

**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2007**

**TITLE OF PAPER : LINEAR STATISTICAL METHODS**  
**COURSE CODE : ST204**  
**TIME ALLOWED : 2 (TWO) HOURS**  
**REQUIRMENTS : STATISTICAL TABLES  
AND CALCULATOR**  
**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.  
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN  
GRANTED BY THE INVIGILATOR**

**QUESTION ONE.**

[ 12 + 5 + 4 + 4 marks ]

- 1.1 Define Simple Linear Regression Model and state the main features of the model.
- 1.2 Explain why normality assumption is important in regression model.
- 1.3 Assume that  $X = 0$  is within the scope of the model defined in part (a). What is the implication for the regression function if  $\beta_0 = 0$ ? How would the regression function plot on a graph?
- 1.4 What is the implication for the regression function if  $\beta_1 = 0$ ? How would the regression function plot on a graph?

**QUESTION TWO.**

[ 5 + 10 + 10 marks ]

- 2.1 Discuss Regression Analysis and Analysis of Variance in terms of their differences.
- 2.2 Define the single-factor ANOVA model and discuss the important features of the model.
- 2.3 Explain how we can decompose the total deviation into two components: variation between treatments and variation within treatments, in a single factor analysis of variance model.

**QUESTION THREE.**

[ 8 + 17 marks ]

- 3.1 State the factor effect model of two-factor studies and its important features.
- 3.2 A rehabilitation centre researcher was interested in examining between physical fitness prior to surgery of persons undergoing corrective knee surgery and time required in physical therapy until successful rehabilitation. Patient records in the rehabilitation centre were examined, 24 male subjects who had undergone similar corrective knee surgery during the past year were selected for the study. The number of days required for successful completion of physical therapy and the prior fitness status for each patient follow.

Prior Physical	Number of Days									
Fitness Status	1	2	3	4	5	6	7	8	9	10
Below Average	29	42	38	40	43	40	30	42		
Average	30	35	39	28	31	31	29	35	29	33
Above Average	26	32	21	20	23	22				

- a. Identify the dependent variable, factor studied and factor levels.
- b. Complete the computation of the ANOVA table and conduct the F test. Clearly state all the steps in the test including the conclusion.
- c. What appears to be the nature of the relationship between physical therapy status and duration of required physical therapy?

**QUESTION FOUR.**

[ 2 + 2 + 4 + 1 + 5 + 5 + 3 + 3 marks ]

The following output was obtained from running the model,  $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$  using SPSS:

**ANOVA**

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3510.205	1	3510.205	20.753	.003(a)
	Residual	1184.018	7	169.145		
	Total	4694.222	8			

**Coefficients**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	333.919	47.255		7.066	.000
	x	-8.932	1.961	-.865	-4.556	.003

- 4.1 State the fitted regression line.
- 4.2 State the null and alternative hypotheses for the F-test of the above ANOVA table.
- 4.3 Perform the F-test and clearly state the conclusion.
- 4.4 What is the estimated value of  $\sigma^2$ ?
- 4.5 Test  $\beta_2 = -10$  against  $\beta_2 \neq -10$  at  $\alpha = 0.05$ .
- 4.6 Test  $\beta_1 = 330$  against  $\beta_1 > 330$  at  $\alpha = 0.01$ .
- 4.7 Compute coefficient of determination,  $r^2$  and interpret the result.
- 4.8 Compute coefficient of correlation,  $r$  and explain the nature and strength of the relationship between dependent and independent variables.

**QUESTION FIVE.**

[ 10 + 4 + 7 + 4 marks ]

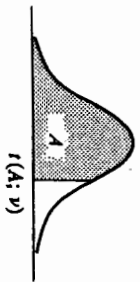
A real estate company specialising in sales of farms would like to know if their salespeople's sale can be predicted based on the number of years with the company. A random sample of sales is taken for 10 salespeople having years of experience ranging from 1 to 10 years and is given below:

Years of experience	1	2	3	4	5	6	7	8	9	10
Number of sales	3	2	5	4	6	8	9	9	12	10

- 5.1 Fit the regression line,  $Y_i = \beta_0 + \beta_1 X_i$ .
- 5.2 Interpret the estimated values of  $\beta_0$  and  $\beta_1$ .
- 5.3 Estimate  $\sigma^2$  and construct a 95% confidence interval for  $\sigma^2$ .
- 5.4 Test  $\beta_1 = 1$  against  $\beta_1 > 1$  at 5% level of significance.

TABLE A.2 Percentiles of the *t* Distribution

Entry is  $t(A; \nu)$  where  $P\{t(\nu) \leq t(A; \nu)\} = A$



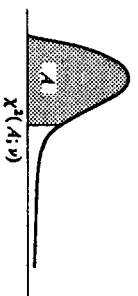
$\nu$	A									
	.60	.70	.80	.85	.90	.95	.975	.98	.985	.99
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358
$\infty$	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326

TABLE A.2 (continued) Percentiles of the *t* Distribution

$\nu$	A									
	.98	.985	.99	.9925	.995	.9975	.9995	.9995	.9995	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590	1589.464	3183.099	6365.900
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598	57.152	81.881	108.842
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924	19.161	24.999	32.909
4	2.999	3.298	3.747	4.088	4.604	5.398	8.610	12.151	15.999	21.448
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869	9.947	12.707	16.750
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959	8.347	10.591	13.707
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408	7.401	9.591	12.591
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041	6.781	8.171	9.781
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781	5.871	7.071	8.471
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587	5.571	6.671	7.871
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437	5.371	6.471	7.671
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318	5.271	6.371	7.571
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221	5.171	6.271	7.471
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140	5.071	6.171	7.371
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073	4.971	6.071	7.271
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015	4.915	5.971	7.171
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965	4.865	5.921	7.121
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922	4.815	5.871	7.071
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883	4.771	5.821	7.021
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849	4.731	5.781	6.981
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819	4.691	5.741	6.941
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792	4.651	5.701	6.901
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768	4.611	5.661	6.861
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745	4.571	5.621	6.821
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725	4.531	5.581	6.781
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707	4.491	5.541	6.741
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690	4.451	5.501	6.701
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674	4.411	5.461	6.661
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659	4.371	5.421	6.621
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646	4.331	5.381	6.581
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551	4.231	5.281	6.481
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460	4.131	5.181	6.381
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373	4.031	5.081	6.281
$\infty$	2.054	2.170	2.326	2.432	2.576	2.807	3.291	3.931	5.031	6.231

TABLE A.3 Percentiles of the  $\chi^2$  Distribution

Entry is  $\chi^2(A; \nu)$  where  $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

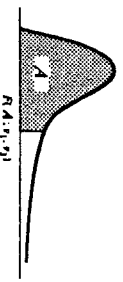


$\nu$	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995	
1	0.0	0.00993	0.0157	0.02982	0.04973	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60	12.84
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84	14.86
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86	16.81
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75	18.55
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55	20.28
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28	22.04
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96	23.59
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59	25.19
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19	26.76
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76	28.30
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30	29.82
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82	31.32
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32	32.80
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80	34.27
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27	35.72
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72	37.16
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16	38.58
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58	39.99
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00	41.40
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40	42.80
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80	44.18
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18	45.56
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56	46.93
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93	48.29
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29	49.64
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64	50.99
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99	52.34
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34	53.67
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67	55.00
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77	70.42
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49	83.30
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95	95.02
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2	108.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3	120.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3	132.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2	144.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1945), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is  $F(A; \nu_1, \nu_2)$  where  $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{F_{1-A; \nu_2, \nu_1}}{1 - A; \nu_1, \nu_2}$$



TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.3
.95	161	200	216	225	230	234	237	239	241	241
.975	648	800	864	900	922	937	948	957	963	965
.99	4,052	5,000	5,403	5,635	5,764	5,859	5,928	5,981	6,022	6,052
.995	16,211	20,000	21,613	22,500	23,056	23,437	23,715	23,925	24,091	24,211
.999	403,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	605,280
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.38
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.9
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	48.5
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	27.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	18.7
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.72
.995	16.2	12.4	10.9	9.52	9.16	8.89	8.68	8.51	8.31	8.31
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df											
	10	12	15	20	24	30	60	120	$\infty$			
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20		
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	63.3		
.95	242	244	246	248	249	250	252	253	254	254		
.975	969	977	985	993	997	1,001	1,010	1,014	1,018	1,018		
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366	6,366		
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	25,464		
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620	636,620		
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44		
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	9.49		
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5		
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5		
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5		
.995	199	199	199	199	199	199	199	199	199	200		
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	999.5		
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27		
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13	5.13		
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53	8.53		
.975	14.4	14.3	14.3	14.3	14.2	14.1	14.0	13.9	13.9	13.9		
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1	26.1		
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8	41.8		
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	123.5		
4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19		
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76	3.76		
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63	5.63		
.975	8.94	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	8.26		
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	13.5		
.995	21.0	20.7	20.4	20.2	20.2	20.2	19.9	19.6	19.5	19.3		
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	44.1		
5	.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15		
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11	3.11		
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37	4.37		
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	6.02		
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	9.02		
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	12.1		
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	23.8		
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12		
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72	2.72		
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67	3.67		
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	4.85		
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88	6.88		
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	8.88		
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	15.7		
7	.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10		
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47	2.47		
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23	3.23		
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14	4.14		
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	5.65		
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	7.08		
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	11.7		

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.50	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	$\infty$		
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09	
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29	
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93	
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67	
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08	
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07	
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06	
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54	
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06	
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72	
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	
15	.50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05	
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	
	.99	4.42	4.25	4.07	3.88	3.79	3.71	3.55	3.47	3.38	
20	.50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03	
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09	
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03	
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	
.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43		
	.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.50	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.90	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.95	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.975	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.99	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.995	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
.999									
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.50	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.90	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.95	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.975	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.99	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.995	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
.999									
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.50	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.90	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.95	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.22	2.12
.975	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.99	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.995	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
.999									
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.50	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.90	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.95	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.11	2.01
.975	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.99	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.995	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10
.999									

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
30	0.935	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02		
.50	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46		
.90	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62		
.95	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79		
.975	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01		
.99	3.54	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18		
.995	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59		
.999											
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01		
.50	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29		
.90	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39		
.95	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48		
.975	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60		
.99	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69		
.995	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89		
.999											
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01		
.50	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19		
.90	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25		
.95	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31		
.975	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38		
.99	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43		
.995	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54		
.999											
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00		
.50	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00		
.90	1.83	1.73	1.67	1.57	1.52	1.46	1.32	1.22	1.00		
.95	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00		
.975	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00		
.99	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00		
.995	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00		
.999											

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