UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2006

TITLE OF PAPER: TOPICS IN STATISTICS

COURSE CODE : ST 405

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : THIS PAPER HAS FIVE QUESTIONS.
ANSWER ANY FOUR(4) QUESTIONS.
EACH QUESTION CARRIES 15 MARKS.

REQUIREMENTS: Scientific Calculator

PLEASE DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GRANTEI BY THE INVIGILATOR

QUESTION ONE

(a) The first ten sample autocorrelation coefficients of 400 random numbers are:
$r_1 = 0.02, r_2 = 0.05, r_3 = -0.09, r_4 = 0.08, r_5 = -0.02, r_6 = 0.00, r_7 = 0.12, r_8 = 0.06, r_9 = 0.02, r_{10} =$
Is there any evidence of non-randomness?

(b) Sixteen successive observations on a given time series are as follows:
1.6, 0.8, 1.2, 0.5, 0.9, 1.1, 1.1, 0.6, 1.5, 0.8, 0.9, 1.2, 0.5, 1.3, 0.8, 1.2

(i) Is the series stationary?
(ii) Calculate r_1.
(iii) When is a time series (x_t) weakly stationary?

(3+2+5+5)Mark

QUESTION TWO

(a) Explain what is meant by:

(i) Deseasonalised series.
(ii) Seasonal variation
(iii) Moving average

(b) The following data are on the production of maize('000 tons in a Country:

YEAR	1951	1952	1953	1954	1955	1956
PRODUCTION ('000 tons)	4	3	4	5	9	9
YEAR	1957	1958	1959	1960	1961	1962
PRODUCTION ('000 tons)	15	10	26	17	18	31
YEAR	1963	1964	1965			
PRODUCTION ('000 tons)	35	34	40			

(i) Draw the time plot of the data.
(ii) Using the Least Square Method ,fit the linear trend Y=a+bT, when T=t-8
(iii) Calculate the trend value for each year.
(iv) Forecast the production for 1970 using the fitted trend.

(1+1+1+2+5+3+2)Mar

QUESTION THREE

(a) Show that for an autoregressive process of order 2 given as $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t$, the variance of X_t is given by

$$Var(X_t) = \left(\frac{1-\alpha_2}{1+\alpha_2}\right)\left[\frac{\sigma_\varepsilon^2}{(1-\alpha_2)^2 - \alpha_1^2}\right].$$

(b) Prove that the general solution of the equation $\rho_k - \alpha_1\rho_{k-1} - \alpha_2\rho_{k-2} = 0$ is

$$\rho_k = \frac{(1-\mu_2^2)\mu_1^{k+1} - (1-\mu_1^2)\mu_2^{k+1}}{(1-\mu_1\mu_2)(\mu_1 - \mu_2)}.$$

(8+7)Mar

QUESTION FOUR

(a) The discrete parameter stationary process $\{X_t\}$ is generated by $X_t - \lambda X_{t-1} = w_t; t = 0, \pm 1, \pm 2.....$ Where λ is a constant $|\lambda| \leq 1$ and w_t satisfies the equation $w_t - \mu w_{t-1} = \varepsilon_t$. μ being a constant $(|\mu| < 1)$ ar (ε_t) is a purely random process with mean zero and variance σ^2. Show that $\{X_t\}$ is a stationary AR (2) process. Hence or otherwise determine the spectral density function.

(b) Suppose a stationary process $\{X_t\}$ can be represented in following two equivalent forms; $X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + ... = \varepsilon_t$ and $X_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ...$ .Let $\pi(B) = \sum \alpha_j B^j$ and $\psi(B) = \sum \beta_j B^j$,

Show that $\pi(B) = \psi^{-1}(B)$.

(5+5+5)Mar

QUESTION FIVE

(a) Given the Markov process $X_t = \phi_1 X_{t-1} + \varepsilon_t$. Show that the canonic factorisation of the Spectral density function for this process is

$$f(\omega) = \frac{\sigma^2}{2\pi\left(1 + \phi_1^2 - 2\phi_1 \cos\omega\right)}.$$

(b) Find the spectral density function of the following moving average processes:

(i) $X_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2}$.

(ii) $X_t = \varepsilon_t + 0.5\varepsilon_{t-1} + 0.3\varepsilon_{t-2}$

(9+3+3)Mar