53 | 2.40 | 2.26 | 1.95 | 1.77 | 1.54 | 1.54 | |
| .999 | | | | | | | | | | | |
| ∞ | 0.934 | 0.945 | 0.956 | 0.967 | 0.972 | 0.978 | 0.989 | 0.994 | 1.00 | 1.00 | |
| .50 | 1.60 | 1.55 | 1.49 | 1.42 | 1.38 | 1.34 | 1.24 | 1.17 | 1.00 | 1.00 | |
| .90 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.32 | 1.22 | 1.00 | 1.00 | |
| .95 | 2.05 | 1.94 | 1.83 | 1.71 | 1.64 | 1.57 | 1.39 | 1.27 | 1.00 | 1.00 | |
| .975 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.47 | 1.32 | 1.00 | 1.00 | |
| .99 | 2.52 | 2.36 | 2.19 | 2.00 | 1.90 | 1.79 | 1.53 | 1.36 | 1.00 | 1.00 | |
| .995 | 2.96 | 2.74 | 2.51 | 2.27 | 2.13 | 1.99 | 1.66 | 1.45 | 1.00 | 1.00 | |
| .999 | | | | | | | | | | | |

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