UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION 2006

COURSE TITLE	:	OPERATIONS RESEARCH I
COURSE CODE	:	ST 307
TIME ALLOWED	:	TWO (3) HOURS
INSTRUCTION	:	ANSWER ANY FOUR (4) QUESTIONS. EACH QUESTION CARRIES 25 MARKS.
REQUIREMENTS	:	CALCULATOR

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Question 1

(a) Define the following terms in the context of linear programming:

(i) Decision variables;
(ii) Objective function;
(iii) Functional constraints.

(b) A manufacturing firm has discontinued production of a certain unprofitable product line. This created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

Machine type	*Available time (in machine hours per week)*
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is

Machine type	*Product 1*	*Product 2*	*Product 3*
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week.

The unit profit would be $30, $12, and $15, respectively, on products 1, 2, and 3.

Formulate the linear programming model for determining how much of each product the firm should produce to maximize profit.

Question 2

(a) How do you determine the following when solving a linear programming problem using the simplex method:

(i) Multiple optimal solutions;
(ii) Infeasible solution;
(iii) Shadow prices;

(b) Consider the following problem:

Maximize $Z = 2X_1 + X_2$,

subject to

$$\begin{aligned} X_2 &\le 10 \\ 2X_1 + 5X_2 &\le 60 \\ X_1 + X_2 &\le 18 \\ 3X_1 + X_2 &\le 44 \end{aligned}$$

and

$X_1 \ge 0, X_2 \ge 0$.

(i) Construct the initial simplex tableau, introducing slack variables and so forth as needed for applying the simplex method.

(ii) Solve the problem by simplex method.

Question 3

A company has three plants producing a certain product that is to be shipped to four distribution centres. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance from each plant to the respective distributing centres is given below in miles:

		Distribution Centre			
		1	2	3	4
Plant	1	800	1,300	400	700
	2	1,100	1,400	600	1,000
	3	600	1,200	800	900

The freight cost for each shipment is $100 plus 50 cents per mile.

The company wishes to determine how much should be shipped from each plant to each of the distribution centers to minimize the total shipping costs.

(a) Formulate this problem as a transformation problem by constructing the appropriate cost and requirements table.

(b) Use the northwest corner rule to obtain an initial basic feasible solution.

(c) Use the transportation simplex method to obtain the optimal solution.

Question 4

Consider the following problem:

Maximize $Z = -5X_1 + 5X_2 + 13X_3$,

Subject to

$$
\begin{aligned}
-X_1 + X_2 + 3X_3 &\leq 20 \\
12X_1 + 4X_2 + 10X_3 &\leq 90
\end{aligned}
$$

and

$$X_j \geq 0 \ (j = 1,2,3).$$

Letting X_4 and X_5 be the slack variables for the respective constraints, the simplex method yields the following *final* set of equations:

$$
\begin{aligned}
&(0) \quad Z + 2X_3 + 5X_4 = 100 \\
&(1) \quad -X_1 + X_2 + 3X_3 + X_4 = 20 \\
&(2) \quad 16X_1 - 2X_3 - 4X_4 + X_5 = 10
\end{aligned}
$$

You are now to conduct sensitivity analysis by *independently* investigating each of the nine changes in the original model indicated below. For each change, use the sensitivity analysis procedure to convert this set of equations (in tabular form) to the proper form for identifying and evaluating the current basic solution, and then test this solution for *feasibility* and for *optimality*.

(a) Change the right-hand side of constraint 1 to $b_1 = 30$.

(b) Change the right-hand side of constraint 2 to $b_1 = 70$.

(c) Change the right-hand sides to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$.

(d) Change the coefficient of X_3 in the objective function to $c_3 = 8$.

(e) Change the coefficients of X_1 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$.

(f) Change the coefficients of X_2 to $\begin{bmatrix} c_2 \\ a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$.

(g) Introduce a new variable X_6 with coefficients $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$.

(h) Introduce a new constraint $2X_1 + 3X_2 + 5X_3 \leq 50$. (Denote its slack variable by X_6.)

(i) Change constraint 2 to $10X_1 + 5X_2 + 10X_3 \leq 100$.

Question 5

Consider the game having the following payoff table:

		II			
		1	2	3	4
	1	5	0	3	1
I	2	2	4	3	2
	3	3	2	0	4

Use the approach described in the lecture to formulate the problem of finding the optimal mixed strategies according to the minimax criterion as a *linear programming* problem.