UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2006

TITLE OF PAPER	**: SAMPLE SURVEY THEORY**
COURSE CODE	**: ST 306**
TIME ALLOWED	**: TWO (2) HOURS**
REQUIREMENTS	**: CALCULATOR AND FORMULA SHEET**
INSTRUCTIONS	**: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

Question 1

According to The Swazi Observer's (17/11/2003) headline story;

"3 out of 4 HIV positive in random test"

Three out of four people who tested for HIV at Lavumisa during the Family Life Association of Swaziland's campaign for Voluntary Counselling and Testing (VCT) on Saturday were found positive.

The results were instant, manifesting a high rate of HIV infection, not only in the drought-ravaged area, but through out Swaziland.......She said only one of the tested residents of Lavumisa was found to be negative of the virus that causes the dreaded AIDS disease. FLAS could not manage to test more than four Lavumisa residents, although the number of residents who queued outside the VCT caravan......exceeded ten.

Comment on the above analysis by this newspaper. (20 marks)

Question 2

a) It has been generally noted that all Household Income and Expenditure Surveys are affected by income underreporting. What rationale would you use to estimate the average household income and total income for Swaziland and why? (5 Marks)

b) Suppose the mean hours watching television per day for 250 sampled students is 2.192 hours, within an element variance of 1.008. If the *fpc* is 0.866, calculate the 95% confidence interval for the parameter (assume SRSWOR) and total number of hours spent by students watching television per day. Also calculate the error of estimating this total. (15 Marks)

Question 3

A psychologist working with a group of mentally retarded adults desires to estimate their average reaction time to a certain stimulus. She feels that men and women probably will show a difference in reaction times, so she wants to stratify on sex. The group of 96 people contains 43 men. In previous studies of this type researchers have found that the times range from 5 to 20 seconds for men and 3 to 14 seconds for women. The cost of sampling is the same for both strata. Using Neyman allocation, find the approximate sample size necessary to estimate the average reaction time for the group to within 1 second. (20 marks)

Question 4

A cluster sample design of park visitors is undertaken. Here we have a number of visitors to a park with week of visit as the primary sampling unit (the park is closed on Friday, so only 6 possible days per week). Out of 10 possible weeks to be sampled, weeks 2, 6, 8, and 10 are selected. Then (observed number of visitors is given in parentheses):

- For week 2, (327) visitors enumerated.
- For week 6, (639) visitors enumerated.
- For week 8, (101) visitors enumerated.
- For week 10, (706) visitors enumerated.

a) Estimate the total number of visitors for the 10 week period with a 95% confidence interval. (10 marks)

b) Estimate the average number of visitors for the 10 week period with a 95% confidence interval. (10 marks)

I. Cumulative Standard Normal Distribution[a]

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	.00	.01	.02	.03	.04	z
.0	.50000	.50399	.50798	.51197	.51595	.0
.1	.53983	.54379	.54776	.55172	.55567	.1
.2	.57926	.58317	.58706	.59095	.59483	.2
.3	.61791	.62172	.62551	.62930	.63307	.3
.4	.65542	.65910	.66276	.66640	.67003	.4
.5	.69146	.69497	.69847	.70194	.70540	.5
.6	.72575	.72907	.73237	.73565	.73891	.6
.7	.75803	.76115	.76424	.76730	.77035	.7
.8	.78814	.79103	.79389	.79673	.79954	.8
.9	.81594	.81859	.82121	.82381	.82639	.9
1.0	.84134	.84375	.84613	.84849	.85083	1.0
1.1	.86433	.86650	.86864	.87076	.87285	1.1
1.2	.88493	.88686	.88877	.89065	.89251	1.2
1.3	.90320	.90490	.90658	.90824	.90988	1.3
1.4	.91924	.92073	.92219	.92364	.92506	1.4
1.5	.93319	.93448	.93574	.93699	.93822	1.5
1.6	.94520	.94630	.94738	.94845	.94950	1.6
1.7	.95543	.95637	.95728	.95818	.95907	1.7
1.8	.96407	.96485	.96562	.96637	.96711	1.8
1.9	.97128	.97193	.97257	.97320	.97381	1.9
2.0	.97725	.97778	.97831	.97882	.97932	2.0
2.1	.98214	.98257	.98300	.98341	.93882	2.1
2.2	.98610	.98645	.98679	.98713	.98745	2.2
2.3	.98928	.98956	.98983	.99010	.99036	2.3
2.4	.99180	.99202	.99224	.99245	.99266	2.4
2.5	.99379	.99396	.99413	.99430	.99446	2.5
2.6	.99534	.99547	.99560	.99573	.99585	2.6
2.7	.99653	.99664	.99674	.99683	.99693	2.7
2.8	.99744	.99752	.99760	.99767	.99774	2.8
2.9	.99813	.99819	.99825	.99831	.99836	2.9
3.0	.99865	.99869	.99874	.99878	.99882	3.0
3.1	.99903	.99906	.99910	.99913	.99916	3.1
3.2	.99931	.99934	.99936	.99938	.99940	3.2
3.3	.99952	.99953	.99955	.99957	.99958	3.3
3.4	.99966	.99968	.99969	.99970	.99971	3.4
3.5	.99977	.99978	.99978	.99979	.99980	3.5
3.6	.99984	.99985	.99985	.99986	.99986	3.6
3.7	.99989	.99990	.99990	.99990	.99991	3.7
3.8	.99993	.99993	.99993	.99994	.99994	3.8
3.9	.99995	.99995	.99996	.99996	.99996	3.9

[a]Reproduced with permission from *Probability and Statistics in Engineering and Management Science*, 3rd edition, by W. W. Hines and D. C. Montgomery, Wiley, New York, 1990.

I. Cumulative Standard Normal Distribution (*continued*)

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\,du$$

z	.05	.06	.07	.08	.09	z
.0	.51994	.52392	.52790	.53188	.53586	.0
.1	.55962	.56356	.56749	.57142	.57534	.1
.2	.59871	.60257	.60642	.61026	.61409	.2
.3	.63683	.64058	.64431	.64803	.65173	.3
.4	.67364	.67724	.68082	.68438	.68793	.4
.5	.70884	.71226	.71566	.71904	.72240	.5
.6	.74215	.74537	.74857	.75175	.75490	.6
.7	.77337	.77637	.77935	.78230	.78523	.7
.8	.80234	.80510	.80785	.81057	.81327	.8
.9	.82894	.83147	.83397	.83646	.83891	.9
1.0	.85314	.85543	.85769	.85993	.86214	1.0
1.1	.87493	.87697	.87900	.88100	.88297	1.1
1.2	.89435	.89616	.89796	.89973	.90147	1.2
1.3	.91149	.91308	.91465	.91621	.91773	1.3
1.4	.92647	.92785	.92922	.93056	.93189	1.4
1.5	.93943	.90462	.94179	.94295	.94408	1.5
1.6	.95053	.95154	.95254	.95352	.95448	1.6
1.7	.95994	.96080	.96164	.96246	.96327	1.7
1.8	.96784	.96856	.96926	.96995	.97062	1.8
1.9	.97441	.97500	.97558	.97615	.97670	1.9
2.0	.97982	.98030	.98077	.98124	.98169	2.0
2.1	.98422	.98461	.98500	.98537	.98574	2.1
2.2	.98778	.98809	.98840	.98870	.98899	2.2
2.3	.99061	.99086	.99111	.99134	.99158	2.3
2.4	.99286	.99305	.99324	.99343	.99361	2.4
2.5	.99461	.99477	.99492	.99506	.99520	2.5
2.6	.99598	.99609	.99621	.99632	.99643	2.6
2.7	.99702	.99711	.99720	.99728	.99736	2.7
2.8	.99781	.99788	.99795	.99801	.99807	2.8
2.9	.99841	.99846	.99851	.99856	.99861	2.9
3.0	.99886	.99889	.99893	.99897	.99900	3.0
3.1	.99918	.99921	.99924	.99926	.99929	3.1
3.2	.99942	.99944	.99946	.99948	.99950	3.2
3.3	.99960	.99961	.99962	.99964	.99965	3.3
3.4	.99972	.99973	.99974	.99975	.99976	3.4
3.5	.99981	.99981	.99982	.99983	.99983	3.5
3.6	.99987	.99987	.99988	.99988	.99989	3.6
3.7	.99991	.99992	.99992	.99992	.99992	3.7
3.8	.99994	.99994	.99995	.99995	.99995	3.8
3.9	.99996	.99996	.99996	.99997	.99997	3.9

$$s^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{V}(\hat{\mu})_{srs} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{V}(\hat{\tau})_{srs} = N^2\hat{V}(\hat{\mu})_{srs}$$

$$\hat{p}_{srs} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{srs} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{pps} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i}\right) \qquad \hat{V}(\hat{\tau})_{pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i} - \hat{\tau}_{pps}\right)^2$$

$$\hat{\mu}_{pps} = \frac{1}{N}\hat{\tau}_{pps} \qquad \hat{V}(\hat{\mu})_{pps} = \frac{1}{N^2}\hat{V}(\hat{\tau})_{pps}$$

$$\hat{\mu}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{\mu})_{sys} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{sys} = N\hat{\mu}_{sys} \qquad \hat{V}(\hat{\tau})_{sys} = N^2\hat{V}(\hat{\mu})_{sys}$$

$$\hat{p}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{sys} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\mu}_{rsys} = \sum_{i=1}^{ns} \frac{\hat{\mu}_i}{ns} \qquad \hat{V}(\hat{\mu})_{rsys} = \left(\frac{N-n}{N}\right)\sum_{i=1}^{ns} \frac{(\hat{\mu}_i - \hat{\mu}_{rsys})^2}{ns(ns-1)}$$

$$\hat{\tau}_{rsys} = N\hat{\mu}_{rsys} \qquad \hat{V}(\hat{\tau})_{rsys} = N^2\hat{V}(\hat{\mu})_{rsys}$$

$$\hat{\mu}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i-n_i}{N_i}\right)\frac{s_i^2}{n_i}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{V}(\hat{\tau})_{str} = N^2\hat{V}(\hat{\mu})_{str}$$

$$\hat{p}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p}_i \qquad \hat{V}(\hat{p})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i-n_i}{N_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}_i)}{n_i-1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{i=1}^{L} w_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{pstr} = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{i=1}^{L} w_i s_i^2 + \frac{1}{n^2}\sum_{i=1}^{L}(1-w_i)s_i^2$$

$$r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$$

$$\hat{V}(r) = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{(n-1)}$$

$$\hat{\rho} = \frac{c\hat{o}v(x,y)}{s_x s_y}$$

$$\hat{V}(r) = \frac{1-(n/N)}{n}\left(\frac{1}{\mu_x^2}\right)(s_y^2 + r^2 s_x^2 - 2r\hat{\rho}s_x s_y)$$

$$\hat{\tau}_{ratio} = r\tau_x$$

$$\hat{V}(\hat{\tau})_{ratio} = \tau_x^2 \hat{V}(r)$$

$$\hat{\mu}_{ratio} = r\mu_x$$

$$\hat{V}(\hat{\mu})_{ratio} = \mu_x^2 \hat{V}(r)$$

$$Y_i = \beta_0 + \beta_1(X_i) + \varepsilon_i$$

$$\sum_{i=1}^{n}(y_i - rx_i)^2 = \sum_{i=1}^{n} y_i^2 + r^2 \sum_{i=1}^{n} x_i^2 - 2r \sum_{i=1}^{n} y_i x_i$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$b_1 = \hat{\rho}(s_y/s_x)$$

$$b_1 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\hat{\rho} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y}$$

$$\hat{\mu}_{reg} = \bar{y} + b_1(\mu_x - \bar{x})$$

$$\hat{V}(\hat{\mu})_{reg} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n(n-1)}$$

$$\hat{y}_i = b_0 + b_1(x_i)$$

$$\hat{V}(\hat{\mu})_{reg} \approx \left(\frac{N-n}{N}\right)\frac{MSE}{n}$$

$$\hat{\mu}_{diff} = \bar{y} + (\mu_x - \bar{x})$$

$$\hat{V}(\hat{\mu})_{diff} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(d_i-\bar{d})^2}{n(n-1)}$$

$$\sum_{i=1}^{n}(d_i - \bar{d})^2 = \sum_{i=1}^{n} d_i^2 - n\bar{d}^2$$

$$\hat{RE}\left(\frac{E1}{E2}\right) = \frac{\hat{V}(E2)}{\hat{V}(E1)}$$

$$\hat{\mu}_{cts1} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}$$

$$\hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i-\bar{y}m_i)^2}{n\bar{M}^2(n-1)}$$

$$\hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)(s_y^2 + \hat{\mu}_{cts1}^2 s_m^2 - 2\hat{\mu}_{cts1}\hat{\rho}s_y s_m)$$

$$\hat{\tau}_{cts1(1)} = M\hat{\mu}_{cts1}$$

$$\hat{V}(\hat{\tau})_{cts1(1)} = M^2\hat{V}(\hat{\mu})_{cts1}$$

$$\hat{\tau}_{cts1(2)} = N\bar{y}_t = N\left(\frac{\sum_{i=1}^{n} y_i}{n}\right) \qquad \hat{V}(\hat{\tau})_{cts1(2)} = \left(\frac{N-n}{N}\right)\left(\frac{N^2}{n}\right)\frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)}$$

$$\bar{m} = \frac{\sum_{i=1}^{n} m_i}{n} \qquad \sum_{i=1}^{n}(y_i - \bar{y}m_i)^2 = \sum_{i=1}^{n} y_i^2 + \bar{y}^2\sum_{i=1}^{n} m_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i m_i$$

$$\hat{p}_{cts1} = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i} \qquad \hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)\frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

$$\Pi_i = \frac{m_i}{M} \qquad \hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)(s_a^2 + \hat{p}^2 s_m^2 - 2\hat{p}\hat{\rho}s_a s_m)$$

$$\hat{\tau}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\Pi_i} \qquad \sum_{i=1}^{n}(a_i - \hat{p}m_i)^2 = \sum_{i=1}^{n} a_i^2 + \hat{p}^2\sum_{i=1}^{n} m_i^2 - 2\hat{p}\sum_{i=1}^{n} a_i m_i$$

$$\hat{\tau}_{cts1,pps} = \frac{M}{n}\sum_{i=1}^{n}\bar{y}_i \qquad \hat{V}(\hat{\tau})_{cts1,pps} = \frac{M^2}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\tau})^2$$

$$\hat{\mu}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i \qquad \hat{V}(\hat{\mu})_{cts1,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu})^2$$

$$\hat{\mu}_{cts2} = \left(\frac{N}{M}\right)\frac{\sum_{i=1}^{n} M_i\bar{y}_i}{n} \qquad \hat{V}(\hat{\mu})_{cts2} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_b^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_b^2 = \frac{\sum_{i=1}^{n}(M_i\bar{y}_i - \bar{M}\hat{\mu})^2}{n-1} \qquad s_i^2 = \frac{\sum_{i=1}^{m_i}(y_{ij}-\bar{y}_i)^2}{m_i-1}$$

$$\hat{\tau}_{cts2} = M\hat{\mu}_{cts2} \qquad \hat{V}(\hat{\tau})_{cts2} = M^2\hat{V}(\hat{\mu})_{cts2}$$

$$\hat{\mu}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{\mu})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\bar{y}_i - \hat{\mu}_{cts2,r})^2}{n-1} \qquad s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\hat{p}_i - \hat{p}_{cts2,r})^2}{n-1}$$

$$\hat{p}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{\hat{p}_i(1-}{m_i-}\right.$$

$\hat{\mu}_{cts2,pps} = \frac{1}{n}\sum_{i=1}^{n} \bar{y}_i$ $\hat{V}(\hat{\mu})_{cts2,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu}_{cts2,pps})^2$

$\hat{\tau}_{cts2,pps} = M\hat{\mu}_{cts2,pps}$ $\hat{V}(\hat{\tau}) = M^2\hat{V}(\hat{\mu})_{cts2,pps}$

n for μ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for τ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4N^2)+\sigma^2}$

n for p (SRS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

n for μ (SYS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for p (SYS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

$k \leq \frac{N}{n}$ $k' = k(ns)$

n for μ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

n for τ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4N^2)+\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR μ:

$n_i = n\left(\frac{N_i\sigma_i/\sqrt{c_i}}{\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}}\right)$ $n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{i=1}^{L} N_i\sigma_i\sqrt{c_i}\right)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_i}\right)$ $n = \frac{\left(\sum_{i=1}^{L} N_i\sigma_i\right)^2}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i}{N}\right)$ $n = \frac{\sum_{i=1}^{L} N_i\sigma_i^2}{N^2(B^2/4)+(1/N)\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR τ: change $N^2(B^2/4)$ to $N^2(B^2/4N^2)$

Allocations for STR p:

$$n_i = n\left(\frac{N_i\sqrt{p_i(1-p_i)/c_i}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right)$$

$$n = \frac{\sum_{i=1}^{L} N_i^2 p_i(1-p_i)/w_i}{N^2(B^2/4)+\sum_{i=1}^{L} N_i p_i(1-p_i)}$$

n for μ (ratio): $n = \frac{N\sigma^2}{N(B^2/4)+\sigma^2}$

n for τ (ratio): $n = \frac{N\sigma^2}{N(B^2/4N^2)+\sigma^2}$

n for μ (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

n for τ (CTS1(1)): $n = \frac{N\sigma_c^2}{N(B^2/4N^2)+\sigma_c^2}$

n for τ (CTS1(2)): $n = \frac{N\sigma_t^2}{N(B^2/4N^2)+\sigma_t^2}$

$s_c^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}m_i)^2}{(n-1)}$ with $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}$ $s_t^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)}$ with $\bar{y}_t = \frac{\sum_{i=1}^{n} y_i}{n}$

n for p (CTS1): $n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$

$s_c^2 = \frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$