UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2006

TITLE OF PAPER:	DISTRIBUTION THEORY
COURSE CODE :	ST 301
TIME ALLOWED :	TWO(2) HOURS
INSTRUCTIONS :	THIS PAPER HAS FIVE QUESTIONS. ANSWER ANY FOUR(4) QUESTIONS. EACH QUESTION CARRIES 15 MARKS.
REQUIREMENTS:	Scientific Calculator

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QUESTION ONE

(a) The Weibull density function is given by $f(y)=\begin{cases} 1/\alpha * m * y^{m-1} e^{-y^{m}/\alpha} \\ 0 \end{cases}$ for y>0, where α and m are positive constants. This density function is often used as a model for the lengths of life of physical systems. Suppose Y has the Weibull density above, find the density function of U= Y^m.

(b) Identify the distribution of a random variable X with the following probability density function $f_X(x;v)=\{\Gamma\left(\frac{v+1}{2}\right)\left(1+\frac{X^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}\}/\left(v\pi\Gamma\left(\frac{v}{2}\right)\right)^{\frac{1}{2}}, -\infty < x < \infty; 0 < v$ and is zero elsewhere and V is the degree of freedom. What is the expected value and variance of this random variable.

(9+6)Marks

QUESTION TWO

(a) Define the cumulant generating function.

(b) Given that $f(x)=\binom{n}{x}p^x(1-p)^{n-x}$ and $\mu_k' = E(X^k)$, show that

$$\mu_{k+1}' = np\mu_k' + p(1-p)\frac{d\mu_k'}{dp}$$

(c) If a large grass lawn contains on average 1 weed per $600\,cm^2$, what will be the distribution of X, the number of weeds in an area of $400\,cm^2$?

(3+7+5)Marks

QUESTION THREE

(a) Let X be a discrete type of random variable, define the probability generating function π(z) and show that (i) $E(X) = \pi'(1)$ and

(ii) Var (X)= $\pi''(1) + \pi'(1) - (\pi'(1))^2$ where $\pi'(1) = \frac{d\pi(z)}{dz}$.

(b) Find the moment generating function of a random variable X with probability density function $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, x>0 \\ 0, otherwise \end{cases}$

(10+5)Marks

QUESTION FOUR

If the random variable X has the probability density function, f(x), given as $f(x)=\frac{x^{\frac{n}{2}-1}e^{-\frac{x}{2}}}{\Gamma\left(\frac{n}{2}\right)2^{n/2}}, 0<x<\infty$, then X is distributed as chi-square with n degrees of freedom.

(a) Show that $g(x)=xe^{-x/2}/4, 0<x<\infty$ is a chi-square variate with 4 degrees of freedom. Hence, determine the mean and variance of the random variable X.

(b) Show that the moment generating function of the random variable X having a chi-square distribution is given as $(1-2t)^{-n/2}, t<1/2$. Assume that $\int_0^\infty x^{p-1}e^{-\lambda x}dx=\Gamma(p)/\lambda$, $\lambda>0$.]

(10+5)Marks

QUESTION FIVE

(a) Given that the random variable W has a beta distribution of the first kind with parameters m and n, write down the probability density function of this random variable. Hence, or otherwise obtain is expected value.

(b) Given that $Y=2x/\beta$ and that X has a gamma distribution with parameters α and β. Show that the random variable Y is a chi-square variate.

(8+7)Marks