

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2006

TITLE OF PAPER : LINEAR STATISTICAL METHODS
COURSE CODE : ST204
TIME ALLOWED : 2 (TWO) HOURS
**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**
**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[15 + 6 + 4 marks]

- 1.1 State the formal statement of the Simple Linear Regression Model. Find the point estimators of two regression coefficients using Method of Least Squares.
- 1.2 Discuss the importance of the assumption of normality in regression model.
- 1.3 State the sampling distribution of regression coefficients, β_0 and β_1 .

QUESTION TWO.

[2 + 2 + 4 + 1 + 5 + 5 + 3 + 3 marks]

The following output was obtained from running the model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ using SPSS:

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	26.450	1	26.450	37.451	.000
	Residual	5.650	8	.706		
	Total	32.100	9			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	10.100	.797		12.668	.000
	X	1.150	.188	.908	6.120	.000

- 2.1 State the fitted regression line.
- 2.2 State the null and alternative hypotheses for the F-test of the above ANOVA table.
- 2.3 Perform the F-test and clearly state the conclusion.
- 2.4 What is the estimated value of σ^2 ?
- 2.5 Test $\beta_0 = 10$ against $\beta_0 \neq 10$ at $\alpha = 0.05$.
- 2.6 Test $\beta_1 = 1$ against $\beta_1 > 1$ at $\alpha = 0.01$.
- 2.7 Compute coefficient of determination, r^2 and interpret the result.
- 2.8 Compute coefficient of correlation, r and explain the nature and strength of the relationship between dependent and independent variables.

QUESTION THREE.

[6 + 4 + 1 + 4 + 1 + 1 + 8 marks]

- 3.1 Define the Factor Effects Model for two-factor studies.
- 3.2 Suppose there is only one case for each treatment. What would be the new form of the model defined in question 3.1? Explain.
- 3.3 Hyperactivity in children usually is treated by counselling, or by drugs, or both. An experiment was designed to evaluate the effectiveness of these different treatments. The data on the attention span, that is, how long each child was able to concentrate on a specific task were collected and analyzed using SPSS. The following ANOVA table is a part of the output from that analysis:

ANOVA TABLE

Source of Variation	Sum of Squares	df	Mean Square	F
Between treatments	102.25	3	34.08	5.31
Factor A	6.13	1	6.13	0.95
Factor B	78.13	1	78.13	12.17
A X B	18.00	1	18.00	2.80
Within treatments	179.75	28	6.42	
Total	282.00	31		

Answer the following questions:

- How many children were used in this experiment?
- What are the treatments in this experiment?
- Which one is the Factor A?
- Which one is the Factor B?
- Using 5% level of significance, describe only the conclusions (based on F-test) in terms of effectiveness of the drug, effectiveness of the counselling and effectiveness of their interaction.

QUESTION FOUR.

[8 + 4 + 2 + 6 + 5 marks]

We believe the existence of a relationship between years of education and salary potential. The following data represent the current annual salary and the total number of years of higher education (after high school) for a sample of 10 men who have been employed for 5 years. Assume that the estimated value of σ^2 is 5.45 .

Annual Salary (in E1000)	20	18	14	28	11	23	30	15	31	10
Years of Higher Education	4	4	2	6	0	5	7	4	8	0

- Fit the regression line, $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$.
- Interpret the estimated values of β_1 and β_2 .
- What would be the annual salary of a men who worked 5 years but never been in any school?
- Test the goodness of fit of the model. Write and explain clearly all the steps to conduct the test.
- Construct a 95% confidence interval for β_1 and interpret the interval.

QUESTION FIVE.

[10 + 3 + 12 marks]

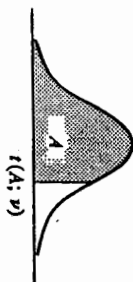
- 5.1 State the Cell Means Model for single-factor studies with all its important features.
- 5.2 A researcher evaluating the effects of a drug designed an experiment using three different drug doses (small, medium, and large). A separate sample of subjects was tested for each drug dose, and the researcher obtained the following scores:

Small	Medium	Large
14	16	24
19	20	18
13	15	20
17	18	18
18	19	22
21	23	24

- a. Identify the dependent variable, factor studied and factor levels.
- b. Complete the computation of the ANOVA table and conduct the F test. Clearly state all the steps in the test including the conclusion.

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



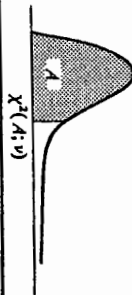
ν	A					
	.60	.70	.80	.85	.90	.95
1	0.325	0.727	1.376	1.963	3.078	6.314
2	0.289	0.617	1.061	1.386	1.886	2.920
3	0.277	0.584	0.978	1.250	1.638	2.353
4	0.271	0.569	0.941	1.190	1.533	2.132
5	0.267	0.559	0.920	1.156	1.476	2.015
6	0.265	0.553	0.906	1.134	1.440	1.943
7	0.263	0.549	0.896	1.119	1.415	1.895
8	0.262	0.546	0.889	1.108	1.397	1.860
9	0.261	0.543	0.883	1.100	1.383	1.833
10	0.260	0.542	0.879	1.093	1.372	1.812
11	0.260	0.540	0.876	1.088	1.363	1.796
12	0.259	0.539	0.873	1.083	1.356	1.782
13	0.259	0.537	0.870	1.079	1.350	1.771
14	0.258	0.537	0.868	1.076	1.345	1.761
15	0.258	0.536	0.866	1.074	1.341	1.753
16	0.258	0.535	0.865	1.071	1.337	1.746
17	0.257	0.534	0.863	1.069	1.333	1.740
18	0.257	0.534	0.862	1.067	1.330	1.734
19	0.257	0.533	0.861	1.066	1.328	1.729
20	0.257	0.533	0.860	1.064	1.325	1.725
21	0.257	0.532	0.859	1.063	1.323	1.721
22	0.256	0.532	0.858	1.061	1.321	1.717
23	0.256	0.532	0.858	1.060	1.319	1.714
24	0.256	0.531	0.857	1.059	1.318	1.711
25	0.256	0.531	0.856	1.058	1.316	1.708
26	0.256	0.531	0.856	1.058	1.315	1.706
27	0.256	0.531	0.855	1.057	1.314	1.703
28	0.256	0.530	0.855	1.056	1.313	1.701
29	0.256	0.530	0.854	1.055	1.311	1.699
30	0.256	0.530	0.854	1.055	1.310	1.697
40	0.255	0.529	0.851	1.050	1.303	1.684
60	0.254	0.527	0.848	1.045	1.296	1.671
120	0.254	0.526	0.845	1.041	1.289	1.658
∞	0.253	0.524	0.842	1.036	1.282	1.645

TABLE A.2 (concluded) Percentiles of the *t* Distribution

ν	A									
	.98	.985	.99	.9925	.995	.9975	.9995			
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590			
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598			
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924			
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610			
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869			
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959			
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408			
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041			
9	2.398	2.572	2.821	2.998	3.250	3.690	4.781			
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587			
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437			
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318			
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221			
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140			
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073			
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015			
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965			
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922			
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883			
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849			
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819			
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792			
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768			
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745			
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725			
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707			
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690			
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674			
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659			
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646			
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551			
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460			
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373			
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291			

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

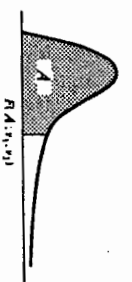


ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.00157	0.00982	0.02393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{F(1-A; \nu_2, \nu_1)}{1 - A}$$



TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
1	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
.95	161	200	216	225	230	234	237	239	241
.975	648	800	864	900	922	937	948	957	963
.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
.995	16,211	20,000	21,615	22,500	23,036	23,437	23,715	23,925	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.84
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
1	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20		
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3		
.95	242	244	246	248	249	250	252	253	254		
.975	969	977	985	993	997	1,001	1,010	1,014	1,018		
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366		
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464		
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620		
2	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44		
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49		
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5		
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5		
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5		
.995	199	199	199	199	199	199	199	199	199		
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5		
3	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27		
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13		
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53		
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9		
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1		
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8		
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5		
4	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19		
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76		
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63		
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26		
.99	14.5	13.9	13.4	13.2	13.1	13.0	12.7	12.6	12.5		
.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3		
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1		
5	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15		
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11		
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37		
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02		
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02		
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1		
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8		
6	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12		
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72		
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67		
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85		
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88		
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88		
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7		
7	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10		
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47		
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23		
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14		
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65		
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08		
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7		

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09	
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29	
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93	
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67	
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08	
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07	
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06	
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54	
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06	
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72	
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	
15	.50	0.977	0.989	1.00	1.01	1.02	1.03	1.04	1.04	1.05	
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87	
20	.50	0.966	0.977	0.989	1.00	1.01	1.02	1.03	1.03	1.03	
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09	
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03	
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.50	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85
.90	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.95	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.975	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.99	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.995	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
.999									
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.50	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.90	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.95	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.975	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.99	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.995	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
.999									
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.50	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.90	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.95	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.975	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.99	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.995	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
.999									
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.50	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.90	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.95	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.975	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.99	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.995	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10
.999									

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
30	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02		
.50	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46		
.90	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62		
.95	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79		
.975	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01		
.99	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18		
.995	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59		
.999											
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01		
.50	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29		
.90	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39		
.95	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48		
.975	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60		
.99	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69		
.995	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89		
.999											
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01		
.50	1.63	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19		
.90	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25		
.95	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31		
.975	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38		
.99	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43		
.995	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54		
.999											
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00		
.50	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00		
.90	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00		
.95	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00		
.975	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00		
.99	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00		
.995	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00		
.999											

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