

UNIVERSITY OF SWAZILAND
FINAL EXAMINATION PAPER 2005

TITLE OF PAPER: OPERATIONS RESEARCH II

COURSE CODE : ST 408

TIME ALLOWED : TWO (2) HOURS

INSTRUCTIONS : ANSWER ANY FOUR (4) QUESTIONS.

REQUIREMENTS: CALCULATOR

**PLEASE DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

Question 1

Customers arrivals at Bo's gas station follow a Poisson process with mean arrival rate of one customer every 4 minutes and the service time follow an exponential distribution with mean service time of 2.5 minutes per customer.

- What is the probability that at anytime the gas station will be idle?
- What is the average number of customers in the system?
- What is the average queue length?
- What is the average time a customer spends in the system?
- What is the average time a customer waits in line to be served?

Question 2.

(a) Show that the optimum order quantity y^* for a single period model without set up cost satisfies the following expression.

$$\int_0^{y^*} f(D)dD = \frac{p-c}{p+h}$$

Where p = shortage cost per unit, c = purchasing/production cost per unit and h = holding cost per unit.

(b) The demand for an item during a single period occurs according to an exponential distribution with mean 10, that is

$$f(D) = \frac{1}{10} e^{-\frac{D}{10}}, \quad D > 0$$

Assuming that demand occurs instantaneously at the beginning of the period and that the per unit holding cost (h) and shortage cost (p) for the period are 1 and 3 respectively. The purchasing cost is 2 per unit. Find the optimal order quantity given an initial inventory of 2 units. What is the optimal order quantity if the initial inventory is 5 units?

Question 3.

(a) For an Economic Production Lot Size model, show that the minimum cost production quantity is given by:

$$Q^* = \sqrt{\frac{2DC_o}{\left(1 - \frac{D}{P}\right)C_h}}$$

(b) The Wilson Publishing Company produces books for the retail market. Demand for a current book is expected to occur at a constant annual rate of 7200 copies. The cost of one copy of the book is \$14.50. The holding cost is based on an 18% annual rate, and production setup costs are \$150 per setup. The equipment on which the book is produced has an annual production volume of 25000 copies. There are 250 working days per year and the lead time for a production run is 15 days. Use the production lot size model to compute the following values:

- (i) Minimum- cost production lot size
- (ii) Number of production runs per year
- (iii) Cycle time
- (iv) Length of production run.
- (v) Total annual cost
- (vi) Reorder point

Question 4.

(a) Define each of the following terms as used in Markov Chains:

- (i) Transition probabilities
- (ii) Absorbing state
- (iii) Steady probabilities

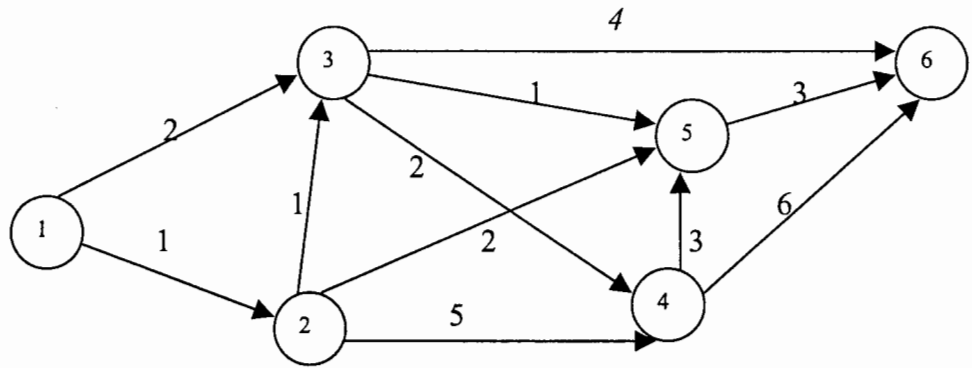
(b) Consider the following Markov system defined by the transition matrix with three states $n = 0, 1$ and 2 :

$$\begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/4 & 1/4 \\ 1/5 & 3/5 & 1/5 \end{pmatrix}$$

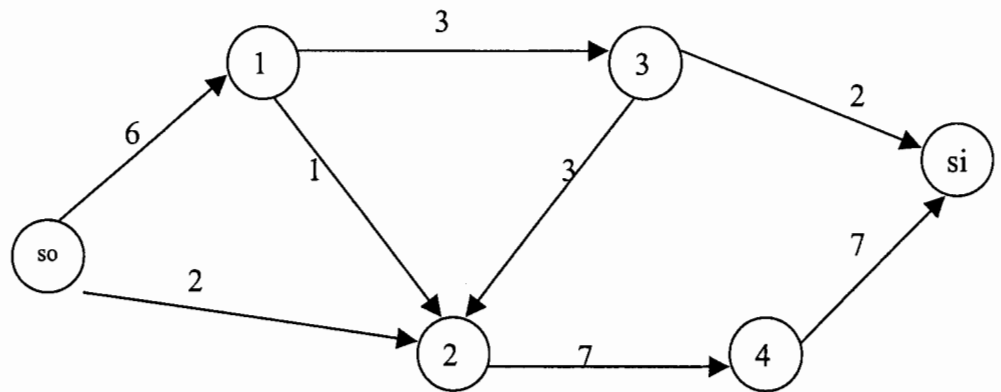
- (i) Find the steady state probabilities of the system.
- (ii) Compute the mean recurrence time for each of the states.

Question 5.

(a) Find the shortest route(s) from city 1 to city 6



(b) Consider the following network flow below



- (i) Find the maximal flow from source (so) to sink (si).
- (ii) Express the above maximal flow problem as a linear programming problem.