UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2007

TITLE OF PAPER:	**DISTRIBUTION THEORY**
COURSE CODE :	**ST 301**
TIME ALLOWED :	**TWO (2) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS FIVE QUESTIONS.** **ANSWER ANY FOUR (4) QUESTIONS.** **EACH QUESTION CARRIES 15 MARKS.**
REQUIREMENTS:	**Scientific calculator and statistical table**

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QUESTION ONE

(a) Suppose X_1 and X_2 are independent random variables with probability density function $f_1(x_1), x_1 \in A_1$ and $f_2(x_2), x_2 \in A_2$ respectively. Let $Y_1 = U_1(X_1)$ and $Y_2 = U_2(X_2)$. For which $x_1 = \omega_1(y_1)$ and $x_2 = \omega_2(y_2)$ be the inverses of the transformation so that $y_1 \in B_1$ and $y_2 \in B_2$. If $A = A_1 * A_2$ is mapped onto $B = B_1 * B_2$. Show that Y_1 and Y_2 are independent random variables.

(b) If X is a standard normal variable, find the probability density function of $Y = X^2$.

QUESTION TWO

(a) Let $X_i, i = 1, 2, \ldots\ldots n$ be independent EXP (β) random variables. Show that $Y = \sum_{i=1}^{n} X_i \sim GAM(n, \beta)$.

(b) Suppose $X_1 \sim GAM(\alpha, 1)$ and $X_2 \sim GAM(\beta, 1)$, use the moment generation technique to find the distribution of $Y_1 = X_1 + X_2$.

(c) Let x~UNIF (0, 1), given that the probability density function of X is $f_X(x) = 1, 0 < x < 1$. Find the distribution function of Y=-2lnx.

QUESTION THREE

(a) The probability density function of a random variable x is given as $f(x) = cx^2, 0 \le x \le 1$ and zero else -where. Find (i) the constant c

(ii) F(x) and evaluate P(X<1/2).

(b) Let the cumulative distribution function of X be

$$F(x) = \begin{cases} 0, x \le 0 \\ \frac{x}{8}, 0 < x < 2 \\ \frac{x^2}{16}, 2 \le x < 4 \\ 1, x \ge 4 \end{cases}$$

(i) Find the probability density function of X

(ii) Evaluate $P(X \geq 1 / X \leq 3)$

QUESTION FOUR

(a) (i) Define the K-th moment of random variable X.

(ii) Find the k-th moment bf a random variable X whose probability density function is given as $f_X(x) = \begin{cases} \frac{1}{10}, 20 < x < 30 \\ 0, otherwise \end{cases}$.

(b) Given that g(x,0)=o and that

$D_\omega[g(x,\omega)] = -\lambda g(x,\omega) + \lambda g(x-1,\omega)$, for x=1,2,3.........

If $g(0,\omega) = \ell^{-\lambda\omega}$, show that $g(x,\omega) = \frac{(\lambda\omega)^2 \ell^{-\lambda\omega}}{x!}$

QUESTION FIVE

Given that X is a random variable with distribution function $F_X(t)$ and $Y = g(x)$, where g is strictly monotonic decreasing function. Show that

$F_Y(t) = 1 - F_X\left(g^{-1}(t)\right) + P_X\left(g^{-1}(t)\right)$. Hence or otherwise derive the distribution function for Y, when Y=a+bx for all b<0.