

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2006

TITLE OF PAPER : LINEAR STATISTICAL METHODS
COURSE CODE : ST204
TIME ALLOWED : 2 (TWO) HOURS
**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**
**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[6 + 6 + 13 marks]

- 1.1 State a Simple Linear Regression Model; define all the terms present in the model.
- 1.2 Discuss some important features of the regression model stated in 1.1.
- 1.3 Using the Method of Least Squares, find the point estimators of the slope and intercept of the regression model defined in 1.1

QUESTION TWO.

[4 + 8 + 2 + 3 + 8 marks]

The following output was obtained from running the model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ using SPSS:

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	278.90	1	278.90	166.01	.000
	Residual	16.77	10	1.68		
	Total	295.67	11			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	-1.234	1.080		-1.14	.013
	X	23.404	1.815	19.908	12.90	.000

- 2.1 State the fitted regression line.
- 2.2 Test the goodness of fit of the model. Write and explain clearly all the steps to conduct the test.
- 2.3 What is the estimated value of σ^2 ?
- 2.4 Test $\beta_1 = 0$ against $\beta_1 \neq 0$ at $\alpha = 0.05$.
- 2.5 Test $\beta_1 = 25$ against $\beta_1 \neq 25$ at $\alpha = 0.05$.

QUESTION THREE.

[15 + 10 marks]

A teacher wishes to try three different teaching techniques, to replace the traditional teaching practices, in order to improve the students' understanding level of Mathematics. The students are randomly assigned to three teaching techniques. After one complete term, the increase in each student's mathematics marks from the previous term marks is recorded.

Method A	Method B	Method C
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4

3.1 Obtain the analysis of variance table.

3.2 Using 5% level of significance, test the effectiveness of the teaching methods.

QUESTION FOUR.

[10 + 4 + 3 + 8 marks]

The head of the local traffic department is concerned about the increase in traffic accidents in recent years. He is convinced that the increased number of bars in the town has a direct influence on the accident rate. He has compiled the following information over the past 9 years:

Number accidents	18	14	17	22	27	31	23	26	30
Number of bars	6	6	7	8	9	9	8	9	10

4.1 Fit the regression line, $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$.

4.2 Interpret the estimated values of β_1 and β_2 .

4.3 What would be the number of accidents if two bars were to close down?

4.4 Compute the correlation coefficient, r and check whether the value of r is confirming the claim of the head of the traffic department.

QUESTION FIVE.

[3 + 5 + 12 + 5 marks]

A clinical psychologist wished to compare three methods for reducing hostility levels in university students. A certain test was used to measure the degree of hostility. The following ANOVA table was obtained from the data:

ANOVA Table

Source of variation	df	SS	MS	F
Between samples	2	8.64	4.32	1.10
Within samples	8	31.28	3.91	
Total	10	39.92		

5.1 What is the dependent variable here?

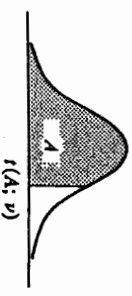
5.2 Identify the factor studied. What are the factor levels?

5.3 Conduct a test to determine if there are differences among mean scores for the three methods. Use a level of significance of $\alpha = 0.05$.

5.4 When informed of the findings, the chief psychologist said: "Well, I told you before that these methods are not worth to test" Does this comment follow from the findings of the experiment? Explain.

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t \leq t(A; \nu)\} = A$



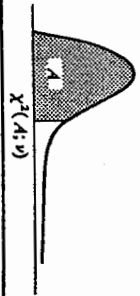
ν	A													
	.60	.70	.80	.85	.90	.95	.975	.985	.99	.995				
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.253	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.612	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.517	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.449	2.634	2.896	3.085	3.355	3.833	5.041
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.398	2.574	2.821	2.998	3.250	3.690	4.781
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.359	2.527	2.764	2.932	3.169	3.581	4.587
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.328	2.491	2.718	2.879	3.106	3.497	4.437
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.303	2.461	2.681	2.836	3.055	3.428	4.318
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.282	2.436	2.650	2.801	3.012	3.372	4.221
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160	2.264	2.415	2.624	2.771	2.977	3.326	4.140
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.249	2.397	2.602	2.746	2.947	3.286	4.073
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.235	2.382	2.583	2.724	2.921	3.252	4.015
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.224	2.368	2.567	2.706	2.898	3.222	3.965
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.214	2.356	2.552	2.689	2.878	3.197	3.922
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.205	2.346	2.539	2.674	2.861	3.174	3.883
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.197	2.336	2.528	2.661	2.845	3.153	3.849
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.189	2.328	2.518	2.649	2.831	3.135	3.819
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.183	2.320	2.508	2.639	2.819	3.119	3.792
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.177	2.313	2.500	2.629	2.807	3.104	3.768
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.172	2.307	2.492	2.620	2.797	3.091	3.745
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.167	2.301	2.485	2.612	2.787	3.078	3.725
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.162	2.296	2.479	2.605	2.779	3.067	3.707
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.158	2.291	2.473	2.598	2.771	3.057	3.690
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.154	2.286	2.467	2.592	2.763	3.047	3.674
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.150	2.282	2.462	2.586	2.756	3.038	3.659
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.147	2.278	2.457	2.581	2.750	3.030	3.646
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.123	2.250	2.423	2.542	2.704	2.971	3.551
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.099	2.223	2.390	2.504	2.660	2.915	3.460
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.076	2.196	2.358	2.468	2.617	2.860	3.373
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.054	2.170	2.326	2.432	2.576	2.807	3.291
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960							

TABLE A.2 (concluded) Percentiles of the *t* Distribution

ν	A													
	.98	.985	.99	.9925	.995	.9975	.9995	.9995	.9995	.9995				
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590							
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598							
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924							
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610							
5	2.612	3.003	3.365	3.634	4.032	4.773	6.869							
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959							
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408							
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041							
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781							
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587							
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437							
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318							
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221							
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140							
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073							
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015							
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965							
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922							
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883							
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849							
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819							
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792							
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768							
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745							
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725							
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707							
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690							
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674							
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659							
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646							
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551							
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460							
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373							
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291							

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

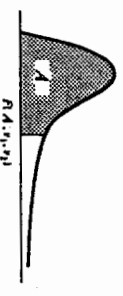


ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.02982	0.04973	0.08158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.03006	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	33.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{F(1-A; \nu_2, \nu_1)}{1 - A; \nu_2, \nu_1}$$



TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		1	2	3	4	5	6	7	8	9	
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03	
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	
	.95	161	200	216	225	230	234	237	239	241	
	.975	648	800	864	900	922	937	948	957	963	
	.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022	
	.995	16,211	20,000	21,613	22,500	23,056	23,437	23,715	23,925	24,091	
	.999	403,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	
	2	.50	0.657	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
		.90	8.33	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975		38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	
.99		98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	
.995		199	199	199	199	199	199	199	199	199	
.999		998.3	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	
3		.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
		.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	
	.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	
	.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	
	4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
		.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975		12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	
.99		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	
.995		31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	
.999		74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	
5		.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
		.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	
	.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	
	.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	
	6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
		.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
		.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	
.99		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
.995		18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	
.999		35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	
7		.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
		.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
		.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	
	.99	12.2	9.53	8.45	7.85	7.46	7.19	6.99	6.84	6.72	
	.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51	
	.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	A	Numerator df													
		10	12	15	20	24	30	60	120	∞					
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20					
	.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3					
	.95	242	244	246	248	249	250	252	253	254					
	.975	969	977	985	993	997	1,001	1,010	1,018	1,024					
	.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,359	6,396					
	.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464					
	.999	603,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620					
	2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44				
		.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49				
		.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5				
.975		39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5					
.99		99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5					
.995		199	199	199	199	199	199	199	199	199					
.999		999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5					
3		.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27				
		.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13				
		.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53				
	.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9					
	.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1					
	.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8					
	.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5					
	4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19				
		.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76				
		.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63				
.975		8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26					
.99		14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5					
.995		21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3					
.999		48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1					
5		.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15				
		.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11				
		.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37				
	.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02					
	.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02					
	.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1					
	.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8					
	6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12				
		.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72				
		.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67				
.975		5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85					
.99		7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88					
.995		10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88					
.999		18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7					
7		.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10				
		.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47				
		.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23				
	.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14					
	.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65					
	.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08					
	.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7					

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.39	3.36
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.83	6.37	6.18	6.03	5.91
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.18	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26

Den. df	Numerator df													
	10	12	15	20	24	30	60	120	∞					
8	.50	1.02	1.03	1.04	1.05	1.06	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.37	2.36	2.35	2.34	2.33	2.32	2.31
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93	2.91	2.89	2.87	2.85
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67	3.63	3.59	3.55	3.51
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	4.79	4.71	4.63	4.55
9	.50	1.01	1.02	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.11	1.12	1.13
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	2.15	2.14	2.13	2.12
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	2.68	2.65	2.62	2.59
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	3.28	3.23	3.18	3.13
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	4.24	4.16	4.08	4.00
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06	2.05	2.04	2.03	2.02
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54	2.50	2.46	2.42	2.38
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	3.03	2.97	2.91	2.85
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	3.84	3.75	3.66	3.57
12	.50	0.989	1.00	1.01	1.02	1.03	1.05	1.05	1.06	1.07	1.08	1.09	1.10	1.11
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	1.88	1.86	1.84	1.82
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	2.27	2.24	2.21	2.18
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.78	2.72	2.66	2.60	2.54	2.48
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	3.28	3.20	3.12	3.04
15	.50	0.977	0.989	1.00	1.01	1.02	1.03	1.05	1.05	1.06	1.07	1.08	1.09	1.10
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	1.74	1.72	1.70	1.68
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	2.03	1.99	1.95	1.91
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	2.34	2.28	2.22	2.16
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87	2.78	2.69	2.60	2.51
20	.50	0.966	0.977	0.989	1.00	1.01	1.02	1.03	1.05	1.05	1.06	1.07	1.08	1.09
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	1.59	1.57	1.55	1.53
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	1.78	1.73	1.67	1.61
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09	2.03	1.97	1.91	1.84
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	2.33	2.24	2.15	2.06
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	1.50	1.47	1.44	1.41
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	1.67	1.61	1.55	1.49
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	1.88	1.81	1.74	1.67
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	2.12	2.03	1.94	1.84

TABLE A.3 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.50	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.90	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.95	5.37	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.975	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.50	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.90	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.95	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.975	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.50	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.90	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.95	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.975	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.50	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.90	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.95	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.975	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
30	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02		
.50	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46		
.90	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62		
.95	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79		
.975	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01		
.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18		
.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59		
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01		
.50	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29		
.90	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39		
.95	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48		
.975	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60		
.995	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69		
.999	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89		
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01		
.50	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19		
.90	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25		
.95	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31		
.975	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38		
.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43		
.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54		
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00		
.50	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00		
.90	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00		
.95	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00		
.975	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00		
.995	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00		
.999	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00		

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