

UNIVERSITY OF SWAZILAND
SUPPLEMENTARY EXAMINATION PAPER 2005

TITLE OF PAPER : **OPERATIONS RESEARCH II**

COURSE CODE : **ST 408**

TIME ALLOWED : **TWO (2) HOURS**

INSTRUCTIONS : **THIS PAPER HAS FIVE QUESTIONS.**
ANSWER ANY FOUR(4) QUESTIONS.

REQUIREMENTS: **Scientific Calculator**

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QUESTION ONE

(a) *Itemize and discuss the components of a queuing system.*

(b) *For a single sever queue system show that*

$$P_n = (1-\rho)\rho^n; n \geq 0, \text{ where } \rho = \lambda/\mu.$$

Hence or otherwise obtain the expected number of units in the system.

QUESTION TWO

The transition matrix of a two state Markov chain is given as:

$$P = \begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{pmatrix}$$

- (i) *Derive the n-step transition matrix, using the method of moments.*
- (ii) *Find the limiting distribution.*
- (iii) *Derive the probability mass function (p.m.f) of the first passage time and the recurrence time. Obtain their respective means.*

QUESTION THREE

- (a) Show that the optimum order quantity y^* for a single period model without set up cost satisfies the expression:

$$\int_0^{y^*} f(D) dD = \frac{p-c}{p+h}$$

Where p =shortage cost per unit; c =purchasing production cost per unit and h =holding cost per unit.

- (b) The demand for an item during a single period occurs according to an exponential distribution with mean 10, that is $f(D) = \frac{1}{10} e^{-\frac{D}{10}}$, $D > 0$.

Assuming that demand occurs instantaneously at the beginning of the period and that the per unit holding cost (h) and shortage cost (P) for the period are 1 and 3 respectively. The purchasing cost is 2 per unit. Find the optimal order quantity given an initial inventory of 2 units. What is the optimal order quantity if the initial inventory is 5 units?

QUESTION FOUR

- (a) For an Economic Production Lot size model, show that the minimum cost production quantity is given

by : $Q^* = \sqrt{\frac{2DC_o}{(1-p)c_h}}$

- (b) A Publishing company produces books for the retail market. Demand for a current book is expected to occur at a constant annual rate of 7200copies. The cost of one copy of the book is \$14.50. The holding cost is based on an 18% annual rate, and production set up cost is \$150 per setup. The equipment on which the book is produced has an annual production volume of 2500copies. There are 250 working days per year and the lead time for a production run is 15days. Use the production lot size model to compute the following values:
- (i) Minimum cost production lot size.
 - (ii) Number of production runs per year.
 - (iii) Cycle time.
 - (iv) Length of production run.
 - (v) Total annual cost.
 - (vi) Reorder point.

QUESTION FIVE

- (a) Define a Markov Chain.
- (b) Distinguish between the following:
- (i) A transient and a recurrent state.
 - (ii) An ergodic state and absorbing state.
- (c) Explain what is meant by recurrent event and state the properties associated with it.
- (d) Show that for any $r < s < n$, the Chapman-Kolmogorov relationship is given as $P_{r,n} = P_{r,s} P_{s,n}$