UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2006

TITLE OF PAPER:	**INFERENTIAL STATISTICS**
COURSE CODE :	**ST 232**
TIME ALLOWED :	**TWO (2) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS FIVE QUESTIONS.** **ANSWER ANY FOUR(4) QUESTIONS.** **EACH QUESTION CARRIES 15 MARKS.**
REQUIREMENTS:	**Scientific Calculator**

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QUESTION ONE

(a) The random variable $Z_1, Z_2, \ldots\ldots, Z_n$ is from a normal distribution with mean $\frac{(n-1)S^2}{\sigma^2}\mu$ and variance σ^2. If $\sum_{i=1}^{n}(Z_i-\bar{Z})^2 \Big/ \sigma^2 = \frac{(n-1)S^2}{\sigma^2}$, show that for n=2, $\frac{(n-1)S^2}{\sigma^2}$ has a chi-square distribution with 1-degree of freedom.

(b) If $\hat{\theta}$ is an estimator of θ and B is the bias, show that the mean square error of $\hat{\theta}$, is MSE($\hat{\theta}$)=Var($\hat{\theta}$) - B^2. Where Var($\hat{\theta}$) is the variance of $\hat{\theta}$.

(6+9)Marks

QUESTION TWO

A study was conducted to assess the amount of chemical residues found in the brain tissue of pelicans. In a test for DDT, random samples of n_1=10 Juveniles and n_2=13 Nestlings gave the results as:

JUVENILES	NESTLINGS
$\bar{y}_1$=0.041	$\bar{y}_2$=0.026
s_1=0.017	s_2=0.006

Test the hypothesis that there is no difference between mean amounts of DDT found in Juveniles and Nestlings against the alternative that the Juveniles have a larger mean at 5% level of significance.

(15)Marks

QUESTION THREE

Let $Y_1, Y_2, \ldots\ldots, Y_n$ be the order statistics of a random sample of size n from a distribution of the continuous type. Given

that $G_r(y) = \sum_{k=r}^{n} \binom{n}{k} [F(y)]^k (1-F(y))^{n-k}$, prove

that $g_r(y) = \frac{n![F(y)]^{r-1}}{(r-1)!(n-r)!} [1-F(y)]^{n-r} (f(y))$, for a<y<b.

(15)Marks

QUESTION FOUR

Let P be the forced vital capacity (the volume of air a person can expel from his lungs) for a male freshman. Seventeen observations of P, which have been recorded, are:

3.7 3.8 4.0 4.3 4.7 4.8 4.9 5.0 5.2 5.4 5.6 5.6 5.6 5.7 6.2 6.8 7.6.

(a) Find the median, the first and third quartiles for this ordered distribution.

(b) Find the 35^{th} and 65^{th} percentiles for this ordered distribution.

(3+3+3+3+3)Marks

QUESTION FIVE

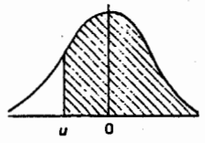
(a) Given a random sample $x_1, x_2, \ldots\ldots\ldots x_n$ from a population with $N(\mu_1, \sigma^2)$ and another sample $y_1, y_2, \ldots\ldots\ldots, y_n$ from a population with $N(\mu_2, \sigma^2)$. Derive an expression for 90% confidence for $\mu_1 - \mu_2$, when σ^2 is known.

(b) The random sample $z_1, z_2, \ldots\ldots\ldots, z_n$ has mean $E(z_i) = \mu$ and variance $V(z_i) = \sigma^2$. Prove that $S^2 = \dfrac{\sum_{i=1}^{n}(z - \bar{z})^2}{n-1}$ is an unbiased estimation for σ^2.

(9+6)Marks

The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-x^2/2}\, dx$,

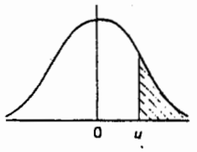
the probability that $U > u$, where $U \sim N(0,1)$.



−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00	u
0.99997	0.99997	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99995	0.99995	−3.9
0.99995	0.99995	0.99995	0.99994	0.99994	0.99994	0.99994	0.99993	0.99993	0.99993	−3.8
0.99992	0.99992	0.99992	0.99992	0.99991	0.99991	0.99990	0.99990	0.99990	0.99989	−3.7
0.99989	0.99988	0.99988	0.99987	0.99987	0.99986	0.99986	0.99985	0.99985	0.99984	−3.6
0.99983	0.99983	0.99982	0.99981	0.99981	0.99980	0.99979	0.99978	0.99978	0.99977	−3.5
0.99976	0.99975	0.99974	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99966	−3.4
0.99965	0.99964	0.99962	0.99961	0.99960	0.99958	0.99957	0.99955	0.99953	0.99952	−3.3
0.99950	0.99948	0.99946	0.99944	0.99942	0.99940	0.99938	0.99936	0.99934	0.99931	−3.2
0.99929	0.99926	0.99924	0.99921	0.99918	0.99916	0.99913	0.99910	0.99906	0.99903	−3.1
0.99900	0.99896	0.99893	0.99889	0.99886	0.99882	0.99878	0.99874	0.99869	0.99865	−3.0
0.99861	0.99856	0.99851	0.99846	0.99841	0.99836	0.99831	0.99825	0.99819	0.99813	−2.9
0.99807	0.99801	0.99795	0.99788	0.99781	0.99774	0.99767	0.99760	0.99752	0.99744	−2.8
0.99736	0.99728	0.99720	0.99711	0.99702	0.99693	0.99683	0.99674	0.99664	0.99653	−2.7
0.99643	0.99632	0.99621	0.99609	0.99598	0.99585	0.99573	0.99560	0.99647	0.99534	−2.6
0.99520	0.99506	0.99492	0.99477	0.99461	0.99446	0.99430	0.99413	0.99396	0.99379	−2.5
0.99361	0.99343	0.99324	0.99305	0.99286	0.99266	0.99245	0.99224	0.99202	0.99180	−2.4
0.99158	0.99134	0.99111	0.99086	0.99061	0.99036	0.99010	0.98983	0.98956	0.98928	−2.3
0.98899	0.98870	0.98840	0.98809	0.98778	0.98745	0.98713	0.98679	0.98645	0.98610	−2.2
0.98574	0.98537	0.98500	0.98461	0.98422	0.98382	0.98341	0.98300	0.98257	0.98214	−2.1
0.98169	0.98124	0.98077	0.98030	0.97982	0.97932	0.97882	0.97831	0.97778	0.97725	−2.0
0.97670	0.97615	0.97558	0.97500	0.97441	0.97381	0.97320	0.97257	0.97193	0.97128	−1.9
0.97062	0.96995	0.96926	0.96856	0.96784	0.96712	0.96638	0.96562	0.96485	0.96407	−1.8
0.96327	0.96246	0.96164	0.96080	0.95994	0.95907	0.95818	0.95728	0.95637	0.95543	−1.7
0.95449	0.95352	0.95254	0.95154	0.95053	0.94950	0.94845	0.94738	0.94630	0.94520	−1.6
0.94408	0.94296	0.94179	0.94062	0.93943	0.93822	0.93699	0.93574	0.93448	0.93319	−1.5
0.93189	0.93056	0.92922	0.92785	0.92647	0.92507	0.92364	0.92220	0.92073	0.91924	−1.4
0.91774	0.91621	0.91466	0.91308	0.91149	0.90988	0.90824	0.90658	0.90490	0.90320	−1.3
0.90147	0.89973	0.89796	0.89617	0.89435	0.89251	0.89065	0.88877	0.88686	0.88493	−1.2
0.88298	0.88100	0.87900	0.87698	0.87493	0.87286	0.87076	0.86864	0.86650	0.86433	−1.1
0.86214	0.85993	0.85769	0.85543	0.85314	0.85083	0.84860	0.84614	0.84375	0.84134	−1.0
0.83891	0.83646	0.83398	0.83147	0.82894	0.82639	0.82381	0.82121	0.81859	0.81594	−0.9
0.81327	0.81057	0.80785	0.80511	0.80234	0.79965	0.79673	0.79389	0.79103	0.78814	−0.8
0.78624	0.78230	0.77935	0.77637	0.77337	0.77035	0.76731	0.76424	0.76115	0.75804	−0.7
0.75490	0.75175	0.74857	0.74537	0.74215	0.73891	0.73565	0.73237	0.72907	0.72575	−0.6
0.72240	0.71904	0.71566	0.71226	0.70884	0.70540	0.70194	0.69847	0.69497	0.69146	−0.5
0.68793	0.68439	0.68082	0.67724	0.67364	0.67003	0.66640	0.66276	0.65910	0.65542	−0.4
0.65173	0.64803	0.64431	0.64058	0.63683	0.63307	0.62930	0.62552	0.62172	0.61791	−0.3
0.61409	0.61026	0.60642	0.60257	0.59871	0.59483	0.59095	0.58706	0.58317	0.57926	−0.2
0.57535	0.57142	0.56750	0.56356	0.55962	0.55567	0.56172	0.54776	0.54380	0.53983	−0.1
0.53586	0.53188	0.52790	0.52392	0.51994	0.51595	0.51197	0.50798	0.50399	0.50000	−0.0

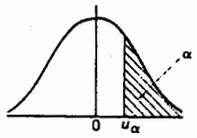
The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-x^2/2}\, dx$,

the probability that $U > u$, where $U \sim N(0,1)$.



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00667	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00416	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

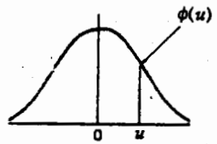
The u_α values tabulated are such that $\Pr(U > u_\alpha) = \alpha$, where $U \sim N(0,1)$



α	u_α	α	u_α	α	u_α	α	u_α
0.50	0.00000	0.34	0.41246	0.18	0.91537	0.025	1.96000
0.49	0.02507	0.33	0.43991	0.17	0.95416	0.020	2.05375
0.48	0.05015	0.32	0.46770	0.16	0.99446	0.010	2.32635
0.47	0.07527	0.31	9.49585	0.15	1.03643	0.009	2.36562
0.46	0.10004	0.30	0.52440	0.14	1.08032	0.008	2.40891
0.45	0.12566	0.29	0.55338	0.13	1.12639	0.007	2.46726
0.44	0.15097	0.28	0.58284	0.12	1.17499	0.006	2.51214
0.43	0.17637	0.27	0.61281	0.11	1.22653	0.005	2.57583
0.42	0.20189	0.26	0.64335	0.10	1.28155	0.004	2.65207
0.41	0.22754	0.25	0.67449	0.09	1.34076	0.003	2.74778
0.40	0.25335	0.24	0.70630	0.08	1.40507	0.002	2.87816
0.39	0.27932	0.23	0.73885	0.07	1.47579	0.001	3.09023
0.38	0.30548	0.22	0.77219	0.06	1.55477	0.0005	3.29053
0.37	0.33185	0.21	0.80642	0.05	1.64485	0.0001	3.71902
0.36	0.35846	0.20	0.84162	0.04	1.75069	0.00005	3.89060
0.35	0.38532	0.19	0.87790	0.03	1.88079	0.00001	4.26489

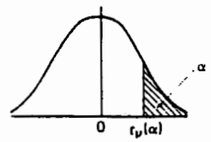
Table 6 ORDINATES OF THE STANDARDISED NORMAL DISTRIBUTION

The function tabulated is $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$.



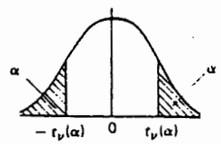
u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.39894	0.39695	0.39104	0.38139	0.36827	0.35207	0.33322	0.31225	0.28969	0.26609
1.0	0.24197	0.21785	0.19419	0.17137	0.14973	0.12952	0.11092	0.09405	0.07895	0.06562
2.0	0.05399	0.04398	0.03547	0.02833	0.02239	0.01753	0.01358	0.01042	0.00792	0.00595
3.0	0.00443	0.00327	0.00238	0.00172	0.00123	0.00087	0.00061	0.00042	0.00029	0.00020
4.0	0.00013	0.00009	0.00006	0.00004	0.00002	0.00002	0.00001	0.00001	0.00000	0.00000

ONE-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)) = \alpha$,
for ν degrees of freedom.

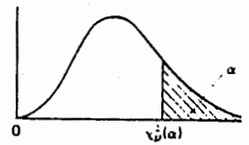
TWO-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)$ or $T_\nu < -t_\nu(\alpha)) = 2\alpha$,
for ν degrees of freedom.

ν	α = 0.4 2α = 0.8	0.25 0.5	0.1 0.2	0.05 0.1	0.025 0.05	0.01 0.02	0.005 0.01	0.0025 0.005	0.001 0.002	0.0005 0.001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.320	318.310	636.620
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.966
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The values tabulated are $\chi^2_\nu(\alpha)$, where
$\Pr(\chi^2_\nu > \chi^2_\nu(\alpha)) = \alpha$, for ν degrees of freedom.



For $\nu > 30$ take $\chi^2_\nu(\alpha) = \nu\left[1 - \frac{2}{9\nu} + u_\alpha\sqrt{\frac{2}{9\nu}}\right]^3$ where u_α is such that $\Pr(U > u_\alpha) = \alpha$, and $U \sim N(0,1)$.

0.995	0.990	0.975	0.950	0.900	0.750	0.500	α / ν
392704.10^{-10}	157088.10^{-9}	982069.10^{-9}	393214.10^{-8}	0.0157908	0.1015308	0.454936	1
0.0100251	0.0201007	0.0506356	0.102587	0.210721	0.575364	1.38629	2
0.0717218	0.114832	0.215795	0.351846	0.584374	1.212534	2.36597	3
0.206989	0.297109	0.484419	0.710723	1.063623	1.92256	3.35669	4
0.411742	0.554298	0.831212	1.145476	1.61031	2.67460	4.35146	5
0.675727	0.872090	1.23734	1.63538	2.20413	3.45460	5.34812	6
0.989256	1.239043	1.68987	2.16735	2.83311	4.25485	6.34581	7
1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	8
1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	9
2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	10
2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.3410	11
3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.3403	12
3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.3398	13
4.07467	4.66043	5.62873	6.57063	7.78953	10.1653	13.3393	14
4.60092	5.22935	6.26214	7.26094	8.54676	11.0365	14.3389	15
5.14221	5.81221	6.90766	7.96165	9.31224	11.9122	15.3385	16
5.69722	6.40776	7.56419	8.67176	10.0852	12.7919	16.3382	17
6.26480	7.01491	8.23075	9.39046	10.8649	13.6753	17.3379	18
6.84397	7.63273	8.90652	10.1170	11.6509	14.5620	18.3377	19
7.43384	8.26040	9.59078	10.8508	12.4426	15.4518	19.3374	20
8.03365	8.89720	10.28293	11.5913	13.2396	16.3444	20.3372	21
8.64272	9.54249	10.9823	12.3380	14.0415	17.2396	21.3370	22
9.26043	10.19567	11.6886	13.0905	14.8480	18.1373	22.3369	23
9.88623	10.8564	12.4012	13.8484	15.6587	19.0373	23.3367	24
10.5197	11.5240	13.1197	14.6114	16.4734	19.9393	24.3366	25
11.1602	12.1981	13.8439	15.3792	17.2919	20.8434	25.3365	26
11.8076	12.8785	14.5734	16.1514	18.1139	21.7494	26.3363	27
12.4613	13.5647	15.3079	16.9279	18.9392	22.6572	27.3362	28
13.1211	14.2565	16.0471	17.7084	19.7677	23.5666	28.3361	29
13.7867	14.9535	16.7908	18.4927	20.5992	24.4776	29.3360	30
20.7065	22.1643	24.4330	26.5093	29.0506	33.6603	39.3353	40
27.9907	29.7067	32.3574	34.7643	37.6886	42.9421	49.3349	50
35.5345	37.4849	40.4817	43.1880	46.4589	52.2938	59.3347	60
43.2752	45.4417	48.7576	51.7393	55.3289	61.6983	69.3345	70
51.1719	53.5401	57.1532	60.3915	64.2778	71.1446	79.3343	80
59.1963	61.7541	65.6466	69.1260	73.2911	80.6247	89.3342	90
67.3276	70.0649	74.2219	77.9295	82.3581	90.1332	99.3341	100

ν \ α	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944	10.828
2	2.77259	4.60517	5.99146	7.37776	9.21034	10.5966	13.816
3	4.10834	6.25139	7.81473	9.34840	11.3449	12.8382	16.266
4	5.38527	7.77944	9.48773	11.1433	13.2767	14.8603	18.467
5	6.62568	9.23636	11.0705	12.8325	15.0863	16.7496	20.515
6	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476	22.458
7	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777	24.322
8	10.2189	13.3616	15.5073	17.5345	20.0902	21.9550	26.125
9	11.3888	14.6837	16.9190	19.0228	21.6660	23.5894	27.877
10	12.5489	15.9872	18.3070	20.4832	23.2093	25.1882	29.588
11	13.7007	17.2750	19.6751	21.9200	24.7250	26.7568	31.264
12	14.8454	18.5493	21.0261	23.3367	26.2170	28.2995	32.909
13	15.9839	19.8119	22.3620	24.7356	27.6882	29.8195	34.528
14	17.1169	21.0641	23.6848	26.1189	29.1412	31.3194	36.123
15	18.2451	22.3071	24.9958	27.4884	30.5779	32.8013	37.697
16	19.3689	23.5418	26.2962	28.8454	31.9999	34.2672	39.252
17	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185	40.790
18	21.6049	25.9894	28.8693	31.5264	34.8053	37.1565	42.312
19	22.7178	27.2036	30.1435	32.8523	36.1909	38.5823	43.820
20	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968	45.315
21	24.9348	29.6151	32.6706	35.4789	38.9322	41.4011	46.797
22	26.0393	30.8133	33.9244	36.7807	40.2894	42.7957	48.268
23	27.1413	32.0069	35.1725	38.0756	41.6384	44.1813	49.728
24	28.2412	33.1962	36.4150	39.3641	42.9798	45.5585	51.179
25	29.3389	34.3816	37.6525	40.6465	44.3141	46.9279	52.618
26	30.4346	35.5632	38.8851	41.9232	45.6417	48.2899	54.052
27	31.5284	36.7412	40.1133	43.1945	46.9629	49.6449	55.476
28	32.6205	37.9159	41.3371	44.4608	48.2782	50.9934	56.892
29	33.7109	39.0875	42.5570	45.7223	49.5879	52.3356	58.301
30	34.7997	40.2560	43.7730	46.9792	50.8922	53.6720	59.703
40	45.6160	51.8051	55.7585	59.3417	63.6907	66.7660	73.402
50	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900	86.661
60	66.9815	74.3970	79.0819	83.2977	88.3794	91.9517	99.607
70	77.5767	85.5270	90.5312	95.0232	100.425	104.215	112.317
80	88.1303	96.5782	101.879	106.629	112.329	116.321	124.839
90	98.6499	107.565	113.145	118.136	124.116	128.299	137.208
100	109.141	118.498	124.342	129.561	135.807	140.169	149.449