

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2006

TITLE OF PAPER : STATISTICAL INFERENCE II

COURSE CODE : ST 303

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES

**INSTRUCTIONS : ANSWER QUESTION ONE AND ANY OTHER
TWO QUESTIONS**

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INVIGILATOR**

Question 1

- a) Justify the use of expected values in the calculation of the Chi-Square test statistic for discrete data. (10 marks)
- b) Explain the relationship between the t-test for independent samples and the ANOVA. Also give a theoretical reason for employing the F-test in the analysis of multiple-group experiment, instead of the t-test for independent samples. (10 marks)

Question 2

Let X have a binomial distribution $bi(100, \frac{1}{4})$. Find the best critical region for testing the simple hypothesis $H_0 : p = \frac{1}{4}$ against simple hypothesis $H_1 : p = \frac{1}{2}$ at $\alpha = 0.05$. Also derive the power function for this hypothesis test. (20 marks)

Question 3

Suppose we suspect that a variable has a distribution with probability density function

$$f(x) = \begin{cases} (k+1)x^k & (0 \leq x \leq 1, k > -1), \\ 0 & (\text{otherwise}) \end{cases}$$

and that the following values are observed

0.2 0.4 0.5 0.7 0.8 0.8 0.9 0.9

- a) Estimate the parameter k by method of moments
- b) Estimate the parameter k by method of maximum likelihood

(20 Marks)

Question 4

Let X be $N(\mu, 8100)$. To test $H_0: \mu = 530$ against the alternative hypothesis $H_1: \mu < 530$. Given a random sample of size $n = 36$, let the critical region be defined by $C = \{\bar{x} : \bar{x} \leq 510.77\}$, where $\bar{x} = 509$ is the observed random sample mean.

- a) Find the p-value for this test. (8 marks)
- b) If $H_1: \mu = 509$, find the probabilities of making the Type II error and of correctly rejecting H_0 . (12 marks)

I. Cumulative Standard Normal Distribution*

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	.00	.01	.02	.03	.04	z
.0	.50000	.50399	.50798	.51197	.51595	.0
.1	.53983	.54379	.54776	.55172	.55567	.1
.2	.57926	.58317	.58706	.59095	.59483	.2
.3	.61791	.62172	.62551	.62930	.63307	.3
.4	.65542	.65910	.66276	.66640	.67003	.4
.5	.69146	.69497	.69847	.70194	.70540	.5
.6	.72575	.72907	.73237	.73565	.73891	.6
.7	.75803	.76115	.76424	.76730	.77035	.7
.8	.78814	.79103	.79389	.79673	.79954	.8
.9	.81594	.81859	.82121	.82381	.82639	.9
1.0	.84134	.84375	.84613	.84849	.85083	1.0
1.1	.86433	.86650	.86864	.87076	.87285	1.1
1.2	.88493	.88686	.88877	.89065	.89251	1.2
1.3	.90320	.90490	.90658	.90824	.90988	1.3
1.4	.91924	.92073	.92219	.92364	.92506	1.4
1.5	.93319	.93448	.93574	.93699	.93822	1.5
1.6	.94520	.94630	.94738	.94845	.94950	1.6
1.7	.95543	.95637	.95728	.95818	.95907	1.7
1.8	.96407	.96485	.96562	.96637	.96711	1.8
1.9	.97128	.97193	.97257	.97320	.97381	1.9
2.0	.97725	.97778	.97831	.97882	.97932	2.0
2.1	.98214	.98257	.98300	.98341	.98382	2.1
2.2	.98610	.98645	.98679	.98713	.98745	2.2
2.3	.98928	.98956	.98983	.99010	.99036	2.3
2.4	.99180	.99202	.99224	.99245	.99266	2.4
2.5	.99379	.99396	.99413	.99430	.99446	2.5
2.6	.99534	.99547	.99560	.99573	.99585	2.6
2.7	.99653	.99664	.99674	.99683	.99693	2.7
2.8	.99744	.99752	.99760	.99767	.99774	2.8
2.9	.99813	.99819	.99825	.99831	.99836	2.9
3.0	.99865	.99869	.99874	.99878	.99882	3.0
3.1	.99903	.99906	.99910	.99913	.99916	3.1
3.2	.99931	.99934	.99936	.99938	.99940	3.2
3.3	.99952	.99953	.99955	.99957	.99958	3.3
3.4	.99966	.99968	.99969	.99970	.99971	3.4
3.5	.99977	.99978	.99978	.99979	.99980	3.5
3.6	.99984	.99985	.99985	.99986	.99986	3.6
3.7	.99989	.99990	.99990	.99990	.99991	3.7
3.8	.99993	.99993	.99993	.99994	.99994	3.8
3.9	.99995	.99995	.99996	.99996	.99996	3.9

*Reproduced with permission from *Probability and Statistics in Engineering and Management Science*, 3rd edition, by W. W. Hines and D. C. Montgomery, Wiley, New York, 1990.

I. Cumulative Standard Normal Distribution (continued)

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	.05	.06	.07	.08	.09	z
.0	.51994	.52392	.52790	.53188	.53586	.0
.1	.55962	.56356	.56749	.57142	.57534	.1
.2	.59871	.60257	.60642	.61026	.61409	.2
.3	.63683	.64058	.64431	.64803	.65173	.3
.4	.67364	.67724	.68082	.68438	.68793	.4
.5	.70884	.71226	.71566	.71904	.72240	.5
.6	.74215	.74537	.74857	.75175	.75490	.6
.7	.77337	.77637	.77935	.78230	.78523	.7
.8	.80234	.80510	.80785	.81057	.81327	.8
.9	.82894	.83147	.83397	.83646	.83891	.9
1.0	.85314	.85543	.85769	.85993	.86214	1.0
1.1	.87493	.87697	.87900	.88100	.88297	1.1
1.2	.89435	.89616	.89796	.89973	.90147	1.2
1.3	.91149	.91308	.91465	.91621	.91773	1.3
1.4	.92647	.92785	.92922	.93056	.93189	1.4
1.5	.93943	.94062	.94179	.94295	.94408	1.5
1.6	.95053	.95154	.95254	.95352	.95448	1.6
1.7	.95994	.96080	.96164	.96246	.96327	1.7
1.8	.96784	.96856	.96926	.96995	.97062	1.8
1.9	.97441	.97500	.97558	.97615	.97670	1.9
2.0	.97982	.98030	.98077	.98124	.98169	2.0
2.1	.98422	.98461	.98500	.98537	.98574	2.1
2.2	.98778	.98809	.98840	.98870	.98899	2.2
2.3	.99061	.99086	.99111	.99134	.99158	2.3
2.4	.99286	.99305	.99324	.99343	.99361	2.4
2.5	.99461	.99477	.99492	.99506	.99520	2.5
2.6	.99598	.99609	.99621	.99632	.99643	2.6
2.7	.99702	.99711	.99720	.99728	.99736	2.7
2.8	.99781	.99788	.99795	.99801	.99807	2.8
2.9	.99841	.99846	.99851	.99856	.99861	2.9
3.0	.99886	.99889	.99893	.99897	.99900	3.0
3.1	.99918	.99921	.99924	.99926	.99929	3.1
3.2	.99942	.99944	.99946	.99948	.99950	3.2
3.3	.99960	.99961	.99962	.99964	.99965	3.3
3.4	.99972	.99973	.99974	.99975	.99976	3.4
3.5	.99981	.99981	.99982	.99983	.99983	3.5
3.6	.99987	.99987	.99988	.99988	.99989	3.6
3.7	.99991	.99992	.99992	.99992	.99992	3.7
3.8	.99994	.99994	.99995	.99995	.99995	3.8
3.9	.99996	.99996	.99996	.99997	.99997	3.9

II. Percentage Points of the t Distribution^a

$\nu \backslash \alpha$.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.019	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

^aAdapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.