

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2006

TITLE OF PAPER : **NONPARAMETRIC ANALYSIS**
COURSE CODE : **ST 409**
TIME ALLOWED : **TWO (2) HOURS**
REQUIREMENTS : **CALCULATOR AND STATISTICAL TABLES**
INSTRUCTIONS : **ANSWER QUESTION ONE AND ANY OTHER
TWO QUESTIONS**

Question 1

- a) What distinguishes t-test for correlated groups from Wilcoxon signed-rank test for paired observations? Explain, giving some examples. (5 Marks)
- b) The following data represent the time, in minutes, that 10 randomly selected patients had to wait at a doctor's office before being seen by the doctor:

15 21 19 32 28 26 25 25 35 24

Test the doctor's claim that the median waiting time for her patients is not more than 20 minutes before being admitted to the examination room. (15 Marks)

Question 2

For data that come from a pet shop database a researcher groups data for analysis into two sets. The grouping variable **chain** (1 = chain store, 2 = privately owned store, 3 = cooperative owned store) and the response variable **fishnum** (number of fish on display). Below are scores for 12 stores;

Chain	3	3	3	3	1	1	2	1	2	1	2	1
Fishnum	32	41	31	38	21	13	17	22	24	11	17	20

Do the three different types of pet shops have the same median number of fish displayed? Use 5% level of significance. (20 Marks)

Question 3

Physicians depend on laboratory results when managing medical problems such as diabetes or epilepsy. In a test for glucose tolerance, three laboratories were each sent 5 identical blood samples from a person who had drunk 50 milligrams of glucose dissolved in water. The laboratory results are listed here:

Lab1	120.1	110.7	108.9	104.2	100.4
Lab2	98.3	112.1	107.7	107.9	99.2
Lab3	103.0	108.5	101.1	110.0	105.4

Assuming that the results are not normally distributed do they indicate a significant difference in readings for the three laboratories? Use $\alpha = 0.05$ (20 Marks)

Question 4

The number of hours of study for an examination and the grades received by a random sample of 10 students are:

No. of Hours	8	5	11	13	10	5	18	15	2	8
Grades	56	44	79	72	70	54	94	85	33	65

Use an appropriate nonparametric test to evaluate the independence between the number of hours studied and grades received in the examination. Use $\alpha = 0.05$

(20 Marks)

TABLE A1 Normal Distribution*

p	Selected values									
	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00										
0.01	-2.3263	-2.2904	-2.2571	-2.2262	-2.1973	-2.1701	-2.1444	-2.1201	-2.0969	-2.0749
0.02	-2.0537	-2.0335	-2.0141	-1.9954	-1.9774	-1.9600	-1.9431	-1.9268	-1.9110	-1.8957
0.03	-1.8808	-1.8663	-1.8522	-1.8384	-1.8250	-1.8119	-1.7991	-1.7866	-1.7744	-1.7624
0.04	-1.7507	-1.7392	-1.7279	-1.7169	-1.7060	-1.6954	-1.6849	-1.6747	-1.6646	-1.6546
0.05	-1.6449	-1.6352	-1.6258	-1.6164	-1.6072	-1.5982	-1.5893	-1.5805	-1.5718	-1.5632
0.06	-1.5548	-1.5464	-1.5382	-1.5301	-1.5220	-1.5141	-1.5063	-1.4985	-1.4909	-1.4833
0.07	-1.4758	-1.4684	-1.4611	-1.4538	-1.4466	-1.4395	-1.4325	-1.4255	-1.4187	-1.4118
0.08	-1.4051	-1.3984	-1.3917	-1.3852	-1.3787	-1.3722	-1.3658	-1.3595	-1.3532	-1.3469
0.09	-1.3408	-1.3346	-1.3285	-1.3225	-1.3165	-1.3106	-1.3047	-1.2988	-1.2930	-1.2873
0.10	-1.2816	-1.2759	-1.2702	-1.2646	-1.2591	-1.2536	-1.2481	-1.2426	-1.2372	-1.2319
0.11	-1.2265	-1.2212	-1.2160	-1.2107	-1.2055	-1.2004	-1.1952	-1.1901	-1.1850	-1.1800
0.12	-1.1750	-1.1700	-1.1650	-1.1601	-1.1552	-1.1503	-1.1455	-1.1407	-1.1359	-1.1311
0.13	-1.1264	-1.1217	-1.1170	-1.1123	-1.1077	-1.1031	-1.0985	-1.0939	-1.0893	-1.0848
0.14	-1.0803	-1.0758	-1.0714	-1.0669	-1.0625	-1.0581	-1.0537	-1.0494	-1.0450	-1.0407
0.15	-1.0364	-1.0322	-1.0279	-1.0237	-1.0194	-1.0152	-1.0110	-1.0069	-1.0027	-0.9986
0.16	-0.9945	-0.9904	-0.9863	-0.9822	-0.9782	-0.9741	-0.9701	-0.9661	-0.9621	-0.9581
0.17	-0.9542	-0.9502	-0.9463	-0.9424	-0.9385	-0.9346	-0.9307	-0.9269	-0.9230	-0.9192
0.18	-0.9154	-0.9116	-0.9078	-0.9040	-0.9002	-0.8965	-0.8927	-0.8890	-0.8853	-0.8816
0.19	-0.8779	-0.8742	-0.8705	-0.8669	-0.8633	-0.8596	-0.8560	-0.8524	-0.8488	-0.8452
0.20	-0.8416	-0.8381	-0.8345	-0.8310	-0.8274	-0.8239	-0.8204	-0.8169	-0.8134	-0.8099
0.21	-0.8064	-0.8030	-0.7995	-0.7961	-0.7926	-0.7892	-0.7858	-0.7824	-0.7790	-0.7756
0.22	-0.7722	-0.7688	-0.7655	-0.7621	-0.7588	-0.7554	-0.7521	-0.7488	-0.7454	-0.7421
0.23	-0.7388	-0.7356	-0.7323	-0.7290	-0.7257	-0.7225	-0.7192	-0.7160	-0.7128	-0.7095
0.24	-0.7063	-0.7031	-0.6999	-0.6967	-0.6935	-0.6903	-0.6871	-0.6840	-0.6808	-0.6776

Selected values
 $Z_{0.0001} = -3.7190$ $Z_{0.0005} = -3.2905$ $Z_{0.0025} = -1.9600$ $Z_{0.05} = -1.6449$
 $Z_{0.9999} = 3.7190$ $Z_{0.9995} = 3.2905$ $Z_{0.975} = 1.9600$ $Z_{0.95} = 1.6449$

TABLE A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.25	-0.6745	-0.6713	-0.6682	-0.6651	-0.6620	-0.6588	-0.6557	-0.6526	-0.6495	-0.6464
0.26	-0.6433	-0.6403	-0.6372	-0.6341	-0.6311	-0.6280	-0.6250	-0.6219	-0.6189	-0.6158
0.27	-0.6128	-0.6098	-0.6068	-0.6038	-0.6008	-0.5978	-0.5948	-0.5918	-0.5888	-0.5858
0.28	-0.5828	-0.5799	-0.5769	-0.5740	-0.5710	-0.5681	-0.5651	-0.5622	-0.5592	-0.5563
0.29	-0.5534	-0.5505	-0.5476	-0.5446	-0.5417	-0.5388	-0.5359	-0.5330	-0.5302	-0.5273
0.30	-0.5244	-0.5215	-0.5187	-0.5158	-0.5129	-0.5101	-0.5072	-0.5044	-0.5015	-0.4987
0.31	-0.4959	-0.4930	-0.4902	-0.4874	-0.4845	-0.4817	-0.4789	-0.4761	-0.4733	-0.4705
0.32	-0.4677	-0.4649	-0.4621	-0.4593	-0.4565	-0.4538	-0.4510	-0.4482	-0.4454	-0.4427
0.33	-0.4399	-0.4372	-0.4344	-0.4316	-0.4289	-0.4261	-0.4234	-0.4207	-0.4179	-0.4152
0.34	-0.4125	-0.4097	-0.4070	-0.4043	-0.4016	-0.3989	-0.3961	-0.3934	-0.3907	-0.3880
0.35	-0.3853	-0.3826	-0.3799	-0.3772	-0.3745	-0.3719	-0.3692	-0.3665	-0.3638	-0.3611
0.36	-0.3585	-0.3558	-0.3531	-0.3505	-0.3478	-0.3451	-0.3425	-0.3398	-0.3372	-0.3345
0.37	-0.3319	-0.3292	-0.3266	-0.3239	-0.3213	-0.3186	-0.3160	-0.3134	-0.3107	-0.3081
0.38	-0.3055	-0.3029	-0.3002	-0.2976	-0.2950	-0.2924	-0.2898	-0.2871	-0.2845	-0.2819
0.39	-0.2793	-0.2767	-0.2741	-0.2715	-0.2689	-0.2663	-0.2637	-0.2611	-0.2585	-0.2559
0.40	-0.2533	-0.2508	-0.2482	-0.2456	-0.2430	-0.2404	-0.2378	-0.2353	-0.2327	-0.2301
0.41	-0.2275	-0.2250	-0.2224	-0.2198	-0.2173	-0.2147	-0.2121	-0.2096	-0.2070	-0.2045
0.42	-0.2019	-0.1993	-0.1968	-0.1942	-0.1917	-0.1891	-0.1866	-0.1840	-0.1815	-0.1789
0.43	-0.1764	-0.1738	-0.1713	-0.1687	-0.1662	-0.1637	-0.1611	-0.1586	-0.1560	-0.1535
0.44	-0.1510	-0.1484	-0.1459	-0.1434	-0.1408	-0.1383	-0.1358	-0.1332	-0.1307	-0.1282
0.45	-0.1257	-0.1231	-0.1206	-0.1181	-0.1156	-0.1130	-0.1105	-0.1080	-0.1055	-0.1030
0.46	-0.1004	-0.0979	-0.0954	-0.0929	-0.0904	-0.0878	-0.0853	-0.0828	-0.0803	-0.0778
0.47	-0.0753	-0.0728	-0.0702	-0.0677	-0.0652	-0.0627	-0.0602	-0.0577	-0.0552	-0.0527
0.48	-0.0502	-0.0476	-0.0451	-0.0426	-0.0401	-0.0376	-0.0351	-0.0326	-0.0301	-0.0276
0.49	-0.0251	-0.0226	-0.0201	-0.0175	-0.0150	-0.0125	-0.0100	-0.0075	-0.0050	-0.0025
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

SOURCE. Generated by R. L. Iman. Used with permission.

* The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

TABLE A2 Chi-Squared Distribution^a

	$p = 0.750$	0.900	0.950	0.975	0.990	0.995	0.999
$k = 1$	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	28.24	33.20	36.42	39.37	42.98	45.56	51.18
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.4
z_p	0.675	1.282	1.645	1.960	2.326	2.576	3.090

For $k > 100$ use the approximation $w_p = (\frac{2}{3})(z_p + \sqrt{2k-1})^2$, or the more accurate $w_p = k \left(1 - \frac{2}{9k} + z_p \sqrt{\frac{2}{9k}} \right)^3$, where z_p is the value from the standardized normal distribution shown in the bottom of the table.

SOURCE: Abridged from Table 8, Vol. 1 of Pearson and Hartley (1976), with permission from the *Biometrika*, Trustees.

^a The entries in this table are quantiles w_p of a chi-squared random variable W with k degrees of freedom, selected so $P(W \leq w_p) = p$ and $P(W > w_p) = 1 - p$.

TABLE A3 (Continued)

<i>n</i>	<i>y</i>	<i>p</i> = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000
	11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002
	14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020
	15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* *Y* has the binomial distribution with parameters *n* and *p*. The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for *p* ranging from 0.05 to 0.95.

For *n* larger than 20, the *r*th quantile *y_r* of a binomial random variable may be approximated using $y_r = np + z_r \sqrt{np(1-p)}$, where *z_r* is the *r*th quantile of a standard normal random variable, obtained from Table A1.

TABLE A4 Exact Confidence Intervals for the Binomial Parameter p

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
1	0	0.000	0.950	0.000	0.975	0.000	0.995
	1	0.050	1.000	0.025	1.000	0.005	1.000
2	0	0.000	0.776	0.000	0.842	0.000	0.929
	1	0.025	0.975	0.013	0.987	0.003	0.997
	2	0.224	1.000	0.158	1.000	0.071	1.000
3	0	0.000	0.632	0.000	0.708	0.000	0.829
	1	0.017	0.865	0.008	0.906	0.002	0.959
	2	0.135	0.983	0.094	0.992	0.041	0.998
	3	0.368	1.000	0.292	1.000	0.171	1.000
4	0	0.000	0.527	0.000	0.602	0.000	0.734
	1	0.013	0.751	0.006	0.806	0.001	0.889
	2	0.098	0.902	0.068	0.932	0.029	0.971
	3	0.249	0.987	0.194	0.994	0.111	0.999
	4	0.473	1.000	0.398	1.000	0.266	1.000
5	0	0.000	0.451	0.000	0.522	0.000	0.653
	1	0.010	0.657	0.005	0.716	0.001	0.815
	2	0.076	0.811	0.053	0.853	0.023	0.917
	3	0.189	0.924	0.147	0.947	0.083	0.977
	4	0.343	0.990	0.284	0.995	0.185	0.999
	5	0.549	1.000	0.478	1.000	0.347	1.000
6	0	0.000	0.393	0.000	0.459	0.000	0.586
	1	0.009	0.582	0.004	0.641	0.001	0.746
	2	0.063	0.729	0.043	0.777	0.019	0.856
	3	0.153	0.847	0.118	0.882	0.066	0.934
	4	0.271	0.937	0.223	0.957	0.144	0.981
	5	0.418	0.991	0.359	0.996	0.254	0.999
	6	0.607	1.000	0.541	1.000	0.414	1.000
7	0	0.000	0.348	0.000	0.410	0.000	0.531
	1	0.007	0.521	0.004	0.579	0.001	0.685
	2	0.053	0.659	0.037	0.710	0.016	0.797
	3	0.129	0.775	0.099	0.816	0.055	0.882
	4	0.225	0.871	0.184	0.901	0.118	0.945
	5	0.341	0.947	0.290	0.963	0.203	0.984
	6	0.479	0.993	0.421	0.996	0.315	0.999
	7	0.652	1.000	0.590	1.000	0.469	1.000

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
8	0	0.000	0.312	0.000	0.369	0.000	0.484
	1	0.006	0.471	0.003	0.526	0.001	0.632
	2	0.046	0.600	0.032	0.651	0.014	0.742
	3	0.111	0.711	0.085	0.755	0.047	0.830
	4	0.193	0.807	0.157	0.843	0.100	0.900
	5	0.289	0.889	0.245	0.915	0.170	0.953
	6	0.400	0.954	0.349	0.968	0.258	0.986
	7	0.529	0.994	0.474	0.997	0.368	0.999
9	8	0.688	1.000	0.631	1.000	0.516	1.000
	0	0.000	0.283	0.000	0.336	0.000	0.445
	1	0.006	0.429	0.003	0.482	0.001	0.585
	2	0.041	0.550	0.028	0.600	0.012	0.693
	3	0.098	0.655	0.075	0.701	0.042	0.781
	4	0.169	0.749	0.137	0.788	0.087	0.854
	5	0.251	0.831	0.212	0.863	0.146	0.913
	6	0.345	0.902	0.299	0.925	0.219	0.958
	7	0.450	0.959	0.400	0.972	0.307	0.988
10	8	0.571	0.994	0.518	0.997	0.415	0.999
	9	0.717	1.000	0.664	1.000	0.555	1.000
	0	0.000	0.259	0.000	0.308	0.000	0.411
	1	0.005	0.394	0.003	0.445	0.001	0.544
	2	0.037	0.507	0.025	0.556	0.011	0.648
	3	0.087	0.607	0.067	0.652	0.037	0.735
	4	0.150	0.696	0.122	0.738	0.077	0.809
	5	0.222	0.778	0.187	0.813	0.128	0.872
	6	0.304	0.850	0.262	0.878	0.191	0.923
	7	0.393	0.913	0.348	0.933	0.265	0.963
11	8	0.493	0.963	0.444	0.975	0.352	0.989
	9	0.606	0.995	0.555	0.997	0.456	0.999
	10	0.741	1.000	0.692	1.000	0.589	1.000
	0	0.000	0.238	0.000	0.285	0.000	0.382
	1	0.005	0.364	0.002	0.413	0.000	0.509
	2	0.033	0.470	0.023	0.518	0.010	0.608
	3	0.079	0.564	0.060	0.610	0.033	0.693
	4	0.135	0.650	0.109	0.692	0.069	0.767
	5	0.200	0.729	0.167	0.766	0.115	0.831
	6	0.271	0.800	0.234	0.833	0.169	0.885
	7	0.350	0.865	0.308	0.891	0.233	0.931
8	0.436	0.921	0.390	0.940	0.307	0.967	
9	0.530	0.967	0.482	0.977	0.392	0.990	
10	0.636	0.995	0.587	0.998	0.491	1.000	
11	0.762	1.000	0.715	1.000	0.618	1.000	

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
12	0	0.000	0.221	0.000	0.265	0.000	0.357
	1	0.004	0.339	0.002	0.385	0.000	0.477
	2	0.030	0.438	0.021	0.484	0.009	0.573
	3	0.072	0.527	0.055	0.572	0.030	0.655
	4	0.123	0.609	0.099	0.651	0.062	0.728
	5	0.181	0.685	0.152	0.723	0.103	0.792
	6	0.245	0.755	0.211	0.789	0.152	0.848
	7	0.315	0.819	0.277	0.848	0.208	0.897
	8	0.391	0.877	0.349	0.901	0.272	0.938
	9	0.473	0.928	0.428	0.945	0.345	0.970
	10	0.562	0.970	0.516	0.979	0.427	0.991
	11	0.661	0.996	0.615	0.998	0.523	1.000
12	0.779	1.000	0.735	1.000	0.643	1.000	
13	0	0.000	0.206	0.000	0.247	0.000	0.335
	1	0.004	0.316	0.002	0.360	0.000	0.449
	2	0.028	0.410	0.019	0.454	0.008	0.541
	3	0.066	0.495	0.050	0.538	0.028	0.621
	4	0.113	0.573	0.091	0.614	0.057	0.691
	5	0.166	0.645	0.139	0.684	0.094	0.755
	6	0.224	0.713	0.192	0.749	0.138	0.811
	7	0.287	0.776	0.251	0.808	0.189	0.862
	8	0.355	0.834	0.316	0.861	0.245	0.906
	9	0.427	0.887	0.386	0.909	0.309	0.943
	10	0.505	0.934	0.462	0.950	0.379	0.972
	11	0.590	0.972	0.546	0.981	0.459	0.992
	12	0.684	0.996	0.640	0.998	0.551	1.000
13	0.794	1.000	0.753	1.000	0.665	1.000	
14	0	0.000	0.193	0.000	0.232	0.000	0.315
	1	0.004	0.297	0.002	0.339	0.000	0.424
	2	0.026	0.385	0.018	0.428	0.008	0.512
	3	0.061	0.466	0.047	0.508	0.026	0.589
	4	0.104	0.540	0.084	0.581	0.053	0.658
	5	0.153	0.610	0.128	0.649	0.087	0.720
	6	0.206	0.675	0.177	0.711	0.127	0.777
	7	0.264	0.736	0.230	0.770	0.172	0.828
	8	0.325	0.794	0.289	0.823	0.223	0.873
	9	0.390	0.847	0.351	0.872	0.280	0.913
	10	0.460	0.896	0.419	0.916	0.342	0.947
	11	0.534	0.939	0.492	0.953	0.411	0.974
	12	0.615	0.974	0.572	0.982	0.488	0.992
	13	0.703	0.996	0.661	0.998	0.576	1.000
14	0.807	1.000	0.768	1.000	0.685	1.000	

TABLE A4 (Continued)

<i>n</i>	<i>Y</i>	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
15	0	0.000	0.181	0.000	0.218	0.000	0.298
	1	0.003	0.279	0.002	0.319	0.000	0.402
	2	0.024	0.363	0.017	0.405	0.007	0.486
	3	0.057	0.440	0.043	0.481	0.024	0.561
	4	0.097	0.511	0.078	0.551	0.049	0.627
	5	0.142	0.577	0.118	0.616	0.080	0.688
	6	0.191	0.640	0.163	0.677	0.117	0.744
	7	0.244	0.700	0.213	0.734	0.159	0.795
	8	0.300	0.756	0.266	0.787	0.205	0.841
	9	0.360	0.809	0.323	0.837	0.256	0.883
	10	0.423	0.858	0.384	0.882	0.312	0.920
	11	0.489	0.903	0.449	0.922	0.373	0.951
	12	0.560	0.943	0.519	0.957	0.439	0.976
	13	0.637	0.976	0.595	0.983	0.514	0.993
	14	0.721	0.997	0.681	0.998	0.598	1.000
15	0.819	1.000	0.782	1.000	0.702	1.000	
16	0	0.000	0.171	0.000	0.206	0.000	0.282
	1	0.003	0.264	0.002	0.302	0.000	0.381
	2	0.023	0.344	0.016	0.383	0.007	0.463
	3	0.053	0.417	0.040	0.456	0.022	0.534
	4	0.090	0.484	0.073	0.524	0.045	0.599
	5	0.132	0.548	0.110	0.587	0.075	0.658
	6	0.178	0.609	0.152	0.646	0.109	0.713
	7	0.227	0.667	0.198	0.701	0.147	0.764
	8	0.279	0.721	0.247	0.753	0.190	0.810
	9	0.333	0.773	0.299	0.802	0.236	0.853
	10	0.391	0.822	0.354	0.848	0.287	0.891
	11	0.452	0.868	0.413	0.890	0.342	0.925
	12	0.516	0.910	0.476	0.927	0.401	0.955
	13	0.583	0.947	0.544	0.960	0.466	0.978
	14	0.656	0.977	0.617	0.984	0.537	0.993
	15	0.736	0.997	0.698	0.998	0.619	1.000
	16	0.829	1.000	0.794	1.000	0.718	1.000

TABLE A4 (Continued)

<i>n</i>	<i>Y</i>	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
17	0	0.000	0.162	0.000	0.195	0.000	0.268
	1	0.003	0.250	0.001	0.287	0.000	0.363
	2	0.021	0.326	0.015	0.364	0.006	0.441
	3	0.050	0.396	0.038	0.434	0.021	0.510
	4	0.085	0.461	0.068	0.499	0.043	0.573
	5	0.124	0.522	0.103	0.560	0.070	0.631
	6	0.166	0.580	0.142	0.617	0.101	0.685
	7	0.212	0.636	0.184	0.671	0.137	0.734
	8	0.260	0.689	0.230	0.722	0.176	0.781
	9	0.311	0.740	0.278	0.770	0.219	0.824
	10	0.364	0.788	0.329	0.816	0.266	0.863
	11	0.420	0.834	0.383	0.858	0.315	0.899
	12	0.478	0.876	0.440	0.897	0.369	0.930
	13	0.539	0.915	0.501	0.932	0.427	0.957
	14	0.604	0.950	0.566	0.962	0.490	0.979
	15	0.674	0.979	0.636	0.985	0.559	0.994
	16	0.750	0.997	0.713	0.999	0.637	1.000
17	0.838	1.000	0.805	1.000	0.732	1.000	
18	0	0.000	0.153	0.000	0.185	0.000	0.255
	1	0.003	0.238	0.001	0.273	0.000	0.346
	2	0.020	0.310	0.014	0.347	0.006	0.422
	3	0.047	0.377	0.036	0.414	0.020	0.488
	4	0.080	0.439	0.064	0.476	0.040	0.549
	5	0.116	0.498	0.097	0.535	0.065	0.605
	6	0.156	0.554	0.133	0.590	0.095	0.658
	7	0.199	0.608	0.173	0.643	0.128	0.707
	8	0.244	0.659	0.215	0.692	0.165	0.753
	9	0.291	0.709	0.260	0.740	0.205	0.795
	10	0.341	0.756	0.308	0.785	0.247	0.835
	11	0.392	0.801	0.357	0.827	0.293	0.872
	12	0.446	0.844	0.410	0.867	0.342	0.905
	13	0.502	0.884	0.465	0.903	0.395	0.935
	14	0.561	0.920	0.524	0.936	0.451	0.960
	15	0.623	0.953	0.586	0.964	0.512	0.980
	16	0.690	0.980	0.653	0.986	0.578	0.994
	17	0.762	0.997	0.727	0.999	0.654	1.000
18	0.847	1.000	0.815	1.000	0.745	1.000	

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
19	0	0.000	0.146	0.000	0.176	0.000	0.243
	1	0.003	0.226	0.001	0.260	0.000	0.331
	2	0.019	0.296	0.013	0.331	0.006	0.404
	3	0.044	0.359	0.034	0.396	0.019	0.468
	4	0.075	0.419	0.061	0.456	0.038	0.527
	5	0.110	0.476	0.091	0.512	0.062	0.582
	6	0.147	0.530	0.126	0.565	0.089	0.633
	7	0.188	0.582	0.163	0.616	0.121	0.681
	8	0.230	0.632	0.203	0.665	0.155	0.726
	9	0.274	0.680	0.244	0.711	0.192	0.768
	10	0.320	0.726	0.289	0.756	0.232	0.808
	11	0.368	0.770	0.335	0.797	0.274	0.845
	12	0.418	0.813	0.384	0.837	0.319	0.879
	13	0.470	0.853	0.435	0.874	0.367	0.911
	14	0.524	0.890	0.488	0.909	0.418	0.938
	15	0.581	0.925	0.544	0.939	0.473	0.962
	16	0.641	0.956	0.604	0.966	0.532	0.981
	17	0.704	0.981	0.669	0.987	0.596	0.994
	18	0.774	0.997	0.740	0.999	0.669	1.000
19	0.854	1.000	0.824	1.000	0.757	1.000	
20	0	0.000	0.139	0.000	0.168	0.000	0.233
	1	0.003	0.216	0.001	0.249	0.000	0.317
	2	0.018	0.283	0.012	0.317	0.005	0.387
	3	0.042	0.344	0.032	0.379	0.018	0.449
	4	0.071	0.401	0.057	0.437	0.036	0.507
	5	0.104	0.456	0.087	0.491	0.058	0.560
	6	0.140	0.508	0.119	0.543	0.085	0.610
	7	0.177	0.558	0.154	0.592	0.114	0.657
	8	0.217	0.606	0.191	0.639	0.146	0.701
	9	0.259	0.653	0.231	0.685	0.181	0.743
	10	0.302	0.698	0.272	0.728	0.218	0.782
	11	0.347	0.741	0.315	0.769	0.257	0.819
	12	0.394	0.783	0.361	0.809	0.299	0.854
	13	0.442	0.823	0.408	0.846	0.343	0.886
	14	0.492	0.860	0.457	0.881	0.390	0.915
	15	0.544	0.896	0.509	0.913	0.440	0.942
	16	0.599	0.929	0.563	0.943	0.493	0.964
	17	0.656	0.958	0.621	0.968	0.551	0.982
	18	0.717	0.982	0.683	0.988	0.613	0.995
	19	0.784	0.997	0.751	0.999	0.683	1.000
	20	0.861	1.000	0.832	1.000	0.767	1.000

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
21	0	0.000	0.133	0.000	0.161	0.000	0.223
	1	0.002	0.207	0.001	0.238	0.000	0.304
	2	0.017	0.271	0.012	0.304	0.005	0.372
	3	0.040	0.329	0.030	0.363	0.017	0.432
	4	0.068	0.384	0.054	0.419	0.034	0.488
	5	0.099	0.437	0.082	0.472	0.055	0.539
	6	0.132	0.487	0.113	0.522	0.080	0.588
	7	0.168	0.536	0.146	0.570	0.108	0.634
	8	0.206	0.583	0.181	0.616	0.138	0.677
	9	0.245	0.628	0.218	0.660	0.171	0.719
	10	0.286	0.672	0.257	0.702	0.205	0.758
	11	0.328	0.714	0.298	0.743	0.242	0.795
	12	0.372	0.755	0.340	0.782	0.281	0.829
	13	0.417	0.794	0.384	0.819	0.323	0.862
	14	0.464	0.832	0.430	0.854	0.366	0.892
	15	0.513	0.868	0.478	0.887	0.412	0.920
	16	0.563	0.901	0.528	0.918	0.461	0.945
	17	0.616	0.932	0.581	0.946	0.512	0.966
	18	0.671	0.960	0.637	0.970	0.568	0.983
	19	0.729	0.983	0.696	0.988	0.628	0.995
	20	0.793	0.998	0.762	0.999	0.696	1.000
21	0.867	1.000	0.839	1.000	0.777	1.000	
22	0	0.000	0.127	0.000	0.154	0.000	0.214
	1	0.002	0.198	0.001	0.228	0.000	0.292
	2	0.016	0.259	0.011	0.292	0.005	0.358
	3	0.038	0.316	0.029	0.349	0.016	0.416
	4	0.065	0.369	0.052	0.403	0.032	0.470
	5	0.094	0.420	0.078	0.454	0.053	0.520
	6	0.126	0.468	0.107	0.502	0.076	0.567
	7	0.160	0.515	0.139	0.549	0.102	0.612
	8	0.196	0.561	0.172	0.593	0.131	0.655
	9	0.233	0.605	0.207	0.636	0.162	0.695
	10	0.271	0.647	0.244	0.678	0.195	0.734
	11	0.311	0.689	0.282	0.718	0.229	0.771
	12	0.353	0.729	0.322	0.756	0.266	0.805
	13	0.395	0.767	0.364	0.793	0.305	0.838
	14	0.439	0.804	0.407	0.828	0.345	0.869
	15	0.485	0.840	0.451	0.861	0.388	0.898
	16	0.532	0.874	0.498	0.893	0.433	0.924
	17	0.580	0.906	0.546	0.922	0.480	0.947
	18	0.631	0.935	0.597	0.948	0.530	0.968
	19	0.684	0.962	0.651	0.971	0.584	0.984
	20	0.741	0.984	0.708	0.989	0.642	0.995

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
23	21	0.802	0.998	0.772	0.999	0.708	1.000
	22	0.873	1.000	0.846	1.000	0.786	1.000
	0	0.000	0.122	0.000	0.148	0.000	0.206
	1	0.002	0.190	0.001	0.219	0.000	0.281
	2	0.016	0.249	0.011	0.280	0.005	0.345
	3	0.037	0.304	0.028	0.336	0.015	0.401
	4	0.062	0.355	0.050	0.388	0.031	0.453
	5	0.090	0.404	0.075	0.437	0.050	0.502
	6	0.120	0.451	0.102	0.484	0.073	0.548
	7	0.152	0.496	0.132	0.529	0.097	0.592
	8	0.186	0.540	0.164	0.573	0.125	0.634
	9	0.222	0.583	0.197	0.615	0.154	0.674
	10	0.258	0.625	0.232	0.655	0.185	0.712
	11	0.296	0.665	0.268	0.694	0.218	0.748
	12	0.335	0.704	0.306	0.732	0.252	0.782
	13	0.375	0.742	0.345	0.768	0.288	0.815
	14	0.417	0.778	0.385	0.803	0.326	0.846
	15	0.460	0.814	0.427	0.836	0.366	0.875
	16	0.504	0.848	0.471	0.868	0.408	0.903
	17	0.549	0.880	0.516	0.898	0.452	0.927
	18	0.596	0.910	0.563	0.925	0.498	0.950
	19	0.645	0.938	0.612	0.950	0.547	0.969
	20	0.696	0.963	0.664	0.972	0.599	0.985
21	0.751	0.984	0.720	0.989	0.655	0.995	
22	0.810	0.998	0.781	0.999	0.719	1.000	
23	0.878	1.000	0.852	1.000	0.794	1.000	
24	0	0.000	0.117	0.000	0.142	0.000	0.198
	1	0.002	0.183	0.001	0.211	0.000	0.271
	2	0.015	0.240	0.010	0.270	0.004	0.332
	3	0.035	0.292	0.027	0.324	0.015	0.387
	4	0.059	0.342	0.047	0.374	0.029	0.438
	5	0.086	0.389	0.071	0.422	0.048	0.485
	6	0.115	0.435	0.098	0.467	0.069	0.530
	7	0.146	0.479	0.126	0.511	0.093	0.573
	8	0.178	0.521	0.156	0.553	0.119	0.614
	9	0.212	0.563	0.188	0.594	0.146	0.653
	10	0.246	0.603	0.221	0.634	0.176	0.690
	11	0.282	0.642	0.256	0.672	0.207	0.726
	12	0.319	0.681	0.291	0.709	0.240	0.760
	13	0.358	0.718	0.328	0.744	0.274	0.793
	14	0.397	0.754	0.366	0.779	0.310	0.824
	15	0.437	0.788	0.406	0.812	0.347	0.854
	16	0.479	0.822	0.447	0.844	0.386	0.881
	17	0.521	0.854	0.489	0.874	0.427	0.907
	18	0.565	0.885	0.533	0.902	0.470	0.931
19	0.611	0.914	0.578	0.929	0.515	0.952	

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
25	20	0.658	0.941	0.626	0.953	0.562	0.971
	21	0.708	0.965	0.676	0.973	0.613	0.985
	22	0.760	0.985	0.730	0.990	0.668	0.996
	23	0.817	0.998	0.789	0.999	0.729	1.000
	24	0.883	1.000	0.858	1.000	0.802	1.000
	0	0.000	0.113	0.000	0.137	0.000	0.191
	1	0.002	0.176	0.001	0.204	0.000	0.262
	2	0.014	0.231	0.010	0.260	0.004	0.321
	3	0.034	0.282	0.025	0.312	0.014	0.374
	4	0.057	0.330	0.045	0.361	0.028	0.424
	5	0.082	0.375	0.068	0.407	0.046	0.470
	6	0.110	0.420	0.094	0.451	0.066	0.514
	7	0.139	0.462	0.121	0.494	0.089	0.555
	8	0.170	0.504	0.150	0.535	0.114	0.595
	9	0.202	0.544	0.180	0.575	0.140	0.634
	10	0.236	0.583	0.211	0.613	0.168	0.670
	11	0.270	0.621	0.244	0.651	0.197	0.705
	12	0.305	0.659	0.278	0.687	0.228	0.739
	13	0.341	0.695	0.313	0.722	0.261	0.772
	14	0.379	0.730	0.349	0.756	0.295	0.803
	15	0.417	0.764	0.387	0.789	0.330	0.832
	16	0.456	0.798	0.425	0.820	0.366	0.860
	17	0.496	0.830	0.465	0.850	0.405	0.886
	18	0.538	0.861	0.506	0.879	0.445	0.911
19	0.580	0.890	0.549	0.906	0.486	0.934	
20	0.625	0.918	0.593	0.932	0.530	0.954	
21	0.670	0.943	0.639	0.955	0.576	0.972	
22	0.718	0.966	0.688	0.975	0.626	0.986	
23	0.769	0.986	0.740	0.990	0.679	0.996	
24	0.824	0.998	0.796	0.999	0.738	1.000	
25	0.887	1.000	0.863	1.000	0.809	1.000	
26	0	0.000	0.109	0.000	0.132	0.000	0.184
	1	0.002	0.170	0.001	0.196	0.000	0.253
	2	0.014	0.223	0.009	0.251	0.004	0.310
	3	0.032	0.272	0.024	0.302	0.013	0.362
	4	0.054	0.318	0.044	0.349	0.027	0.410
	5	0.079	0.363	0.066	0.393	0.044	0.455
	6	0.106	0.405	0.090	0.436	0.064	0.498
	7	0.134	0.447	0.116	0.478	0.085	0.538
	8	0.163	0.487	0.143	0.518	0.109	0.578
	9	0.194	0.526	0.172	0.557	0.134	0.615
	10	0.226	0.564	0.202	0.594	0.161	0.651
	11	0.258	0.602	0.234	0.631	0.189	0.686
	12	0.292	0.638	0.266	0.666	0.218	0.719
	13	0.327	0.673	0.299	0.701	0.249	0.751
	14	0.362	0.708	0.334	0.734	0.281	0.782

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
	15	0.398	0.742	0.369	0.766	0.314	0.811
	16	0.436	0.774	0.406	0.798	0.349	0.839
	17	0.474	0.806	0.443	0.828	0.385	0.866
	18	0.513	0.837	0.482	0.857	0.422	0.891
	19	0.553	0.866	0.522	0.884	0.462	0.915
	20	0.595	0.894	0.564	0.910	0.502	0.936
	21	0.637	0.921	0.607	0.934	0.545	0.956
	22	0.682	0.946	0.651	0.956	0.590	0.973
	23	0.728	0.968	0.698	0.976	0.638	0.987
	24	0.777	0.986	0.749	0.991	0.690	0.996
	25	0.830	0.998	0.804	0.999	0.747	1.000
	26	0.891	1.000	0.868	1.000	0.816	1.000
27	0	0.000	0.105	0.000	0.128	0.000	0.178
	1	0.002	0.164	0.001	0.190	0.000	0.245
	2	0.013	0.215	0.009	0.243	0.004	0.300
	3	0.031	0.263	0.024	0.292	0.013	0.351
	4	0.052	0.308	0.042	0.337	0.026	0.397
	5	0.076	0.351	0.063	0.381	0.042	0.441
	6	0.101	0.392	0.086	0.423	0.061	0.483
	7	0.129	0.432	0.111	0.463	0.082	0.523
	8	0.157	0.471	0.138	0.502	0.104	0.561
	9	0.186	0.509	0.165	0.540	0.128	0.597
	10	0.217	0.547	0.194	0.576	0.154	0.633
	11	0.248	0.583	0.224	0.612	0.181	0.667
	12	0.280	0.618	0.255	0.647	0.209	0.700
	13	0.313	0.653	0.287	0.681	0.238	0.731
	14	0.347	0.687	0.319	0.713	0.269	0.762
	15	0.382	0.720	0.353	0.745	0.300	0.791
	16	0.417	0.752	0.388	0.776	0.333	0.819
	17	0.453	0.783	0.424	0.806	0.367	0.846
	18	0.491	0.814	0.460	0.835	0.403	0.872
	19	0.529	0.843	0.498	0.862	0.439	0.896
	20	0.568	0.871	0.537	0.889	0.477	0.918
	21	0.608	0.899	0.577	0.914	0.517	0.939
	22	0.649	0.924	0.619	0.937	0.559	0.958
	23	0.692	0.948	0.663	0.958	0.603	0.974
	24	0.737	0.969	0.708	0.976	0.649	0.987
	25	0.785	0.987	0.757	0.991	0.700	0.996
	26	0.836	0.998	0.810	0.999	0.755	1.000
	27	0.895	1.000	0.872	1.000	0.822	1.000
28	0	0.000	0.101	0.000	0.123	0.000	0.172
	1	0.002	0.159	0.001	0.183	0.000	0.237
	2	0.013	0.208	0.009	0.235	0.004	0.291
	3	0.030	0.254	0.023	0.282	0.012	0.340
	4	0.050	0.298	0.040	0.327	0.025	0.385

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
	5	0.073	0.339	0.061	0.369	0.041	0.428
	6	0.098	0.380	0.083	0.410	0.059	0.469
	7	0.124	0.419	0.107	0.449	0.079	0.508
	8	0.151	0.457	0.132	0.487	0.100	0.545
	9	0.179	0.494	0.159	0.524	0.123	0.581
	10	0.208	0.530	0.186	0.559	0.148	0.615
	11	0.238	0.565	0.215	0.594	0.173	0.649
	12	0.269	0.600	0.245	0.628	0.200	0.681
	13	0.301	0.634	0.275	0.661	0.228	0.713
	14	0.333	0.667	0.306	0.694	0.257	0.743
	15	0.366	0.699	0.339	0.725	0.287	0.772
	16	0.400	0.731	0.372	0.755	0.319	0.800
	17	0.435	0.762	0.406	0.785	0.351	0.827
	18	0.470	0.792	0.441	0.814	0.385	0.852
	19	0.506	0.821	0.476	0.841	0.419	0.877
	20	0.543	0.849	0.513	0.868	0.455	0.900
	21	0.581	0.876	0.551	0.893	0.492	0.921
	22	0.620	0.902	0.590	0.917	0.531	0.941
	23	0.661	0.927	0.631	0.939	0.572	0.959
	24	0.702	0.950	0.673	0.960	0.615	0.975
	25	0.746	0.970	0.718	0.977	0.660	0.988
	26	0.792	0.987	0.765	0.991	0.709	0.996
	27	0.841	0.998	0.817	0.999	0.763	1.000
	28	0.899	1.000	0.877	1.000	0.828	1.000
29	0	0.000	0.098	0.000	0.119	0.000	0.167
	1	0.002	0.153	0.001	0.178	0.000	0.230
	2	0.012	0.202	0.008	0.228	0.004	0.282
	3	0.029	0.246	0.022	0.274	0.012	0.330
	4	0.049	0.288	0.039	0.317	0.024	0.374
	5	0.070	0.329	0.058	0.358	0.039	0.416
	6	0.094	0.368	0.080	0.397	0.056	0.455
	7	0.119	0.406	0.103	0.435	0.076	0.493
	8	0.145	0.443	0.127	0.472	0.096	0.530
	9	0.172	0.479	0.153	0.508	0.119	0.565
	10	0.201	0.514	0.179	0.543	0.142	0.599
	11	0.229	0.549	0.207	0.577	0.167	0.632
	12	0.259	0.583	0.235	0.611	0.192	0.664
	13	0.289	0.616	0.264	0.643	0.219	0.695
	14	0.320	0.648	0.294	0.675	0.247	0.724
	15	0.352	0.680	0.325	0.706	0.276	0.753
	16	0.384	0.711	0.357	0.736	0.305	0.781
	17	0.417	0.741	0.389	0.765	0.336	0.808
	18	0.451	0.771	0.423	0.793	0.368	0.833
	19	0.486	0.799	0.457	0.821	0.401	0.858
	20	0.521	0.828	0.492	0.847	0.435	0.881
	21	0.557	0.855	0.528	0.873	0.470	0.904

TABLE A4 (Continued)

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
	22	0.594	0.881	0.565	0.897	0.507	0.924
	23	0.632	0.906	0.603	0.920	0.545	0.944
	24	0.671	0.930	0.642	0.942	0.584	0.961
	25	0.712	0.951	0.683	0.961	0.626	0.976
	26	0.754	0.971	0.726	0.978	0.670	0.988
	27	0.798	0.988	0.772	0.992	0.718	0.996
	28	0.847	0.998	0.822	0.999	0.770	1.000
	29	0.902	1.000	0.881	1.000	0.833	1.000
30	0	0.000	0.095	0.000	0.116	0.000	0.162
	1	0.002	0.149	0.001	0.172	0.000	0.223
	2	0.012	0.195	0.008	0.221	0.004	0.274
	3	0.028	0.239	0.021	0.265	0.012	0.320
	4	0.047	0.280	0.038	0.307	0.023	0.363
	5	0.068	0.319	0.056	0.347	0.038	0.404
	6	0.091	0.357	0.077	0.386	0.054	0.443
	7	0.115	0.394	0.099	0.423	0.073	0.480
	8	0.140	0.430	0.123	0.459	0.093	0.516
	9	0.166	0.465	0.147	0.494	0.114	0.550
	10	0.193	0.499	0.173	0.528	0.137	0.583
	11	0.221	0.533	0.199	0.561	0.160	0.616
	12	0.250	0.566	0.227	0.594	0.185	0.647
	13	0.279	0.598	0.255	0.626	0.211	0.677
	14	0.308	0.630	0.283	0.657	0.237	0.707
	15	0.339	0.661	0.313	0.687	0.265	0.735
	16	0.370	0.692	0.343	0.717	0.293	0.763
	17	0.402	0.721	0.374	0.745	0.323	0.789
	18	0.434	0.750	0.406	0.773	0.353	0.815
	19	0.467	0.779	0.439	0.801	0.384	0.840
	20	0.501	0.807	0.472	0.827	0.417	0.863
	21	0.535	0.834	0.506	0.853	0.450	0.886
	22	0.570	0.860	0.541	0.877	0.484	0.907
	23	0.606	0.885	0.577	0.901	0.520	0.927
	24	0.643	0.909	0.614	0.923	0.557	0.946
	25	0.681	0.932	0.653	0.944	0.596	0.962
	26	0.720	0.953	0.693	0.962	0.637	0.977
	27	0.761	0.972	0.735	0.979	0.680	0.988
	28	0.805	0.988	0.779	0.992	0.726	0.996
	29	0.851	0.998	0.828	0.999	0.777	1.000
	30	0.905	1.000	0.884	1.000	0.838	1.000

SOURCE: Generated by R. L. Iman. Used with permission.

TABLE A5 Sample Sizes for Nonparametric Tolerance Limits When $r + m = 1^a$

$1 - \alpha$	$q = 0.500$	0.700	0.750	0.800	0.850	0.900	0.950	0.975	0.980	0.990
0.500	1	2	3	4	5	7	14	28	35	69
0.700	2	4	5	6	8	12	24	48	60	120
0.750	2	4	5	7	9	14	28	55	69	138
0.800	3	5	6	8	10	16	32	64	80	161
0.850	3	6	7	9	12	19	37	75	94	189
0.900	4	7	9	11	15	22	45	91	144	230
0.950	5	9	11	14	19	29	59	119	149	299
0.975	6	11	13	17	23	36	72	146	183	368
0.980	6	11	14	18	25	38	77	155	194	390
0.990	7	13	17	21	29	44	90	182	228	459
0.995	8	15	19	24	33	51	104	210	263	528
0.999	10	20	25	31	43	66	135	273	342	688

^aThe quantity tabled is the sample size n such that $q^n \leq \alpha$, for use in finding the tolerance limits

$$P(X^{(1)} \leq p \text{ of the population}) \geq 1 - \alpha$$

or

$$P(q \text{ of the population} \leq X^{(n)}) \geq 1 - \alpha$$

as described in Section 3.3.

TABLE A6 Sample Sizes for Nonparametric Tolerance Limits When $r + m = 2^a$

$1 - \alpha$	$q = 0.500$	0.700	0.750	0.800	0.850	0.900	0.950	0.975	0.980	0.990
0.500	3	6	7	9	11	17	34	67	84	168
0.700	5	8	10	12	16	24	49	97	122	244
0.750	5	9	10	13	18	27	53	107	134	269
0.800	5	9	11	14	19	29	59	119	149	299
0.850	6	10	13	16	22	33	67	134	168	337
0.900	7	12	15	18	25	38	77	155	194	388
0.950	8	14	18	22	30	46	93	188	236	473
0.975	9	17	20	26	35	54	110	221	277	555
0.980	9	17	21	27	37	56	115	231	290	581
0.990	11	20	24	31	42	64	130	263	330	662
0.995	12	22	27	34	47	72	146	294	369	740
0.999	14	27	33	42	58	89	181	366	458	920

^aThe quantity tabled is the sample size n such that $q^n + nq^{n-1}(1 - q) \leq \alpha$ for use in finding the tolerance limits

$$P(X^{(1)} \leq q \text{ of the population} \leq X^{(n+1-m)}) \geq 1 - \alpha$$

when $r + m = 2$.

TABLE A7 (Continued)

n	p	$m = 2$	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
9	0.001	45	45	45	47	48	49	51	53	54	56	58	60	61	63	65	67	69	71	72	
	0.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	73	75	77	79	81	82
	0.01	45	47	49	51	53	55	58	61	63	66	69	72	74	77	80	83	85	88	91	94
	0.025	46	48	50	53	56	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100
	0.05	47	50	52	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108
10	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
11	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
12	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
13	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
14	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
15	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
16	0.001	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.005	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.01	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.025	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111
	0.05	48	51	55	58	61	64	68	71	74	77	80	83	87	91	94	98	101	104	108	111

TABLE A7 (Continued)

<i>n</i>	<i>p</i>	<i>m</i> = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
17	0.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220	224	
	0.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235	240	245
	0.01	154	158	162	167	172	177	182	187	192	197	203	209	214	220	225	231	236	242	247	253
	0.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253	259	265
	0.05	157	163	169	174	180	187	193	199	206	212	219	226	233	239	246	253	260	267	274	281
18	0.001	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274	281	288
	0.005	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243	248	253
	0.01	171	174	178	183	188	193	198	203	209	214	219	225	231	237	242	248	254	260	266	272
	0.025	172	176	181	186	191	196	202	208	214	220	227	233	239	246	252	258	265	271	278	284
	0.05	174	179	184	190	196	202	207	213	220	227	233	240	247	254	260	267	274	281	288	295
19	0.001	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288	295	302
	0.005	178	185	192	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300	307	314
	0.01	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268	273	279
	0.025	191	194	198	203	208	213	219	224	230	235	242	248	254	260	265	272	278	284	290	296
	0.05	192	195	200	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292	298	304
20	0.001	193	198	204	210	216	223	229	236	243	249	256	263	271	278	285	292	300	307	314	321
	0.005	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314	321	328
	0.01	198	205	212	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326	334	341
	0.025	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293	299	306
	0.05	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310	316	323

For *n* or *m* greater than 20, the *p*th quantile w_p of the Mann-Whitney test statistic may be approximated by

$$w_p = n(N + 1)/2 + z_p \sqrt{nm(N + 1)/12}$$

where z_p is the *p*th quantile of a standard normal random variable, obtained from Table A1, and where $N = m + n$.

*The entries in this table are quantiles w_p of the Mann-Whitney test statistic *T*, given by Equation 5.1.1, for selected values of *p*. Note that $P(T < w_p) \approx p$. Upper quantiles may be found from the equation

$$w_p = n(n + m + 1) - w_{1-p}$$

Critical regions correspond to values of *T* less than (or greater than) but not equal to the appropriate quantile.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes^a

Sample Sizes	$W_{0.90}$	$W_{0.95}$	$W_{0.99}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

SOURCE: Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.

^aThe null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A9 Quantiles of the Squared Ranks Test Statistic^a

<i>n</i>	<i>p</i>	<i>m</i> = 3	4	5	6	7	8	9	10
3	0.005	14	14	14	14	14	14	21	21
	0.01	14	14	14	14	21	21	26	26
	0.025	14	14	21	26	29	30	35	41
	0.05	21	21	26	30	38	42	49	54
	0.10	26	29	35	42	50	59	69	77
	0.90	65	90	117	149	182	221	260	305
	0.95	70	101	129	161	197	238	285	333
	0.975	77	110	138	170	213	257	308	362
	0.99	77	110	149	194	230	285	329	394
	0.995	77	110	149	194	245	302	346	413
4	0.005	30	30	30	39	39	46	50	54
	0.01	30	30	39	46	50	51	62	66
	0.025	30	39	50	54	63	71	78	90
	0.05	39	50	57	66	78	90	102	114
	0.10	50	62	71	85	99	114	130	149
	0.90	111	142	182	222	270	321	375	435
	0.95	119	154	197	246	294	350	413	476
	0.975	126	165	206	255	311	374	439	510
	0.99	126	174	219	270	334	401	470	545
	0.995	126	174	230	281	351	414	494	567
5	0.005	55	55	66	75	79	88	99	110
	0.01	55	66	75	82	90	103	115	127
	0.025	66	79	88	100	114	130	145	162
	0.05	75	88	103	120	135	155	175	195
	0.10	87	103	121	142	163	187	212	239
	0.90	169	214	264	319	379	445	514	591
	0.95	178	228	282	342	410	479	558	639
	0.975	183	235	297	363	433	508	592	680
	0.99	190	246	310	382	459	543	631	727
	0.995	190	255	319	391	478	559	654	754
6	0.005	91	104	115	124	136	152	167	182
	0.01	91	115	124	139	155	175	191	210
	0.025	115	130	143	164	184	208	231	255
	0.05	124	139	164	187	211	239	268	299
	0.10	136	163	187	215	247	280	315	352
	0.90	243	300	364	435	511	592	679	772
	0.95	255	319	386	463	545	634	730	831
	0.975	259	331	406	486	574	670	771	880
	0.99	271	339	424	511	607	706	817	935
	0.995	271	346	431	526	624	731	847	970

SOURCE. Adapted from tables generated by R.L. Iman. Used with permission.

^aThe entries in this table are selected quantiles w_p of the squared ranks test statistic T , given by Equation 5.3.3. Note that $P(T < w_p) \leq p$ and $P(T > w_p) \leq 1 - p$. Critical regions correspond to values less than (or greater than) but not including the appropriate quantile.

TABLE A9 (Continued)

<i>n</i>	<i>p</i>	<i>m</i> = 3	4	5	6	7	8	9	10
7	0.005	140	155	172	195	212	235	257	280
	0.01	155	172	191	212	236	260	287	315
	0.025	172	195	217	245	274	305	338	372
	0.05	188	212	240	274	308	344	384	425
	0.10	203	236	271	308	350	394	440	489
	0.90	335	407	487	572	665	764	871	984
	0.95	347	428	515	608	707	814	929	1051
	0.975	356	443	536	635	741	856	979	1108
	0.99	364	456	560	664	779	900	1032	1172
	0.995	371	467	571	683	803	929	1067	1212
8	0.005	204	236	260	284	311	340	368	401
	0.01	221	249	276	309	340	372	408	445
	0.025	249	276	311	345	384	425	468	513
	0.05	268	300	340	381	426	473	524	576
	0.10	285	329	374	423	476	531	590	652
	0.90	447	536	632	735	846	965	1091	1224
	0.95	464	560	664	776	896	1023	1159	1303
	0.975	476	579	689	807	935	1071	1215	1368
	0.99	485	599	716	840	980	1124	1277	1442
	0.995	492	604	731	863	1005	1156	1319	1489
9	0.005	304	325	361	393	429	466	508	549
	0.01	321	349	384	423	464	508	553	601
	0.025	342	380	423	469	517	570	624	682
	0.05	365	406	457	510	567	626	689	755
	0.10	390	444	501	561	625	694	766	843
	0.90	581	689	803	925	1056	1195	1343	1498
	0.95	601	717	840	972	1112	1261	1420	1587
	0.975	615	741	870	1009	1158	1317	1485	1662
	0.99	624	757	900	1049	1209	1377	1556	1745
	0.995	629	769	916	1073	1239	1417	1601	1798
10	0.005	406	448	486	526	573	620	672	725
	0.01	425	470	513	561	613	667	725	785
	0.025	457	505	560	616	677	741	808	879
	0.05	486	539	601	665	734	806	883	963
	0.10	514	580	649	724	801	885	972	1064
	0.90	742	866	1001	1144	1296	1457	1627	1806
	0.95	765	901	1045	1197	1360	1533	1715	1907
	0.975	778	925	1078	1241	1413	1596	1788	1991
	0.99	793	949	1113	1286	1470	1664	1869	2085
	0.995	798	961	1130	1314	1505	1708	1921	2145

For *n* or *m* greater than 10, the *p*th quantile *w_p* of the squared ranks test statistic may be approximated by

$$w_p = \frac{n(N+1)(2N+1)}{6} + z_p \sqrt{\frac{mn(N+1)(2N+1)(8N+11)}{180}}$$

where *N* = *n* + *m*, and where *z_p* is the *p*th quantile of a standard normal random variable, obtained from Table A1.

TABLE A10 Quantiles of Spearman's ρ^a

n	$p = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p \approx \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glasser and Winter (1961), with corrections, with permission from the *Biometrika* Trustees.

^aThe entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$p = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A11 (Continued)

<i>n</i>	<i>p</i> = 0.900	0.950	0.975	0.990	0.995
38	103 (0.1465)	131 (0.1863)	155 (0.2205)	183 (0.2603)	203 (0.2888)
39	107 (0.1444)	137 (0.1849)	161 (0.2173)	191 (0.2578)	211 (0.2848)
40	110 (0.1372)	142 (0.1821)	168 (0.2154)	198 (0.2538)	220 (0.2821)
41	114 (0.1390)	146 (0.1780)	174 (0.2122)	206 (0.2512)	228 (0.2780)
42	119 (0.1382)	151 (0.1754)	181 (0.2102)	213 (0.2474)	235 (0.2729)
43	123 (0.1362)	157 (0.1739)	187 (0.2071)	221 (0.2447)	245 (0.2713)
44	128 (0.1353)	162 (0.1712)	194 (0.2051)	228 (0.2410)	252 (0.2664)
45	132 (0.1333)	168 (0.1697)	200 (0.2020)	236 (0.2383)	262 (0.2646)
46	135 (0.1304)	173 (0.1671)	207 (0.2000)	245 (0.2367)	271 (0.2618)
47	141 (0.1304)	179 (0.1656)	213 (0.1970)	253 (0.2340)	279 (0.2581)
48	144 (0.1277)	186 (0.1649)	220 (0.1950)	260 (0.2305)	288 (0.2553)
49	150 (0.1276)	190 (0.1616)	228 (0.1939)	268 (0.2279)	296 (0.2517)
50	153 (0.1249)	197 (0.1608)	233 (0.1902)	277 (0.2261)	305 (0.2490)
51	159 (0.1247)	203 (0.1592)	241 (0.1890)	285 (0.2235)	315 (0.2471)
52	162 (0.1222)	208 (0.1569)	248 (0.1870)	294 (0.2217)	324 (0.2443)
53	168 (0.1219)	214 (0.1553)	256 (0.1858)	302 (0.2192)	334 (0.2424)
54	173 (0.1209)	221 (0.1544)	263 (0.1838)	311 (0.2173)	343 (0.2397)
55	177 (0.1192)	227 (0.1529)	269 (0.1811)	319 (0.2148)	353 (0.2377)
56	182 (0.1182)	232 (0.1506)	276 (0.1792)	328 (0.2130)	362 (0.2351)
57	186 (0.1165)	240 (0.1504)	284 (0.1779)	336 (0.2105)	372 (0.2331)
58	191 (0.1155)	245 (0.1482)	291 (0.1760)	345 (0.2087)	381 (0.2305)
59	197 (0.1151)	251 (0.1467)	299 (0.1748)	355 (0.2075)	391 (0.2285)
60	202 (0.1141)	258 (0.1458)	306 (0.1729)	364 (0.2056)	402 (0.2271)

For *n* greater than 60, approximate quantiles of *T* may be obtained from

$$w_p \cong z_p \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

where z_p is from the standard normal distribution given by Table A1. Approximate quantiles of τ may be obtained from

$$w_p \cong z_p \frac{\sqrt{2(2n+5)}}{3\sqrt{n(n-1)}}$$

Critical regions correspond to values of *T* greater than (or less than) but not including the appropriate quantile. Note that the median of *T* is 0. Quantiles for τ are obtained by dividing the quantiles of *T* by $n(n-1)/2$.

SOURCE. Adapted from Table I, Best (1974), with permission from the author.

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

	$W_{0.005}$	$W_{0.01}$	$W_{0.025}$	$W_{0.05}$	$W_{0.10}$	$W_{0.20}$	$W_{0.30}$	$W_{0.40}$	$W_{0.50}$	$\frac{n(n+1)}{2}$
$n = 4$	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	66	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	83	91	98	105	210
21	44	50	59	68	78	91	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	248	266	282	297.5	595
35	160	175	196	214	236	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	266	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903

TABLE A12 (Continued)

	$w_{0.005}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$\frac{n(n+1)}{2}$
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n+1)/4] + z_p \sqrt{n(n+1)(2n+1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE. Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

* The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^+ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T^+ < w_p) \leq p$ and $P(T^+ > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^+ less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic^a

One-Sided Test											
$p = 0.90$						$p = 0.90$	0.95	0.975	0.99	0.995	
Two-Sided Test											
$p = 0.80$						$p = 0.80$	0.90	0.95	0.98	0.99	
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252
Approximation for $n > 40$						$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$	

SOURCE. Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

^aThe entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.

TABLE A14 Quantiles of the Lilliefors Test Statistic for Normality^a

	$p = 0.80$	0.85	0.90	0.95	0.99
Sample size $n = 4$	0.303	0.320	0.344	0.374	0.414
5	0.290	0.302	0.319	0.344	0.398
6	0.268	0.280	0.295	0.321	0.371
7	0.252	0.264	0.280	0.304	0.353
8	0.239	0.251	0.266	0.290	0.333
9	0.227	0.239	0.253	0.275	0.319
10	0.217	0.228	0.241	0.262	0.303
11	0.209	0.219	0.232	0.252	0.291
12	0.201	0.210	0.223	0.243	0.281
13	0.193	0.203	0.215	0.233	0.270
14	0.187	0.196	0.209	0.227	0.264
15	0.181	0.190	0.202	0.219	0.256
16	0.176	0.184	0.195	0.212	0.248
17	0.170	0.179	0.190	0.207	0.241
18	0.166	0.174	0.185	0.201	0.234
19	0.162	0.171	0.181	0.197	0.230
20	0.159	0.167	0.177	0.192	0.223
21	0.155	0.163	0.173	0.188	0.219
22	0.152	0.160	0.170	0.185	0.214
23	0.149	0.156	0.165	0.181	0.210
24	0.145	0.153	0.162	0.177	0.205
25	0.144	0.151	0.159	0.173	0.202
26	0.141	0.147	0.156	0.170	0.198
27	0.138	0.145	0.153	0.166	0.193
28	0.136	0.142	0.151	0.165	0.191
29	0.134	0.140	0.149	0.162	0.188
30	0.132	0.138	0.146	0.159	0.183
≥ 31	<u>0.741</u>	<u>0.775</u>	<u>0.819</u>	<u>0.895</u>	<u>1.035</u>
	d_n	d_n	d_n	d_n	d_n

$$d_n = (\sqrt{n} - 0.01 + 0.83/\sqrt{n})$$

SOURCE: Table L5, Mason and Bell (1986). Used with permission from Marcel Dekker, Inc.

^aThe entries in this table are the approximate quantiles w_p of the Lilliefors test statistic T_1 as defined by Equation 6.2.4. Reject H_0 at the level α if T_1 exceeds $w_{1-\alpha}$ for the particular sample size n .

TABLE A15 Quantiles of the Lilliefors Test Statistic for the Exponential Distribution^a

n	p =										
	0.05	0.10	0.20	0.30	0.50	0.70	0.80	0.90	0.95	0.99	0.999
2	0.3127	0.3200	0.3337	0.3617	0.4337	0.5034	0.5507	0.5934	0.6133	0.6284	0.6317
3	0.2299	0.2544	0.2899	0.3166	0.3645	0.4122	0.4508	0.5111	0.5508	0.6003	0.6296
4	0.2072	0.2281	0.2545	0.2766	0.3163	0.3685	0.4007	0.4442	0.4844	0.5574	0.6215
5	0.1884	0.2052	0.2290	0.2483	0.2877	0.3317	0.3603	0.4045	0.4420	0.5127	0.5814
6	0.1726	0.1882	0.2102	0.2290	0.2645	0.3045	0.3320	0.3732	0.4085	0.4748	0.5497
7	0.1604	0.1750	0.1961	0.2136	0.2458	0.2838	0.3098	0.3481	0.3811	0.4459	0.5181
8	0.1506	0.1646	0.1845	0.2006	0.2309	0.2671	0.2914	0.3274	0.3590	0.4208	0.4913
9	0.1426	0.1561	0.1746	0.1897	0.2186	0.2529	0.2758	0.3101	0.3404	0.3995	0.4679
10	0.1359	0.1486	0.1661	0.1805	0.2082	0.2407	0.2626	0.2955	0.3244	0.3813	0.4473
12	0.1249	0.1364	0.1524	0.1657	0.1912	0.2209	0.2411	0.2714	0.2981	0.3511	0.4132
14	0.1162	0.1268	0.1418	0.1542	0.1778	0.2054	0.2242	0.2525	0.2774	0.3272	0.3858
16	0.1091	0.1191	0.1332	0.1448	0.1669	0.1929	0.2105	0.2371	0.2606	0.3076	0.3632
18	0.1032	0.1127	0.1260	0.1369	0.1578	0.1824	0.1990	0.2242	0.2465	0.2911	0.3441
20	0.0982	0.1073	0.1199	0.1303	0.1501	0.1735	0.1893	0.2132	0.2345	0.2771	0.3277
22	0.0939	0.1025	0.1146	0.1245	0.1434	0.1657	0.1809	0.2038	0.2241	0.2649	0.3135
24	0.0901	0.0984	0.1099	0.1195	0.1376	0.1590	0.1735	0.1954	0.2150	0.2542	0.3010
26	0.0868	0.0947	0.1058	0.1150	0.1324	0.1530	0.1670	0.1881	0.2069	0.2447	0.2899
28	0.0838	0.0914	0.1021	0.1110	0.1278	0.1477	0.1611	0.1815	0.1997	0.2362	0.2799
30	0.0811	0.0885	0.0988	0.1074	0.1236	0.1428	0.1559	0.1756	0.1932	0.2286	0.2709
35	0.0754	0.0822	0.0918	0.0997	0.1148	0.1326	0.1447	0.1630	0.1793	0.2123	0.2517
40	0.0707	0.0771	0.0861	0.0935	0.1077	0.1243	0.1356	0.1528	0.1681	0.1990	0.2361
45	0.0668	0.0729	0.0814	0.0884	0.1017	0.1174	0.1281	0.1443	0.1588	0.1880	0.2231
50	0.0636	0.0693	0.0774	0.0840	0.0966	0.1116	0.1217	0.1371	0.1509	0.1787	0.2121
60	0.0582	0.0635	0.0708	0.0769	0.0885	0.1021	0.1114	0.1255	0.1381	0.1635	0.1943
70	0.0541	0.0589	0.0658	0.0714	0.0821	0.0946	0.1033	0.1164	0.1281	0.1517	^b
80	0.0507	0.0553	0.0616	0.0669	0.0769	0.0887	0.0968	0.1090	0.1200	0.1421	^b
90	0.0479	0.0522	0.0582	0.0632	0.0726	0.0838	0.0914	0.1029	0.1132	0.1341	^b
n = 100	0.0455	0.0496	0.0553	0.0600	0.0690	0.0796	0.0868	0.0977	0.1075	0.1274	^b
Approximation for n > 100	$\frac{0.4550}{\sqrt{n}}$	$\frac{0.4959}{\sqrt{n}}$	$\frac{0.5530}{\sqrt{n}}$	$\frac{0.6000}{\sqrt{n}}$	$\frac{0.6898}{\sqrt{n}}$	$\frac{0.7957}{\sqrt{n}}$	$\frac{0.8678}{\sqrt{n}}$	$\frac{0.9773}{\sqrt{n}}$	$\frac{1.0753}{\sqrt{n}}$	$\frac{1.2743}{\sqrt{n}}$	^b

SOURCE: Adapted from Durbin (1975), with permission from the Biometrika Trustees.

^aThe entries in this table are selected quantiles w_p of the Lilliefors test statistic T_1 as given by Equation 6.2.6. Reject at the level of significance α if T_1 is greater than the $1 - \alpha$ quantile given in the table. The approximation for $n > 100$ is merely the exact value for $n = 100$. More accurate approximations for $n > 100$ may be obtained from Table 54 of Pearson and Hartley (1972).

^bThese quantiles are not presently available.

TABLE A16 Coefficients for the Shapiro-Wilk Test^a

$n \backslash i$	2	3	4	5	6	7	8	9	10
1	0.7071	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.0588	0.5739
2	—	0.0000	0.1667	0.2413	0.2806	0.3031	0.3164	0.3244	0.3291
3	—	—	—	0.0000	0.0875	0.1401	0.1743	0.1976	0.2141
4	—	—	—	—	—	0.0000	0.0561	0.0947	0.1224
5	—	—	—	—	—	—	—	0.0000	0.0399

$n \backslash i$	11	12	13	14	15	16	17	18	19	20
1	0.5601	0.5475	0.5359	0.5251	0.5150	0.5056	0.4968	0.4886	0.4808	0.4734
2	0.3315	0.3325	0.3325	0.3318	0.3306	0.3290	0.3273	0.3253	0.3232	0.3211
3	0.2260	0.2347	0.2412	0.2460	0.2495	0.2521	0.2540	0.2553	0.2561	0.2565
4	0.1429	0.1586	0.1707	0.1802	0.1878	0.1939	0.1988	0.2027	0.2059	0.2085
5	0.0695	0.0922	0.1099	0.1240	0.1353	0.1447	0.1524	0.1587	0.1641	0.1686
6	0.0000	0.0303	0.0539	0.0727	0.0880	0.1005	0.1109	0.1197	0.1271	0.1334
7	—	—	0.0000	0.0240	0.0433	0.0593	0.0725	0.0837	0.0932	0.1013
8	—	—	—	—	0.0000	0.0196	0.0359	0.0496	0.0612	0.0711
9	—	—	—	—	—	—	0.0000	0.0163	0.0303	0.0422
10	—	—	—	—	—	—	—	—	0.0000	0.0140

$n \backslash i$	21	22	23	24	25	26	27	28	29	30
1	0.4643	0.4590	0.4542	0.4493	0.4450	0.4407	0.4366	0.4328	0.4291	0.4254
2	0.3185	0.3156	0.3126	0.3098	0.3069	0.3043	0.3018	0.2992	0.2968	0.2944
3	0.2578	0.2571	0.2563	0.2554	0.2543	0.2533	0.2522	0.2510	0.2499	0.2487
4	0.2119	0.2131	0.2139	0.2145	0.2148	0.2151	0.2152	0.2151	0.2150	0.2148
5	0.1736	0.1764	0.1787	0.1807	0.1822	0.1836	0.1848	0.1857	0.1864	0.1870
6	0.1399	0.1443	0.1480	0.1512	0.1539	0.1563	0.1584	0.1601	0.1616	0.1630
7	0.1092	0.1150	0.1201	0.1245	0.1283	0.1316	0.1346	0.1372	0.1395	0.1415
8	0.0804	0.0878	0.0941	0.0997	0.1046	0.1089	0.1128	0.1162	0.1192	0.1219
9	0.0530	0.0618	0.0696	0.0764	0.0823	0.0876	0.0923	0.0965	0.1002	0.1036
10	0.0263	0.0368	0.0459	0.0539	0.0610	0.0672	0.0728	0.0778	0.0822	0.0862
11	0.0000	0.0122	0.0228	0.0321	0.0403	0.0476	0.0540	0.0598	0.0650	0.0697
12	—	—	0.0000	0.0107	0.0200	0.0284	0.0358	0.0424	0.0483	0.0537
13	—	—	—	—	0.0000	0.0094	0.0178	0.0253	0.0320	0.0381
14	—	—	—	—	—	—	0.0000	0.0084	0.0159	0.0227
15	—	—	—	—	—	—	—	—	0.0000	0.0076

TABLE A16 (Continued)

$i \backslash n$	31	32	33	34	35	36	37	38	39	40
1	0.4220	0.4188	0.4156	0.4127	0.4096	0.4068	0.4040	0.4015	0.3989	0.3964
2	0.2921	0.2898	0.2876	0.2854	0.2834	0.2813	0.2794	0.2774	0.2755	0.2737
3	0.2475	0.2462	0.2451	0.2439	0.2427	0.2415	0.2403	0.2391	0.2380	0.2368
4	0.2145	0.2141	0.2137	0.2132	0.2127	0.2121	0.2116	0.2110	0.2104	0.2098
5	0.1874	0.1878	0.1880	0.1882	0.1883	0.1883	0.1883	0.1881	0.1880	0.1878
6	0.1641	0.1651	0.1660	0.1667	0.1673	0.1678	0.1683	0.1686	0.1689	0.1691
7	0.1433	0.1449	0.1463	0.1475	0.1487	0.1496	0.1505	0.1513	0.1520	0.1526
8	0.1243	0.1265	0.1284	0.1301	0.1317	0.1331	0.1344	0.1356	0.1366	0.1376
9	0.1066	0.1093	0.1118	0.1140	0.1160	0.1179	0.1196	0.1211	0.1225	0.1237
10	0.0899	0.0931	0.0961	0.0988	0.1013	0.1036	0.1056	0.1075	0.1092	0.1108
11	0.0739	0.0777	0.0812	0.0844	0.0873	0.0900	0.0924	0.0947	0.0967	0.0986
12	0.0585	0.0629	0.0669	0.0706	0.0739	0.0770	0.0798	0.0824	0.0848	0.0870
13	0.0435	0.0485	0.0530	0.0572	0.0610	0.0645	0.0677	0.0706	0.0733	0.0759
14	0.0289	0.0344	0.0395	0.0441	0.0484	0.0523	0.0559	0.0592	0.0622	0.0651
15	0.0144	0.0206	0.0262	0.0314	0.0361	0.0404	0.0444	0.0481	0.0515	0.0546
16	0.0000	0.0068	0.0131	0.0187	0.0239	0.0287	0.0331	0.0372	0.0409	0.0444
17	—	—	0.0000	0.0062	0.0119	0.0172	0.0220	0.0264	0.0305	0.0343
18	—	—	—	—	0.0000	0.0057	0.0110	0.0158	0.0203	0.0244
19	—	—	—	—	—	—	0.0000	0.0053	0.0101	0.0146
20	—	—	—	—	—	—	—	—	0.0000	0.0049

$i \backslash n$	41	42	43	44	45	46	47	48	49	50
1	0.3940	0.3917	0.3894	0.3872	0.3850	0.3830	0.3808	0.3789	0.3770	0.3751
2	0.2719	0.2701	0.2684	0.2667	0.2651	0.2635	0.2620	0.2604	0.2589	0.2574
3	0.2357	0.2345	0.2334	0.2323	0.2313	0.2302	0.2291	0.2281	0.2271	0.2260
4	0.2091	0.2085	0.2078	0.2072	0.2065	0.2058	0.2052	0.2045	0.2038	0.2032
5	0.1876	0.1874	0.1871	0.1868	0.1865	0.1862	0.1859	0.1855	0.1851	0.1847
6	0.1693	0.1694	0.1695	0.1695	0.1695	0.1695	0.1695	0.1693	0.1692	0.1691
7	0.1531	0.1535	0.1539	0.1542	0.1545	0.1548	0.1550	0.1551	0.1553	0.1554
8	0.1384	0.1392	0.1398	0.1405	0.1410	0.1415	0.1420	0.1423	0.1427	0.1430
9	0.1249	0.1259	0.1269	0.1278	0.1286	0.1293	0.1300	0.1306	0.1312	0.1317
10	0.1123	0.1136	0.1149	0.1160	0.1170	0.1180	0.1189	0.1197	0.1205	0.1212
11	0.1004	0.1020	0.1035	0.1049	0.1062	0.1073	0.1085	0.1095	0.1105	0.1113
12	0.0891	0.0909	0.0927	0.0943	0.0959	0.0972	0.0986	0.0998	0.1010	0.1020
13	0.0782	0.0804	0.0824	0.0842	0.0860	0.0876	0.0892	0.0906	0.0919	0.0932
14	0.0677	0.0701	0.0724	0.0745	0.0765	0.0783	0.0801	0.0817	0.0832	0.0846
15	0.0575	0.0602	0.0628	0.0651	0.0673	0.0694	0.0713	0.0731	0.0748	0.0764

TABLE A16 (Continued)

n i	41	42	43	44	45	46	47	48	49	50
16	0.0476	0.0506	0.0534	0.0560	0.0584	0.0607	0.0628	0.0648	0.0667	0.0685
17	0.0379	0.0411	0.0442	0.0471	0.0497	0.0522	0.0546	0.0568	0.0588	0.0608
18	0.0283	0.0318	0.0352	0.0383	0.0412	0.0439	0.0465	0.0489	0.0511	0.0532
19	0.0188	0.0227	0.0263	0.0296	0.0328	0.0357	0.0385	0.0411	0.0436	0.0459
20	0.0094	0.0136	0.0175	0.0211	0.0245	0.0277	0.0307	0.0335	0.0361	0.0386
21	0.0000	0.0045	0.0087	0.0126	0.0163	0.0197	0.0229	0.0259	0.0288	0.0314
22	—	—	0.0000	0.0042	0.0081	0.0118	0.0153	0.0185	0.0215	0.0244
23	—	—	—	—	0.0000	0.0039	0.0076	0.0111	0.0143	0.0174
24	—	—	—	—	—	—	0.0000	0.0037	0.0071	0.0104
25	—	—	—	—	—	—	—	—	0.0000	0.0035

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*The entries in this table are the coefficients a_i for use in the Shapiro-Wilk test statistic for normality given by Equation 6.2.9.

TABLE A17 Quantiles of the Shapiro-Wilk Test Statistic^a

n	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
3	0.753	0.756	0.767	0.789	0.959	0.998	0.999	1.000	1.000
4	0.687	0.707	0.748	0.792	0.935	0.987	0.992	0.996	0.997
5	0.686	0.715	0.762	0.806	0.927	0.979	0.986	0.991	0.993
6	0.713	0.743	0.788	0.826	0.927	0.974	0.981	0.986	0.989
7	0.730	0.760	0.803	0.838	0.928	0.972	0.979	0.985	0.988
8	0.749	0.778	0.818	0.851	0.932	0.972	0.978	0.984	0.987
9	0.764	0.791	0.829	0.859	0.935	0.972	0.978	0.984	0.986
10	0.781	0.806	0.842	0.869	0.938	0.972	0.978	0.983	0.986
11	0.792	0.817	0.850	0.876	0.940	0.973	0.979	0.984	0.986
12	0.805	0.828	0.859	0.883	0.943	0.973	0.979	0.984	0.986
13	0.814	0.837	0.866	0.889	0.945	0.974	0.979	0.984	0.986
14	0.825	0.846	0.874	0.895	0.947	0.975	0.980	0.984	0.986
15	0.835	0.855	0.881	0.901	0.950	0.975	0.980	0.984	0.987
16	0.844	0.863	0.887	0.906	0.952	0.976	0.981	0.985	0.987
17	0.851	0.869	0.892	0.910	0.954	0.977	0.981	0.985	0.987
18	0.858	0.874	0.897	0.914	0.956	0.978	0.982	0.986	0.988
19	0.863	0.879	0.901	0.917	0.957	0.978	0.982	0.986	0.988
20	0.868	0.884	0.905	0.920	0.959	0.979	0.983	0.986	0.988
21	0.873	0.888	0.908	0.923	0.960	0.980	0.983	0.987	0.989
22	0.878	0.892	0.911	0.926	0.961	0.980	0.984	0.987	0.989
23	0.881	0.895	0.914	0.928	0.962	0.981	0.984	0.987	0.989
24	0.884	0.898	0.916	0.930	0.963	0.981	0.984	0.987	0.989

TABLE A17 (Continued)

<i>n</i>	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
25	0.888	0.901	0.918	0.931	0.964	0.981	0.985	0.988	0.989
26	0.891	0.904	0.920	0.933	0.965	0.982	0.985	0.988	0.989
27	0.894	0.906	0.923	0.935	0.965	0.982	0.985	0.988	0.990
28	0.896	0.908	0.924	0.936	0.966	0.982	0.985	0.988	0.990
29	0.898	0.910	0.926	0.937	0.966	0.982	0.985	0.988	0.990
30	0.900	0.912	0.927	0.939	0.967	0.983	0.985	0.988	0.990
31	0.902	0.914	0.929	0.940	0.967	0.983	0.986	0.988	0.990
32	0.904	0.915	0.930	0.941	0.968	0.983	0.986	0.988	0.990
33	0.906	0.917	0.931	0.942	0.968	0.983	0.986	0.989	0.990
34	0.908	0.919	0.933	0.943	0.969	0.983	0.986	0.989	0.990
35	0.910	0.920	0.934	0.944	0.969	0.984	0.986	0.989	0.990
36	0.912	0.922	0.935	0.945	0.970	0.984	0.986	0.989	0.990
37	0.914	0.924	0.936	0.946	0.970	0.984	0.987	0.989	0.990
38	0.916	0.925	0.938	0.947	0.971	0.984	0.987	0.989	0.990
39	0.917	0.927	0.939	0.948	0.971	0.984	0.987	0.989	0.991
40	0.919	0.928	0.940	0.949	0.972	0.985	0.987	0.989	0.991
41	0.920	0.929	0.941	0.950	0.972	0.985	0.987	0.989	0.991
42	0.922	0.930	0.942	0.951	0.972	0.985	0.987	0.989	0.991
43	0.923	0.932	0.943	0.951	0.973	0.985	0.987	0.990	0.991
44	0.924	0.933	0.944	0.952	0.973	0.985	0.987	0.990	0.991
45	0.926	0.934	0.945	0.953	0.973	0.985	0.988	0.990	0.991
46	0.927	0.935	0.945	0.953	0.974	0.985	0.988	0.990	0.991
47	0.928	0.936	0.946	0.954	0.974	0.985	0.988	0.990	0.991
48	0.929	0.937	0.947	0.954	0.974	0.985	0.988	0.990	0.991
49	0.929	0.937	0.947	0.955	0.974	0.985	0.988	0.990	0.991
50	0.930	0.938	0.947	0.955	0.974	0.985	0.988	0.990	0.991

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* The entries in this table are quantiles w_p of the Shapiro-Wilk test statistic given by Equation 6.2.9. Reject H_0 at the level p if $T_3 < w_p$.

TABLE A18 A Method for Converting the Shapiro-Wilk Statistic to Approximate Normality

$v \backslash n$ (d_n)	3 (0.7500)	4 (0.6297)	5 (0.5521)	6 (0.4963)	$v \backslash n$ (d_n)	3 (0.7500)	4 (0.6297)	5 (0.5521)	6 (0.4963)
-7.0	-3.29	—	—	—	2.2	0.52	0.74	0.75	0.64
-5.4	-2.81	—	—	—	2.6	0.67	1.00	1.09	1.06
-5.0	-2.68	—	—	—	3.0	0.81	1.23	1.40	1.45
-4.6	-2.54	—	—	—	3.4	0.95	1.44	1.67	1.83
-4.2	-2.40	—	—	—	3.8	1.07	1.65	1.91	2.17
-3.8	-2.25	-3.50	—	—	4.2	1.19	1.85	2.15	2.50
-3.4	-2.10	-3.27	—	—	4.6	1.31	2.03	2.47	2.77
-3.0	-1.94	-3.05	-4.01	—	5.0	1.42	2.19	2.85	3.09
-2.6	-1.77	-2.84	-3.70	—	5.4	1.52	2.34	3.24	3.54
-2.2	-1.59	-2.64	-3.38	—	5.8	1.62	2.48	3.64	—
-1.8	-1.40	-2.44	-3.11	—	6.2	1.72	2.62	—	—
-1.4	-1.21	-2.22	-2.87	—	6.6	1.81	2.75	—	—
-1.0	-1.01	-1.96	-2.56	-3.72	7.0	1.90	2.87	—	—
-0.6	-0.80	-1.66	-2.20	-2.88	7.4	1.98	2.97	—	—
-0.2	-0.60	-1.31	-1.81	-2.27	7.8	2.07	3.08	—	—
0.2	-0.39	-0.94	-1.41	-1.85	8.2	2.15	3.22	—	—
0.6	-0.19	-0.57	-0.97	-1.38	8.6	2.23	3.36	—	—
1.0	0.00	-0.19	-0.51	-0.84	9.0	2.31	—	—	—
1.4	0.18	0.15	-0.06	-0.33	9.4	2.38	—	—	—
1.8	0.35	0.45	0.37	0.18	9.8	2.45	—	—	—

For $3 \leq n \leq 6$, first compute $v = \ln [(T - d_n)/(1 - T)]$ where d_n is given at the top of the table and T is the Shapiro-Wilk statistic. Then enter the table with v and n to find G , which is approximately normal.

TABLE A18 (Continued)

n	b_n	c_n	d_n	n	b_n	c_n	d_n
7	-2.356	1.245	0.4533	29	-6.074	1.934	0.1907
8	-2.696	1.333	0.4186	30	-6.150	1.949	0.1872
9	-2.968	1.400	0.3900				
10	-3.262	1.471	0.3600	31	-6.248	1.965	0.1840
				32	-6.324	1.976	0.1811
11	-3.485	1.515	0.3451	33	-6.402	1.988	0.1781
12	-3.731	1.571	0.3270	34	-6.480	2.000	0.1755
13	-3.936	1.613	0.3111	35	-6.559	2.012	0.1727
14	-4.155	1.655	0.2969				
15	-4.373	1.695	0.2842	36	-6.640	2.024	0.1702
				37	-6.721	2.037	0.1677
16	-4.567	1.724	0.2727	38	-6.803	2.049	0.1656
17	-4.713	1.739	0.2622	39	-6.887	2.062	0.1633
18	-4.885	1.770	0.2528	40	-6.961	2.075	0.1612
19	-5.018	1.786	0.2440				
20	-5.153	1.802	0.2359	41	-7.035	2.088	0.1591
				42	-7.111	2.101	0.1572
21	-5.291	1.818	0.2264	43	-7.188	2.114	0.1552
22	-5.413	1.835	0.2207	44	-7.266	2.128	0.1534
23	-5.508	1.848	0.2157	45	-7.345	2.141	0.1516
24	-5.605	1.862	0.2106				
25	-5.704	1.876	0.2063	46	-7.414	2.155	0.1499
				47	-7.484	2.169	0.1482
26	-5.803	1.890	0.2020	48	-7.555	2.183	0.1466
27	-5.905	1.905	0.1980	49	-7.615	2.198	0.1451
28	-5.988	1.919	0.1943	50	-7.677	2.212	0.1436

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For $7 \leq n \leq 50$, enter the table above with n to find the coefficients b_n , c_n , and d_n . Then compute

$$G = b_n + c_n \ln \{(T - d_n)/(1 - T)\}$$

which is approximately standard normal.

TABLE A19 Quantiles of the Smirnov Test Statistic for Two Samples of Equal Size n^a

One-Sided Test: $p = 0.90$						One-Sided Test: $p = 0.90$				
Two-Sided Test: $p = 0.80$						Two-Sided Test: $p = 0.80$				
	0.95	0.975	0.99	0.995		0.95	0.975	0.99	0.995	
$n = 3$	2/3	2/3			$n = 22$	7/22	8/22	8/22	10/22	10/22
4	3/4	3/4	3/4		23	7/23	8/23	9/23	10/23	10/23
5	3/5	3/5	4/5	4/5	24	7/24	8/24	9/24	10/24	11/24
6	3/6	4/6	4/6	5/6	25	7/25	8/25	9/25	10/25	11/25
7	4/7	4/7	5/7	5/7	26	7/26	8/26	9/26	10/26	11/26
8	4/8	4/8	5/8	5/8	27	7/27	8/27	9/27	11/27	11/27
9	4/9	5/9	5/9	6/9	28	8/28	9/28	10/28	11/28	12/28
10	4/10	5/10	6/10	6/10	29	8/29	9/29	10/29	11/29	12/29
11	5/11	5/11	6/11	7/11	30	8/30	9/30	10/30	11/30	12/30
12	5/12	5/12	6/12	7/12	31	8/31	9/31	10/31	11/31	12/31
13	5/13	6/13	6/13	7/13	32	8/32	9/32	10/32	12/32	12/32
14	5/14	6/14	7/14	7/14	33	8/33	9/33	11/33	12/33	13/33
15	5/15	6/15	7/15	8/15	34	8/34	10/34	11/34	12/34	13/34
16	6/16	6/16	7/16	8/16	35	8/35	10/35	11/35	12/35	13/35
17	6/17	7/17	7/17	8/17	36	9/36	10/36	11/36	12/36	13/36
18	6/18	7/18	8/18	9/18	37	9/37	10/37	11/37	13/37	13/37
19	6/19	7/19	8/19	9/19	38	9/38	10/38	11/38	13/38	14/38
20	6/20	7/20	8/20	9/20	39	9/39	10/39	11/39	13/39	14/39
21	6/21	7/21	8/21	9/21	40	9/40	10/40	12/40	13/40	14/40
Approximation for $n > 40$:						$\frac{1.52}{\sqrt{n}}$	$\frac{1.73}{\sqrt{n}}$	$\frac{1.92}{\sqrt{n}}$	$\frac{2.15}{\sqrt{n}}$	$\frac{2.30}{\sqrt{n}}$

SOURCE. Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

^a The entries in this table are selected quantiles w_p of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m^a

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995	
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99	
$N_1 = 1$	$N_2 = 9$	17/18					
	10	9/10					
$N_1 = 2$	$N_2 = 3$	5/6					
	4	3/4					
	5	4/5	4/5				
	6	5/6	5/6				
	7	5/7	6/7				
	8	3/4	7/8	7/8			
	9	7/9	8/9	8/9			
	10	7/10	4/5	9/10			
	$N_1 = 3$	$N_2 = 4$	3/4	3/4			
		5	2/3	4/5	4/5		
6		2/3	2/3	5/6			
7		2/3	5/7	6/7	6/7		
8		5/8	3/4	3/4	7/8		
9		2/3	2/3	7/9	8/9	8/9	
10		3/5	7/10	4/5	9/10	9/10	
12		7/12	2/3	3/4	5/6	11/12	
$N_1 = 4$		$N_2 = 5$	3/5	3/4	4/5	4/5	
		6	7/12	2/3	3/4	5/6	5/6
	7	17/28	5/7	3/4	6/7	6/7	
	8	5/8	5/8	3/4	7/8	7/8	
	9	5/9	2/3	3/4	7/9	8/9	
	10	11/20	13/20	7/10	4/5	4/5	
	12	7/12	2/3	2/3	3/4	5/6	
	16	9/16	5/8	11/16	3/4	13/16	
$N_1 = 5$	$N_2 = 6$	3/5	2/3	2/3	5/6	5/6	
	7	4/7	23/35	5/7	29/35	6/7	
	8	11/20	5/8	27/40	4/5	4/5	
	9	5/9	3/5	31/45	7/9	4/5	
	10	1/2	3/5	7/10	7/10	4/5	
	15	8/15	3/5	2/3	11/15	11/15	
	20	1/2	11/20	3/5	7/10	3/4	
$N_1 = 6$	$N_2 = 7$	23/42	4/7	29/42	5/7	5/6	
	8	1/2	7/12	2/3	3/4	3/4	
	9	1/2	5/9	2/3	13/18	7/9	
	10	1/2	17/30	19/30	7/10	11/15	
	12	1/2	7/12	7/12	2/3	3/4	
	18	4/9	5/9	11/18	2/3	13/18	
	24	11/24	1/2	7/12	5/8	2/3	

TABLE A20 (Continued)

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 7$	$N_2 = 8$	27/56	33/56	5/8	41/56	3/4
	9	31/63	5/9	40/63	5/7	47/63
	10	33/70	39/70	43/70	7/10	5/7
	14	3/7	1/2	4/7	9/14	5/7
	28	3/7	13/28	15/28	17/28	9/14
$N_1 = 8$	$N_2 = 9$	4/9	13/24	5/8	2/3	3/4
	10	19/40	21/40	23/40	27/40	7/10
	12	11/24	1/2	7/12	5/8	2/3
	16	7/16	1/2	9/16	5/8	5/8
	32	13/32	7/16	1/2	9/16	19/32
$N_1 = 9$	$N_2 = 10$	7/15	1/2	26/45	2/3	31/45
	12	4/9	1/2	5/9	11/18	2/3
	15	19/45	22/45	8/15	3/5	29/45
	18	7/18	4/9	1/2	5/9	11/18
	36	13/36	5/12	17/36	19/36	5/9
$N_1 = 10$	$N_2 = 15$	2/5	7/15	1/2	17/30	19/30
	20	2/5	9/20	1/2	11/20	3/5
	40	7/20	2/5	9/20	1/2	—
$N_1 = 12$	$N_2 = 15$	23/60	9/20	1/2	11/20	7/12
	16	3/8	7/16	23/48	13/24	7/12
	18	13/36	5/12	17/36	19/36	5/9
	20	11/30	5/12	7/15	31/60	17/30
$N_1 = 15$	$N_2 = 20$	7/20	2/5	13/30	29/60	31/60
$N_1 = 16$	$N_2 = 20$	27/80	31/80	17/40	19/40	41/80
Large sample approximation		$1.07\sqrt{\frac{m+n}{mn}}$	$1.22\sqrt{\frac{m+n}{mn}}$	$1.36\sqrt{\frac{m+n}{mn}}$	$1.52\sqrt{\frac{m+n}{mn}}$	$1.63\sqrt{\frac{m+n}{mn}}$

SOURCE. Adapted from Massey (1952), with permission from the Institute of Mathematical Statistics.

^aThe entries in this table are selected quantiles w_p of the Smirnov test statistic T for two samples, defined by Equations 6.3.1, 6.3.2, and 6.3.3. To enter the table let N_1 be the smaller sample size and let N_2 be the larger sample size. Reject H_0 at the level α if T exceeds $w_{1-\alpha}$ as given in this table. If n and m are not covered by this table, use the large sample approximation given at the end of the table, or consult exact tables by Kim and Jennrich, which appear in Harter and Owen (1970) for $n, m \leq 100$.

TABLE A21 The *t* Distribution^a

Degrees of Freedom	$p = 0.6$	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

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^aThe entries in this table are quantiles w_p of the *t* distribution for various degrees of freedom. Quantiles w_p for $p < 0.5$ may be computed from the equation

$$w_p = -w_{1-p}$$

Note that $w_{0.50} = 0$ for all degrees of freedom.

TABLE A22 The F Distribution with k_1 and k_2 Degrees of Freedom (0.75 Quantiles)

$k_2 \backslash k_1$	1	2	3	4	5	6	7	8	9
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56
11	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53
12	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51
13	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49
14	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46
16	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44
17	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43
18	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42
19	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41
21	1.40	1.48	1.48	1.46	1.44	1.43	1.42	1.41	1.40
22	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39
23	1.39	1.47	1.47	1.45	1.43	1.42	1.41	1.40	1.39
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38
25	1.39	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38
26	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37
28	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37
29	1.38	1.45	1.45	1.43	1.41	1.40	1.38	1.37	1.36
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29
∞	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27

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TABLE A22 (Continued)

10	12	15	20	24	30	40	60	120	∞
9.32	9.41	9.49	9.58	9.63	9.67	9.71	9.76	9.80	9.85
3.38	3.39	3.41	3.43	3.43	3.44	3.45	3.46	3.47	3.48
2.44	2.45	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
1.89	1.89	1.89	1.88	1.88	1.88	1.88	1.87	1.87	1.87
1.77	1.77	1.76	1.76	1.75	1.75	1.75	1.74	1.74	1.74
1.69	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.65	1.65
1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.54	1.53	1.53
1.55	1.54	1.53	1.52	1.52	1.51	1.51	1.50	1.49	1.48
1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47	1.46	1.45
1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42
1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38
1.45	1.44	1.43	1.41	1.41	1.40	1.39	1.38	1.37	1.36
1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32
1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.30
1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.29
1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28
1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28
1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28	1.27
1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.26
1.37	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.27	1.25
1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.26	1.25
1.36	1.35	1.33	1.32	1.31	1.30	1.28	1.27	1.26	1.24
1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.25	1.24
1.35	1.34	1.32	1.31	1.30	1.29	1.27	1.26	1.25	1.23
1.35	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.24	1.23
1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.22	1.21	1.19
1.30	1.29	1.27	1.25	1.24	1.22	1.21	1.19	1.17	1.15
1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.13	1.10
1.25	1.24	1.22	1.19	1.18	1.16	1.14	1.12	1.08	1.00

TABLE A22 (Continued) (0.90 Quantiles)

$k_1 \backslash k_2$	1	2	3	4	5	6	7	8	9
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63

TABLE A22 (Continued)

10	12	15	20	24	30	40	60	120	∞
60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.68
1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

TABLE A22 (Continued) (0.95 Quantiles)

$k_2 \backslash k_1$	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

TABLE A22 (Continued)

10	12	15	20	24	30	40	60	120	∞
241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

TABLE A22 (Continued) (0.975 Quantiles)

$k_1 \backslash k_2$	1	2	3	4	5	6	7	8	9
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11

TABLE A22 (Continued)

10	12	15	20	24	30	40	60	120	∞
968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
3.06	2.96	2.80	2.76	2.70	2.64	2.59	2.52	2.46	2.40
2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

TABLE A22 (Continued) (0.99 Quantiles)

$k_1 \backslash k_2$	1	2	3	4	5	6	7	8	9
1	4052	4999.5	5403	5625	5764	5859	5928	5981	6022
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

TABLE A22 (Continued)

10	12	15	20	24	30	40	60	120	∞
6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00