UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2006

TITLE OF PAPER:	**APPLIED LINEAR REGRESSION**
COURSE CODE :	**ST 403**
TIME ALLOWED :	**TWO(2) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS FIVE QUESTIONS.** **ANSWER <u>ANY FOUR(4)</u> QUESTIONS.** **EACH QUESTION CARRIES 15 MARKS.**
REQUIREMENTS:	**Scientific Calculator**

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QUESTION ONE

(a) Suppose A is a symmetric and idempotent matrix and given that X is distributed normal with mean (μ) and variance (V).
Let $E(XX')=V+\mu\mu'$, show that $E(X'AX)=tr(AV)+\mu'A\mu$.

(b) Given that **Y=Xβ + U, for U ~N(O,σ^2). Show that the Least square Estimator of β is unbiased.**

(c) Fill in the missing gap in the ANOVA table below

Source of variation	Sum of Squares	Degree of Freedom	Mean Square	F-ratio
α	SS(α)	r-1	***	***
τ	***	***	***	***
Є	***	(r-1)(t-1)	SS(Є)/DF(Є)	
Total	SS(total)	***		

(5+5+5)Marks

QUESTION TWO

(a) **Consider the following data on consumption expenditure of six persons of which 1,2,3 are low, middle and upper income earners respectively.**

INCOME CLASS	CONSUMPTION EXPENDITURE
LOW	y_{11}, y_{12}, y_{13}
MIDDLE	y_{21}, y_{22}
UPPER	y_{23}

Given that Y'=[16 10 19 11 13 27] , obtain the estimate for the regression coefficients $b' = (\mu, \alpha_1, \alpha_2, \alpha_3)$.

(b) **In a multiple regression model Y=Xβ + U, what is the implication of X having rank k less than n, the number of observations?.**

(10+5)Marks

QUESTION THREE

(a) Given that **Y=Xβ + U, for** $E(uu') = \sigma^2 I_n$ **, show that** $V(\hat{\beta}) = \sigma^2 (X'X)^{-1}$.

(b) **Give three properties of a generalised inverse for a symmetric matrix.**

(9+6)Marks

QUESTION FOUR

Data concerning the weekly fuel consumption (Y), average hourly temperature (x_1) and chill index (x_2) at certain factory has been observed for the last eight years. Fit the regression model on the Fuel consumption data below. Test for the significance of the coefficients at 5% level.

WEEK(i)	AVERAGE HOURLY TEMPERATURE	CHILL INDEX	FUEL CONSUMPTION
1	28	18	12.4
2	28	14	11.7
3	32.5	24	12.4
4	39	22	10.8
5	45.9	8	9.4
6	57.8	16	9.5
7	58.1	1	8
8	62.5	0	7.5

QUESTION FIVE

(a) Assume that the relationship between X and Y is a parabola, such that $Y = b_0 + b_1X + b_2X^2 + U$. Obtain the normal equations for this model. What are the assumptions on the disturbance term in this model?

(b) Explain the meaning of exogenous and endogenous variables in the context of a regression model.

(c) Enumerate three basic assumptions of Analysis of Variance inference.

TABLE 7 (a). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.0	254.3
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.85	2.79	2.61	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.67	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.60	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.49	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.45	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.41	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.38	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.32	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.30	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.27	2.20	2.00	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.24	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.22	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.20	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.19	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.18	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	1.89	1.62
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.14	2.07	1.86	1.59
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.12	2.05	1.84	1.57
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.11	2.03	1.82	1.55
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.09	2.02	1.81	1.53
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.08	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.91	1.83	1.61	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.83	1.75	1.52	1.00

The function tabulated in Table 7 is F_P defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1+\frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\,\nu_1^{\frac{1}{2}\nu_1}\,\nu_2^{\frac{1}{2}\nu_2}\int_{F_P}^{\infty}\frac{F^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1 F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

with $P=5$, $2\frac{1}{2}$, 1 and 0.1. If F is the ratio of a mean square on ν_1 degrees of freedom to an independent mean square on ν_2 degrees of freedom, and if the mean squares have equal expectations, then $P/100$ is the probability that $F \geqslant F_P$. The lower percentage points, that is the value F'_P such that $P/100$ is the probability that $F \leqslant F'_P$ may be found by interchanging ν_1 and ν_2 and using the reciprocal of the tabulated value.

Linear interpolation will usually be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, though occasionally a slight improvement may be effected by using harmonic interpolation. Otherwise, except

TABLE 7 (*b*). 2½ PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$	648	800	864	900	922	937	948	957	969	977	997	1018
2	38·5	39·0	39·2	39·2	39·3	39·3	39·4	39·4	39·4	39·4	39·5	39·5
3	17·4	16·0	15·4	15·1	14·9	14·7	14·6	14·5	14·4	14·3	14·1	13·9
4	12·22	10·65	9·98	9·60	9·36	9·20	9·07	8·98	8·84	8·75	8·51	8·26
5	10·01	8·43	7·76	7·39	7·15	6·98	6·85	6·76	6·62	6·52	6·28	6·02
6	8·81	7·26	6·60	6·23	5·99	5·82	5·70	5·60	5·46	5·37	5·12	4·85
7	8·07	6·54	5·89	5·52	5·29	5·12	4·99	4·90	4·76	4·67	4·42	4·14
8	7·57	6·06	5·42	5·05	4·82	4·65	4·53	4·43	4·30	4·20	3·95	3·67
9	7·21	5·71	5·08	4·72	4·48	4·32	4·20	4·10	3·96	3·87	3·61	3·33
10	6·94	5·46	4·83	4·47	4·24	4·07	3·95	3·85	3·72	3·62	3·37	3·08
11	6·72	5·26	4·63	4·28	4·04	3·88	3·76	3·66	3·53	3·43	3·17	2·88
12	6·55	5·10	4·47	4·12	3·89	3·73	3·61	3·51	3·37	3·28	3·02	2·72
13	6·41	4·97	4·35	4·00	3·77	3·60	3·48	3·39	3·25	3·15	2·89	2·60
14	6·30	4·86	4·24	3·89	3·66	3·50	3·38	3·29	3·15	3·05	2·79	2·49
15	6·20	4·76	4·15	3·80	3·58	3·41	3·29	3·20	3·06	2·96	2·70	2·40
16	6·12	4·69	4·08	3·73	3·50	3·34	3·22	3·12	2·99	2·89	2·63	2·32
17	6·04	4·62	4·01	3·66	3·44	3·28	3·16	3·06	2·92	2·82	2·56	2·25
18	5·98	4·56	3·95	3·61	3·38	3·22	3·10	3·01	2·87	2·77	2·50	2·19
19	5·92	4·51	3·90	3·56	3·33	3·17	3·05	2·96	2·82	2·72	2·45	2·13
20	5·87	4·46	3·86	3·51	3·29	3·13	3·01	2·91	2·77	2·68	2·41	2·09
21	5·83	4·42	3·82	3·48	3·25	3·09	2·97	2·87	2·73	2·64	2·37	2·04
22	5·79	4·38	3·78	3·44	3·22	3·05	2·93	2·84	2·70	2·60	2·33	2·00
23	5·75	4·35	3·75	3·41	3·18	3·02	2·90	2·81	2·67	2·57	2·30	1·97
24	5·72	4·32	3·72	3·38	3·15	2·99	2·87	2·78	2·64	2·54	2·27	1·94
25	5·69	4·29	3·69	3·35	3·13	2·97	2·85	2·75	2·61	2·51	2·24	1·91
26	5·66	4·27	3·67	3·33	3·10	2·94	2·82	2·73	2·59	2·49	2·22	1·88
27	5·63	4·24	3·65	3·31	3·08	2·92	2·80	2·71	2·57	2·47	2·19	1·85
28	5·61	4·22	3·63	3·29	3·06	2·90	2·78	2·69	2·55	2·45	2·17	1·83
29	5·59	4·20	3·61	3·27	3·04	2·88	2·76	2·67	2·53	2·43	2·15	1·81
30	5·57	4·18	3·59	3·25	3·03	2·87	2·75	2·65	2·51	2·41	2·14	1·79
32	5·53	4·15	3·56	3·22	3·00	2·84	2·72	2·62	2·48	2·38	2·10	1·75
34	5·50	4·12	3·53	3·19	2·97	2·81	2·69	2·59	2·45	2·35	2·08	1·72
36	5·47	4·09	3·51	3·17	2·94	2·79	2·66	2·57	2·43	2·33	2·05	1·69
38	5·45	4·07	3·48	3·15	2·92	2·76	2·64	2·55	2·41	2·31	2·03	1·66
40	5·42	4·05	3·46	3·13	2·90	2·74	2·62	2·53	2·39	2·29	2·01	1·64
60	5·29	3·93	3·34	3·01	2·79	2·63	2·51	2·41	2·27	2·17	1·88	1·48
120	5·15	3·80	3·23	2·89	2·67	2·52	2·39	2·30	2·16	2·05	1·76	1·31
∞	5·02	3·69	3·12	2·79	2·57	2·41	2·29	2·19	2·05	1·94	1·64	1·00

when ν_1 and ν_2 are both large, interpolation should be linear in $\nu_1 F_P$ or $\nu_2 F_P$ (this is equivalent to harmonic interpolation). When ν_1 and ν_2 are both large the percentage points may be found from the formula

$$1{\cdot}1513 \log F_P = \tfrac{1}{2} \ln F_P = \frac{x_P\sqrt{h+\lambda}}{h} - \left(\frac{1}{\nu_1-1} - \frac{1}{\nu_2-1}\right)\left(\lambda+\frac{5}{6}\right)$$

where x_P is the P-per cent point of the normal distribution (Table 2), $\lambda=\frac{1}{6}(x_P^2-3)$ and $\frac{2}{h}=\frac{1}{\nu_1-1}+\frac{1}{\nu_2-1}$.
For the values of P given in Table 7, x_P and λ are as follows:

P	5	2½	1	0·1
x_P	+1·6449	1·9600	2·3263	3·0902
λ	−0·0491	+0·1402	0·4020	1·0916

TABLE 7 (*c*). 1 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6366
2	98·5	99·0	99·2	99·2	99·3	99·3	99·4	99·4	99·4	99·4	99·5	99·5
3	34·1	30·8	29·5	28·7	28·2	27·9	27·7	27·5	27·2	27·1	26·6	26·1
4	21·2	18·0	16·7	16·0	15·5	15·2	15·0	14·8	14·5	14·4	13·9	13·5
5	16·26	13·27	12·06	11·39	10·97	10·67	10·46	10·29	10·05	9·89	9·47	9·02
6	13·74	10·92	9·78	9·15	8·75	8·47	8·26	8·10	7·87	7·72	7·31	6·88
7	12·25	9·55	8·45	7·85	7·46	7·19	6·99	6·84	6·62	6·47	6·07	5·65
8	11·26	8·65	7·59	7·01	6·63	6·37	6·18	6·03	5·81	5·67	5·28	4·86
9	10·56	8·02	6·99	6·42	6·06	5·80	5·61	5·47	5·26	5·11	4·73	4·31
10	10·04	7·56	6·55	5·99	5·64	5·39	5·20	5·06	4·85	4·71	4·33	3·91
11	9·65	7·21	6·22	5·67	5·32	5·07	4·89	4·74	4·54	4·40	4·02	3·60
12	9·33	6·93	5·95	5·41	5·06	4·82	4·64	4·50	4·30	4·16	3·78	3·36
13	9·07	6·70	5·74	5·21	4·86	4·62	4·44	4·30	4·10	3·96	3·59	3·17
14	8·86	6·51	5·56	5·04	4·70	4·46	4·28	4·14	3·94	3·80	3·43	3·00
15	8·68	6·36	5·42	4·89	4·56	4·32	4·14	4·00	3·80	3·67	3·29	2·87
16	8·53	6·23	5·29	4·77	4·44	4·20	4·03	3·89	3·69	3·55	3·18	2·75
17	8·40	6·11	5·18	4·67	4·34	4·10	3·93	3·79	3·59	3·46	3·08	2·65
18	8·29	6·01	5·09	4·58	4·25	4·01	3·84	3·71	3·51	3·37	3·00	2·57
19	8·18	5·93	5·01	4·50	4·17	3·94	3·77	3·63	3·43	3·30	2·92	2·49
20	8·10	5·85	4·94	4·43	4·10	3·87	3·70	3·56	3·37	3·23	2·86	2·42
21	8·02	5·78	4·87	4·37	4·04	3·81	3·64	3·51	3·31	3·17	2·80	2·36
22	7·95	5·72	4·82	4·31	3·99	3·76	3·59	3·45	3·26	3·12	2·75	2·31
23	7·88	5·66	4·76	4·26	3·94	3·71	3·54	3·41	3·21	3·07	2·70	2·26
24	7·82	5·61	4·72	4·22	3·90	3·67	3·50	3·36	3·17	3·03	2·66	2·21
25	7·77	5·57	4·68	4·18	3·86	3·63	3·46	3·32	3·13	2·99	2·62	2·17
26	7·72	5·53	4·64	4·14	3·82	3·59	3·42	3·29	3·09	2·96	2·58	2·13
27	7·68	5·49	4·60	4·11	3·78	3·56	3·39	3·26	3·06	2·93	2·55	2·10
28	7·64	5·45	4·57	4·07	3·75	3·53	3·36	3·23	3·03	2·90	2·52	2·06
29	7·60	5·42	4·54	4·04	3·73	3·50	3·33	3·20	3·00	2·87	2·49	2·03
30	7·56	5·39	4·51	4·02	3·70	3·47	3·30	3·17	2·98	2·84	2·47	2·01
32	7·50	5·34	4·46	3·97	3·65	3·43	3·26	3·13	2·93	2·80	2·42	1·96
34	7·45	5·29	4·42	3·93	3·61	3·39	3·22	3·09	2·90	2·76	2·38	1·91
36	7·40	5·25	4·38	3·89	3·58	3·35	3·18	3·05	2·86	2·72	2·35	1·87
38	7·35	5·21	4·34	3·86	3·54	3·32	3·15	3·02	2·83	2·69	2·32	1·84
40	7·31	5·18	4·31	3·83	3·51	3·29	3·12	2·99	2·80	2·66	2·29	1·80
60	7·08	4·98	4·13	3·65	3·34	3·12	2·95	2·82	2·63	2·50	2·12	1·60
120	6·85	4·79	3·95	3·48	3·17	2·96	2·79	2·66	2·47	2·34	1·95	1·38
∞	6·63	4·61	3·78	3·32	3·02	2·80	2·64	2·51	2·32	2·18	1·79	1·00

The function tabulated in Table 7 is F_P defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1+\frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\nu_1^{\frac{1}{2}\nu_1}\nu_2^{\frac{1}{2}\nu_2}\int_{F_P}^{\infty}\frac{F^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

with $P=5$, $2\frac{1}{2}$, 1 and 0·1. If F is the ratio of a mean square on ν_1 degrees of freedom to an independent mean square on ν_2 degrees of freedom, and if the mean squares have equal expectations, then $P/100$ is the probability that $F \geqslant F_P$. The lower percentage points, that is the value F'_P such that $P/100$ is the probability that $F \leqslant F'_P$ may be found by interchanging ν_1 and ν_2 and using the reciprocal of the tabulated value.

Linear interpolation will usually be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, though occasionally a slight improvement may be effected by using harmonic interpolation. Otherwise, except

TABLE 7 (*d*). 0·1 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=$ 1*	4053	5000	5404	5625	5764	5859	5929	5981	6056	6107	6235	6366*
2	998·5	999·0	999·2	999·2	999·3	999·3	999·4	999·4	999·4	999·4	999·5	999·5
3	167·0	148·5	141·1	137·1	134·6	132·8	131·5	130·6	129·2	128·3	125·9	123·5
4	74·14	61·25	56·18	53·44	51·71	50·53	49·66	49·00	48·05	47·41	45·77	44·05
5	47·18	37·12	33·20	31·09	29·75	28·83	28·16	27·65	26·92	26·42	25·14	23·79
6	35·51	27·00	23·70	21·92	20·80	20·03	19·46	19·03	18·41	17·99	16·90	15·75
7	29·25	21·69	18·77	17·20	16·21	15·52	15·02	14·63	14·08	13·71	12·73	11·70
8	25·42	18·49	15·83	14·39	13·48	12·86	12·40	12·05	11·54	11·19	10·30	9·34
9	22·86	16·39	13·90	12·56	11·71	11·13	10·69	10·37	9·87	9·57	8·72	7·81
10	21·04	14·91	12·55	11·28	10·48	9·93	9·52	9·20	8·74	8·44	7·64	6·76
11	19·69	13·81	11·56	10·35	9·58	9·05	8·66	8·35	7·92	7·63	6·85	6·00
12	18·64	12·97	10·80	9·63	8·89	8·38	8·00	7·71	7·29	7·00	6·25	5·42
13	17·82	12·31	10·21	9·07	8·35	7·86	7·49	7·21	6·80	6·52	5·78	4·97
14	17·14	11·78	9·73	8·62	7·92	7·44	7·08	6·80	6·40	6·13	5·41	4·60
15	16·59	11·34	9·34	8·25	7·57	7·09	6·74	6·47	6·08	5·81	5·10	4·31
16	16·12	10·97	9·01	7·94	7·27	6·80	6·46	6·19	5·81	5·55	4·85	4·06
17	15·72	10·66	8·73	7·68	7·02	6·56	6·22	5·96	5·58	5·32	4·63	3·85
18	15·38	10·39	8·49	7·46	6·81	6·35	6·02	5·76	5·39	5·13	4·45	3·67
19	15·08	10·16	8·28	7·27	6·62	6·18	5·85	5·59	5·22	4·97	4·29	3·51
20	14·82	9·95	8·10	7·10	6·46	6·02	5·69	5·44	5·08	4·82	4·15	3·38
21	14·59	9·77	7·94	6·95	6·32	5·88	5·56	5·31	4·95	4·70	4·03	3·26
22	14·38	9·61	7·80	6·81	6·19	5·76	5·44	5·19	4·83	4·58	3·92	3·15
23	14·19	9·47	7·67	6·70	6·08	5·65	5·33	5·09	4·73	4·48	3·82	3·05
24	14·03	9·34	7·55	6·59	5·98	5·55	5·23	4·99	4·64	4·39	3·74	2·97
25	13·88	9·22	7·45	6·49	5·89	5·46	5·15	4·91	4·56	4·31	3·66	2·89
26	13·74	9·12	7·36	6·41	5·80	5·38	5·07	4·83	4·48	4·24	3·59	2·82
27	13·61	9·02	7·27	6·33	5·73	5·31	5·00	4·76	4·41	4·17	3·52	2·75
28	13·50	8·93	7·19	6·25	5·66	5·24	4·93	4·69	4·35	4·11	3·46	2·69
29	13·39	8·85	7·12	6·19	5·59	5·18	4·87	4·64	4·29	4·05	3·41	2·64
30	13·29	8·77	7·05	6·12	5·53	5·12	4·82	4·58	4·24	4·00	3·36	2·59
32	13·12	8·64	6·94	6·01	5·43	5·02	4·72	4·48	4·14	3·91	3·27	2·50
34	12·97	8·52	6·83	5·92	5·34	4·93	4·63	4·40	4·06	3·83	3·19	2·42
36	12·83	8·42	6·74	5·84	5·26	4·86	4·56	4·33	3·99	3·76	3·12	2·35
38	12·71	8·33	6·66	5·76	5·19	4·79	4·49	4·26	3·93	3·70	3·06	2·29
40	12·61	8·25	6·59	5·70	5·13	4·73	4·44	4·21	3·87	3·64	3·01	2·23
60	11·97	7·77	6·17	5·31	4·76	4·37	4·09	3·86	3·54	3·32	2·69	1·89
120	11·38	7·32	5·78	4·95	4·42	4·04	3·77	3·55	3·24	3·02	2·40	1·54
∞	10·83	6·91	5·42	4·62	4·10	3·74	3·47	3·27	2·96	2·74	2·13	1·00

when ν_1 and ν_2 are both large, interpolation should be linear in $\nu_1 F_P$ or $\nu_2 F_P$ (this is equivalent to harmonic interpolation). When ν_1 and ν_2 are both large the percentage points may be found from the formula

$$1{\cdot}1513 \log F_P = \tfrac{1}{2} \ln F_P = \frac{x_P\sqrt{h+\lambda}}{h} - \left(\frac{1}{\nu_1 - 1} - \frac{1}{\nu_2 - 1}\right)\left(\lambda + \frac{5}{6}\right)$$

where x_P is the P-per cent point of the normal distribution (Table 2), $\lambda = \frac{1}{6}(x_P^2 - 3)$ and $\frac{2}{h} = \frac{1}{\nu_1 - 1} + \frac{1}{\nu_2 - 1}$. For the values of P given in Table 7, x_P and λ are as follows:

P	5	$2\frac{1}{2}$	1	0·1
x_P	+1·6449	1·9600	2·3263	3·0902
λ	−0·0491	+0·1402	0·4020	1·0916

* Entries for $\nu_2 = 1$ must be multiplied by 100.