University of Swaziland

Supplementary Examination Paper 2006

Title of Paper: Probability Theory I

Course Code: ST 201

Time Allowed: Three (3) Hours

Instructions: Answer any five questions (each Question carries 20 marks)

Requirements: Use of Calculator is allowed.

Please do not open this paper until permission has been granted by the Invigilator.

QUESTION ONE

(a) Describe circumstances in which a Poisson distribution of probabilities may arise.

(b) The average number of telephone calls handled by an exchange between 2.0pm and 4.0pm is 240.At that time of day the probability of a call per unit time is constant. Considering the time interval between 3.0pm and 3.02pm, show that:

(i) the probability of exactly three calls being put through the exchange is about 0.2;

(ii) it is more likely than not that more than three calls will be handled.

(c) The number of minor injuries a football coach can expect during the course of the game is a random variable having the Poisson distribution with $\lambda=4.4$. Find the probability that during the course of a game there will at most be three injuries.

(3+4+4+4+5)Marks

QUESTION TWO

(a) Sugar is packed in cardboard cartons and it is found that the weight is normally distributed with mean weight of 500 grams and standard

deviation of 10 grams. Find the probability that a packet chosen at random has a weight:

(i) Less than 503 grams.
(ii) Greater than 520 grams.
(iii) Lies between 510 grams and 515 grams.

(b) Of the items produced in a certain factory, 3% are defective. A sample of 25 items is selected for inspection .By using the Binomial distribution:

(i) What is the probability that exactly 4 defectives are found?
(ii) What is the probability that three or more defectives are found?

(4+4+4+4+4)Marks

QUESTION THREE

Suppose a fair coin is tossed four times. Let X be the number of heads obtained, A is the event that at least two heads are obtained and B is the event that even numbers of heads are obtained.

(i) Obtain the probability distribution for X.
(ii) Write out the elements of sets A and B.
(iii) Find P (A), P (B) and P (A or B).

(iv) Are events A and B mutually exclusive or not?

(6+3+3+2+2+2+2)Marks

QUESTION FOUR

A bag contains 8 balls, identical except for colour, of which 5 are red, and 3 are white. A man draws two balls at random, what is the probability that:

(a) (i) One of the balls shown is white and other is red.

(ii) Both balls are of the same colour.

(b) If three balls are drawn at random, what is the probability that exactly 2 balls are red ?.

(c) What would be the probability in question 4(a) if the first ball drawn is replaced before the second one is drawn.

(4+4+4+4+4)Marks

QUESTION FIVE

(a) The random variable X has the cumulative distribution function given as

$F_X(x) = 0, x < 0$

$= \frac{1}{2}, 0 \le x < 2$

$= \frac{5}{6}, 2 \le x < 3$

$= 1, x \ge 3$

Find the probability density function for X.

(b) If X and Y are jointly continuous with a joint probability density function

$$f(X,Y)=\begin{cases} 2(X+Y-2XY^2); 0<x<1, 0<y<1 \\ 0, elsewhere \end{cases}.$$

Determine: the marginal density functions for X and Y.

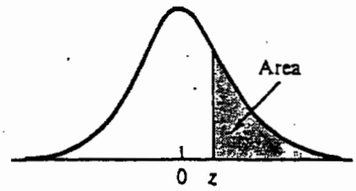
(2+2+2+2+6+6)Marks

QUESTION SIX

A calculator manufacturer buys the same integrated circuit from three different suppliers, named I, II and III. From past experience, 1percent of the circuits supplied by I have been defective, 3percent of those supplied by II have been defective and 4percent of those supplied by III have been defective. Granted that this manufacturer buys 30percent of his circuits from I, 50percent from II and the rest from III. Compute the probability that an integrated circuit checked just before final assembly into a calculator is found to be defective.

(20marks)

Normal curve areas
Standard normal probability in right-hand tail (for negative values of z areas are found by symmetry)



	Second decimal place of z									
z	*.00*	*.01*	*.02*	*.03*	*.04*	*.05*	*.06*	*.07*	*.08*	*.09*
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

Selected χ^2-values

$\chi^2_{.05}$	df	$\chi^2_{.05}$	df
3.84	1	11.07	5
5.99	2	12.59	6
7.81	3	14.07	7
9.49	4	15.51	8

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

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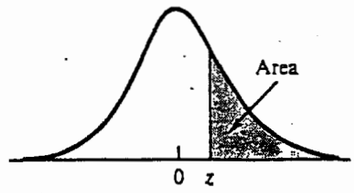

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