UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2007

TITLE OF PAPER:	**Introduction to regression analysis**
COURSE CODE :	**ST 304**
TIME ALLOWED :	**TWO (2) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS FIVE QUESTIONS. ANSWER ANY FOUR (4) QUESTIONS. EACH QUESTION CARRIES 15 MARKS.**
REQUIREMENTS:	**Scientific calculator and statistical table**

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QUESTION ONE

The morning newspaper lists the following used car prices for a foreign compact , with age X measured in years and selling price Y measured in thousands Emalangeni.

X	1	2	3	4	5	6	7	8	9	10
Y	2.45	1.8	2.0	2.0	1.7	1.2	1.15	0.69	0.6	0.47

(a) Determine the equation of the Least Squares regression line.

(b) Construct a 95% confidence interval for the slope of the regression line.

(c) Determine the predicted value for the average selling price of a five-year-old model compact and construct a 95% confidence.

(5+5+5Marks)

QUESTION TWO

The following scores are obtained on a test of dexterity and aggression administered to a random sample of 10 high school seniors:

Student	1	2	3	4	5	6	7	8	9	10
Dexterity	23	29	45	36	49	41	30	15	40	38
Aggression	45	48	16	28	38	21	36	18	31	37

Using Spearman's statistic, test the null hypothesis that the manifestations of dexterity and aggression are independent.

(15Marks)

QUESTION THREE

In a multiple linear regression model Y=Xβ+ U, if all the assumptions necessary for the least squares method hold, except that $E(UU') \neq \sigma^2 I$.

(a) What happens to the estimates of the parameters by the ordinary least squares method?

(b) Suggest an alternative estimating procedure and find the estimates of the parameters and the variance-covariance matrix of the estimates.

(5+10Marks)

QUESTION FOUR

(a) Show that under the first order auto-regressive scheme $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$,

where $u_t \sim NID(0, \sigma_u^2); Cov(\varepsilon_t, \varepsilon_{t-s}) = \rho^s \left(\frac{\sigma_u^2}{1-\rho^2}\right), s \neq 0$.

(5Marks)

(b) Given the following results from the regression model

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ell_t$.

t	1	2	3	4	5	6	7	8	9
$Y-\hat{Y}$	-0.418	-0.350	0.507	-0.374	-0.181	0.652	0.256	0.342	-0.434

Obtain the estimate of ρ and test at 5% level of significance for positive correlation given that n=50.

(10Marks)

QUESTION FIVE

A company sells a special skin cream through fashion stores exclusively. It operates in 15 marketing districts and is interested in predicting district sales.

(i) Fit the regression model $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ell_i$.

(ii) Test whether the sales are related to population and per capita discretionary income at 5% level of significance.

(iii) Obtain the multiple determination coefficients unadjusted and interpret your result.

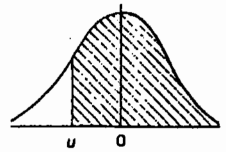
(5+5+5Marks)

District (i)	Sales (Y)	Target Population(X_1)	Per Capita Discretionary Income (X_2)
1	162	274	2.450
2	120	180	3.254
3	223	375	3.802
4	131	205	2.838
5	67	86	2.347
6	169	265	3.782
7	81	98	3.008
8	192	330	2.450
9	116	195	2.137
10	55	53	2.560

11	252	430	4.020
12	232	372	4.427
13	144	236	2.660
14	103	157	2.088
15	212	370	2.605

The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-x^2/2}\, dx$,

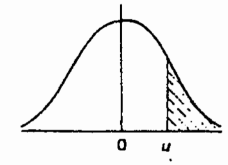
the probability that $U > u$, where $U \sim N(0,1)$.



−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00	u
0.99997	0.99997	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99995	0.99995	−3.9
0.99995	0.99995	0.99995	0.99994	0.99994	0.99994	0.99994	0.99993	0.99993	0.99993	−3.8
0.99992	0.99992	0.99992	0.99992	0.99991	0.99991	0.99990	0.99990	0.99990	0.99989	−3.7
0.99989	0.99988	0.99988	0.99987	0.99987	0.99986	0.99986	0.99985	0.99985	0.99984	−3.6
0.99983	0.99983	0.99982	0.99981	0.99981	0.99980	0.99979	0.99978	0.99978	0.99977	−3.5
0.99976	0.99975	0.99974	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99966	−3.4
0.99965	0.99964	0.99962	0.99961	0.99960	0.99958	0.99957	0.99955	0.99953	0.99952	−3.3
0.99950	0.99948	0.99946	0.99944	0.99942	0.99940	0.99938	0.99936	0.99934	0.99931	−3.2
0.99929	0.99926	0.99924	0.99921	0.99918	0.99916	0.99913	0.99910	0.99906	0.99903	−3.1
0.99900	0.99896	0.99893	0.99889	0.99886	0.99882	0.99878	0.99874	0.99869	0.99865	−3.0
0.99861	0.99856	0.99851	0.99846	0.99841	0.99836	0.99831	0.99825	0.99819	0.99813	−2.9
0.99807	0.99801	0.99795	0.99788	0.99781	0.99774	0.99767	0.99760	0.99752	0.99744	−2.8
0.99736	0.99728	0.99720	0.99711	0.99702	0.99693	0.99683	0.99674	0.99664	0.99653	−2.7
0.99643	0.99632	0.99621	0.99609	0.99598	0.99585	0.99573	0.99560	0.99547	0.99534	−2.6
0.99520	0.99506	0.99492	0.99477	0.99461	0.99446	0.99430	0.99413	0.99396	0.99379	−2.5
0.99361	0.99343	0.99324	0.99305	0.99286	0.99266	0.99245	0.99224	0.99202	0.99180	−2.4
0.99158	0.99134	0.99111	0.99086	0.99061	0.99036	0.99010	0.98983	0.98956	0.98928	−2.3
0.98899	0.98870	0.98840	0.98809	0.98778	0.98745	0.98713	0.98679	0.98645	0.98610	−2.2
0.98574	0.98537	0.98500	0.98461	0.98422	0.98382	0.98341	0.98300	0.98257	0.98214	−2.1
0.98169	0.98124	0.98077	0.98030	0.97982	0.97932	0.97882	0.97831	0.97778	0.97725	−2.0
0.97670	0.97615	0.97558	0.97500	0.97441	0.97381	0.97320	0.97257	0.97193	0.97128	−1.9
0.97062	0.96995	0.96926	0.96856	0.96784	0.96712	0.96638	0.96562	0.96485	0.96407	−1.8
0.96327	0.96246	0.96164	0.96080	0.95994	0.95907	0.95818	0.95728	0.95637	0.95543	−1.7
0.95449	0.95352	0.95254	0.95154	0.95053	0.94950	0.94845	0.94738	0.94630	0.94520	−1.6
0.94408	0.94295	0.94179	0.94062	0.93943	0.93822	0.93699	0.93574	0.93448	0.93319	−1.5
0.93189	0.93056	0.92922	0.92785	0.92647	0.92507	0.92364	0.92220	0.92073	0.91924	−1.4
0.91774	0.91621	0.91466	0.91308	0.91149	0.90988	0.90824	0.90658	0.90490	0.90320	−1.3
0.90147	0.89973	0.89796	0.89617	0.89435	0.89251	0.89065	0.88877	0.88686	0.88493	−1.2
0.88298	0.88100	0.87900	0.87698	0.87493	0.87286	0.87076	0.86864	0.86650	0.86433	−1.1
0.86214	0.85993	0.85769	0.85543	0.85314	0.85083	0.84850	0.84614	0.84375	0.84134	−1.0
0.83891	0.83646	0.83398	0.83147	0.82894	0.82639	0.82381	0.82121	0.81859	0.81594	−0.9
0.81327	0.81057	0.80785	0.80511	0.80234	0.79955	0.79673	0.79389	0.79103	0.78814	−0.8
0.78524	0.78230	0.77935	0.77637	0.77337	0.77035	0.76731	0.76424	0.76115	0.75804	−0.7
0.75490	0.75175	0.74857	0.74537	0.74215	0.73891	0.73565	0.73237	0.72907	0.72575	−0.6
0.72240	0.71904	0.71566	0.71226	0.70884	0.70540	0.70194	0.69847	0.69497	0.69146	−0.5
0.68793	0.68439	0.68082	0.67724	0.67364	0.67003	0.66640	0.66276	0.65910	0.65542	−0.4
0.65173	0.64803	0.64431	0.64058	0.63683	0.63307	0.62930	0.62552	0.62172	0.61791	−0.3
0.61409	0.61026	0.60642	0.60257	0.59871	0.59483	0.59095	0.58706	0.58317	0.57926	−0.2
0.57535	0.57142	0.56750	0.56356	0.55962	0.55567	0.55172	0.54776	0.54380	0.53983	−0.1
0.53586	0.53188	0.52790	0.52392	0.61994	0.51595	0.51197	0.50798	0.50399	0.50000	−0.0

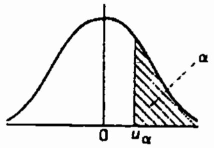
The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-x^2/2}\, dx$,

the probability that $U > u$, where $U \sim N(0,1)$.



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08228
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.06370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00687	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00016	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

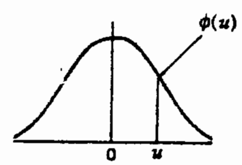
The u_α values tabulated are such that $\Pr(U > u_\alpha) = \alpha$, where $U \sim N(0,1)$



α	u_α	α	u_α	α	u_α	α	u_α
0.50	0.00000	0.34	0.41246	0.18	0.91537	0.025	1.96000
0.49	0.02507	0.33	0.43991	0.17	0.95416	0.020	2.05375
0.48	0.05015	0.32	0.46770	0.16	0.99446	0.010	2.32635
0.47	0.07527	0.31	0.49585	0.15	1.03643	0.009	2.36562
0.46	0.10004	0.30	0.52440	0.14	1.08032	0.008	2.40891
0.45	0.12566	0.29	0.55338	0.13	1.12639	0.007	2.46726
0.44	0.15097	0.28	0.58284	0.12	1.17499	0.006	2.51214
0.43	0.17637	0.27	0.61281	0.11	1.22653	0.005	2.57583
0.42	0.20189	0.26	0.64335	0.10	1.28155	0.004	2.65207
0.41	0.22754	0.25	0.67449	0.09	1.34076	0.003	2.74778
0.40	0.25335	0.24	0.70630	0.08	1.40507	0.002	2.87816
0.39	0.27932	0.23	0.73885	0.07	1.47579	0.001	3.09023
0.38	0.30548	0.22	0.77219	0.06	1.55477	0.0005	3.29053
0.37	0.33185	0.21	0.80642	0.05	1.64485	0.0001	3.71902
0.36	0.35846	0.20	0.84162	0.04	1.75069	0.00005	3.89060
0.35	0.38532	0.19	0.87790	0.03	1.88079	0.00001	4.26489

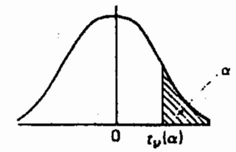
Table 6 ORDINATES OF THE STANDARDISED NORMAL DISTRIBUTION

The function tabulated is $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$.



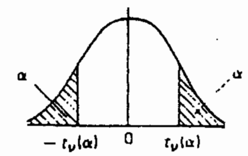
u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.39894	0.39695	0.39104	0.38139	0.36827	0.35207	0.33322	0.31225	0.28969	0.26609
1.0	0.24197	0.21785	0.19419	0.17137	0.14973	0.12952	0.11092	0.09405	0.07895	0.06562
2.0	0.05399	0.04398	0.03547	0.02833	0.02239	0.01753	0.01358	0.01042	0.00792	0.00595
3.0	0.00443	0.00327	0.00238	0.00172	0.00123	0.00087	0.00061	0.00042	0.00029	0.00020
4.0	0.00013	0.00009	0.00006	0.00004	0.00002	0.00002	0.00001	0.00001	0.00000	0.00000

ONE-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)) = \alpha$,
for ν degrees of freedom.

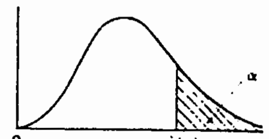
TWO-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)$ or $T_\nu < -t_\nu(\alpha)) = 2\alpha$,
for ν degrees of freedom.

ν	α = 0.4 2α = 0.8	0.25 0.5	0.1 0.2	0.05 0.1	0.025 0.05	0.01 0.02	0.005 0.01	0.0025 0.005	0.001 0.002	0.0005 0.001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.320	318.310	636.620
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.766	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The values tabulated are $\chi^2_\nu(\alpha)$, where
$Pr(\chi^2_\nu > \chi^2_\nu(\alpha)) = \alpha$, for ν degrees of freedom.



0.995	0.990	0.975	0.950	0.900	0.750	0.500	α / ν
392704 . 10^{-10}	157088 . 10^{-9}	982069 . 10^{-9}	393214 . 10^{-8}	0.0157908	0.1015308	0.454936	1
0.0100251	0.0201007	0.0506356	0.102587	0.210721	0.575364	1.38629	2
0.0717218	0.114832	0.215795	0.351846	0.584374	1.212534	2.36597	3
0.206989	0.297109	0.484419	0.710723	1.063623	1.92256	3.35669	4
0.411742	0.554298	0.831212	1.145476	1.61031	2.67460	4.35146	5
0.675727	0.872090	1.23734	1.63538	2.20413	3.45460	5.34812	6
0.989256	1.239043	1.68987	2.16735	2.83311	4.25485	6.34581	7
1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	8
1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	9
2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	10
2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.3410	11
3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.3403	12
3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.3398	13
4.07467	4.66043	5.62873	6.57063	7.78953	10.1653	13.3393	14
4.60092	5.22935	6.26214	7.26094	8.54676	11.0365	14.3389	15
5.14221	5.81221	6.90766	7.96165	9.31224	11.9122	15.3385	16
5.69722	6.40776	7.56419	8.67176	10.0852	12.7919	16.3382	17
6.26480	7.01491	8.23075	9.39046	10.8649	13.6753	17.3379	18
6.84397	7.63273	8.90652	10.1170	11.6509	14.5620	18.3377	19
7.43384	8.26040	9.59078	10.8508	12.4426	15.4518	19.3374	20
8.03365	8.89720	10.28293	11.5913	13.2396	16.3444	20.3372	21
8.64272	9.54249	10.9823	12.3380	14.0415	17.2396	21.3370	22
9.26043	10.19567	11.6886	13.0905	14.8480	18.1373	22.3369	23
9.88623	10.8564	12.4012	13.8484	15.6587	19.0373	23.3367	24
10.5197	11.5240	13.1197	14.6114	16.4734	19.9393	24.3366	25
11.1602	12.1981	13.8439	15.3792	17.2919	20.8434	25.3365	26
11.8076	12.8785	14.5734	16.1514	18.1139	21.7494	26.3363	27
12.4613	13.5647	15.3079	16.9279	18.9392	22.6572	27.3362	28
13.1211	14.2565	16.0471	17.7084	19.7677	23.5666	28.3361	29
13.7867	14.9535	16.7908	18.4927	20.5992	24.4776	29.3360	30
20.7065	22.1643	24.4330	26.5093	29.0505	33.6603	39.3353	40
27.9907	29.7067	32.3574	34.7643	37.6886	42.9421	49.3349	50
35.5345	37.4849	40.4817	43.1880	46.4589	52.2938	59.3347	60
43.2752	45.4417	48.7576	51.7393	55.3289	61.6983	69.3346	70
51.1719	53.5401	57.1532	60.3915	64.2778	71.1445	79.3343	80
59.1963	61.7541	65.6466	69.1260	73.2911	80.6247	89.3342	90
67.3276	70.0649	74.2219	77.9295	82.3581	90.1332	99.3341	100

For $\nu > 30$ take $\chi^2_\nu(\alpha) = \nu\left[1 - \frac{2}{9\nu} + u_\alpha\sqrt{\frac{2}{9\nu}}\right]^3$ where u_α is such that $Pr(U > u_\alpha) = \alpha$, and $U \sim N(0,1)$.

ν / α	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944	10.828
2	2.77259	4.60517	5.99146	7.37776	9.21034	10.5966	13.816
3	4.10834	6.25139	7.81473	9.34840	11.3449	12.8382	16.266
4	5.38527	7.77944	9.48773	11.1433	13.2767	14.8603	18.467
5	6.62568	9.23636	11.0705	12.8325	15.0863	16.7496	20.515
6	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476	22.458
7	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777	24.322
8	10.2189	13.3616	15.5073	17.5345	20.0902	21.9550	26.125
9	11.3888	14.6837	16.9190	19.0228	21.6660	23.5894	27.877
10	12.5489	15.9872	18.3070	20.4832	23.2093	25.1882	29.588
11	13.7007	17.2750	19.6751	21.9200	24.7250	26.7568	31.264
12	14.8454	18.5493	21.0261	23.3367	26.2170	28.2995	32.909
13	15.9839	19.8119	22.3620	24.7356	27.6882	29.8195	34.528
14	17.1169	21.0641	23.6848	26.1189	29.1412	31.3194	36.123
15	18.2451	22.3071	24.9958	27.4884	30.5779	32.8013	37.697
16	19.3689	23.5418	26.2962	28.8454	31.9999	34.2672	39.252
17	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185	40.790
18	21.6049	25.9894	28.8693	31.5264	34.8053	37.1565	42.312
19	22.7178	27.2036	30.1435	32.8523	36.1909	38.5823	43.820
20	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968	45.315
21	24.9348	29.6151	32.6706	35.4789	38.9322	41.4011	46.797
22	26.0393	30.8133	33.9244	36.7807	40.2894	42.7957	48.268
23	27.1413	32.0069	35.1725	38.0756	41.6384	44.1813	49.728
24	28.2412	33.1962	36.4150	39.3641	42.9798	45.5585	51.179
25	29.3389	34.3816	37.6525	40.6465	44.3141	46.9279	52.618
26	30.4346	35.5632	38.8851	41.9232	45.6417	48.2899	54.052
27	31.5284	36.7412	40.1133	43.1945	46.9629	49.6449	55.476
28	32.6205	37.9159	41.3371	44.4608	48.2782	50.9934	56.892
29	33.7109	39.0875	42.5570	45.7223	49.5879	52.3356	58.301
30	34.7997	40.2560	43.7730	46.9792	50.8922	53.6720	59.703
40	45.6160	51.8051	55.7585	59.3417	63.6907	66.7660	73.402
50	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900	86.661
60	66.9815	74.3970	79.0819	83.2977	88.3794	91.9517	99.607
70	77.5767	85.5270	90.5312	95.0232	100.425	104.215	112.317
80	88.1303	96.5782	101.879	106.629	112.329	116.321	124.839
90	98.6499	107.565	113.145	118.136	124.116	128.299	137.208
100	109.141	118.498	124.342	129.561	135.807	140.169	149.449

TABLE 7 (*a*). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$	161·4	199·5	215·7	224·6	230·2	234·0	236·8	238·9	241·9	243·9	249·0	254·3
2	18·5	19·0	19·2	19·2	19·3	19·3	19·4	19·4	19·4	19·4	19·5	19·5
3	10·13	9·55	9·28	9·12	9·01	8·94	8·89	8·85	8·79	8·74	8·64	8·53
4	7·71	6·94	6·59	6·39	6·26	6·16	6·09	6·04	5·96	5·91	5·77	5·63
5	6·61	5·79	5·41	5·19	5·05	4·95	4·88	4·82	4·74	4·68	4·53	4·36
6	5·99	5·14	4·76	4·53	4·39	4·28	4·21	4·15	4·06	4·00	3·84	3·67
7	5·59	4·74	4·35	4·12	3·97	3·87	3·79	3·73	3·64	3·57	3·41	3·23
8	5·32	4·46	4·07	3·84	3·69	3·58	3·50	3·44	3·35	3·28	3·12	2·93
9	5·12	4·26	3·86	3·63	3·48	3·37	3·29	3·23	3·14	3·07	2·90	2·71
10	4·96	4·10	3·71	3·48	3·33	3·22	3·14	3·07	2·98	2·91	2·74	2·54
11	4·84	3·98	3·59	3·36	3·20	3·09	3·01	2·95	2·85	2·79	2·61	2·40
12	4·75	3·89	3·49	3·26	3·11	3·00	2·91	2·85	2·75	2·69	2·51	2·30
13	4·67	3·81	3·41	3·18	3·03	2·92	2·83	2·77	2·67	2·60	2·42	2·21
14	4·60	3·74	3·34	3·11	2·96	2·85	2·76	2·70	2·60	2·53	2·35	2·13
15	4·54	3·68	3·29	3·06	2·90	2·79	2·71	2·64	2·54	2·48	2·29	2·07
16	4·49	3·63	3·24	3·01	2·85	2·74	2·66	2·59	2·49	2·42	2·24	2·01
17	4·45	3·59	3·20	2·96	2·81	2·70	2·61	2·55	2·45	2·38	2·19	1·96
18	4·41	3·55	3·16	2·93	2·77	2·66	2·58	2·51	2·41	2·34	2·15	1·92
19	4·38	3·52	3·13	2·90	2·74	2·63	2·54	2·48	2·38	2·31	2·11	1·88
20	4·35	3·49	3·10	2·87	2·71	2·60	2·51	2·45	2·35	2·28	2·08	1·84
21	4·32	3·47	3·07	2·84	2·68	2·57	2·49	2·42	2·32	2·25	2·05	1·81
22	4·30	3·44	3·05	2·82	2·66	2·55	2·46	2·40	2·30	2·23	2·03	1·78
23	4·28	3·42	3·03	2·80	2·64	2·53	2·44	2·37	2·27	2·20	2·00	1·76
24	4·26	3·40	3·01	2·78	2·62	2·51	2·42	2·36	2·25	2·18	1·98	1·73
25	4·24	3·39	2·99	2·76	2·60	2·49	2·40	2·34	2·24	2·16	1·96	1·71
26	4·23	3·37	2·98	2·74	2·59	2·47	2·39	2·32	2·22	2·15	1·95	1·69
27	4·21	3·35	2·96	2·73	2·57	2·46	2·37	2·31	2·20	2·13	1·93	1·67
28	4·20	3·34	2·95	2·71	2·56	2·45	2·36	2·29	2·19	2·12	1·91	1·65
29	4·18	3·33	2·93	2·70	2·55	2·43	2·35	2·28	2·18	2·10	1·90	1·64
30	4·17	3·32	2·92	2·69	2·53	2·42	2·33	2·27	2·16	2·09	1·89	1·62
32	4·15	3·29	2·90	2·67	2·51	2·40	2·31	2·24	2·14	2·07	1·86	1·59
34	4·13	3·28	2·88	2·65	2·49	2·38	2·29	2·23	2·12	2·05	1·84	1·57
36	4·11	3·26	2·87	2·63	2·48	2·36	2·28	2·21	2·11	2·03	1·82	1·55
38	4·10	3·24	2·85	2·62	2·46	2·35	2·26	2·19	2·09	2·02	1·81	1·53
40	4·08	3·23	2·84	2·61	2·45	2·34	2·25	2·18	2·08	2·00	1·79	1·51
60	4·00	3·15	2·76	2·53	2·37	2·25	2·17	2·10	1·99	1·92	1·70	1·39
120	3·92	3·07	2·68	2·45	2·29	2·18	2·09	2·02	1·91	1·83	1·61	1·25
∞	3·84	3·00	2·60	2·37	2·21	2·10	2·01	1·94	1·83	1·75	1·52	1·00

The function tabulated in Table 7 is F_P defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1+\frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\,\nu_1^{\frac{1}{2}\nu_1}\,\nu_2^{\frac{1}{2}\nu_2}\int_{F_P}^{\infty}\frac{F^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

with $P=5$, $2\frac{1}{2}$, 1 and 0·1. If F is the ratio of a mean square on ν_1 degrees of freedom to an independent mean square on ν_2 degrees of freedom, and if the mean squares have equal expectations, then $P/100$ is the probability that $F \geqslant F_P$. The lower percentage points, that is the value F'_P such that $P/100$ is the probability that $F \leqslant F'_P$ may be found by interchanging ν_1 and ν_2 and using the reciprocal of the tabulated value.

Linear interpolation will usually be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, though occasionally a slight improvement may be effected by using harmonic interpolation. Otherwise, except

TABLE 7 (*b*). 2½ PER CENT POINTS OF THE *F*-DISTRIBUTION

	$\nu_1 =$ 1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 =$ 1	648	800	864	900	922	937	948	957	969	977	997	1018
2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.5	39.5
3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.4	14.3	14.1	13.9
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.84	8.75	8.51	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.62	6.52	6.28	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.46	5.37	5.12	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.76	4.67	4.42	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.30	4.20	3.95	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	3.96	3.87	3.61	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.72	3.62	3.37	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.53	3.43	3.17	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.37	3.28	3.02	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.25	3.15	2.89	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.15	3.05	2.79	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.06	2.96	2.70	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.99	2.89	2.63	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.92	2.82	2.56	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.87	2.77	2.50	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.82	2.72	2.45	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.77	2.68	2.41	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.73	2.64	2.37	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.70	2.60	2.33	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.67	2.57	2.30	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.64	2.54	2.27	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.61	2.51	2.24	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.59	2.49	2.22	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.57	2.47	2.19	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.55	2.45	2.17	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.53	2.43	2.15	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.51	2.41	2.14	1.79
32	5.53	4.15	3.56	3.22	3.00	2.84	2.72	2.62	2.48	2.38	2.10	1.75
34	5.50	4.12	3.53	3.19	2.97	2.81	2.69	2.59	2.45	2.35	2.08	1.72
36	5.47	4.09	3.51	3.17	2.94	2.79	2.66	2.57	2.43	2.33	2.05	1.69
38	5.45	4.07	3.48	3.15	2.92	2.76	2.64	2.55	2.41	2.31	2.03	1.66
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.39	2.29	2.01	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.27	2.17	1.88	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.16	2.05	1.76	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.05	1.94	1.64	1.00

when ν_1 and ν_2 are both large, interpolation should be linear in $\nu_1 F_P$ or $\nu_2 F_P$ (this is equivalent to harmonic interpolation). When ν_1 and ν_2 are both large the percentage points may be found from the formula

$$1.1513 \log F_P = \tfrac{1}{2} \ln F_P = \frac{x_P\sqrt{h+\lambda}}{h} - \left(\frac{1}{\nu_1 - 1} - \frac{1}{\nu_2 - 1}\right)\left(\lambda + \frac{5}{6}\right)$$

where x_P is the P-per cent point of the normal distribution (Table 2), $\lambda = \frac{1}{6}(x_P^2 - 3)$ and $\frac{2}{h} = \frac{1}{\nu_1 - 1} + \frac{1}{\nu_2 - 1}$.
For the values of P given in Table 7, x_P and λ are as follows:

P	5	2½	1	0.1
x_P	+1.6449	1.9600	2.3263	3.0902
λ	−0.0491	+0.1402	0.4020	1.0916

TABLE 7(c). 1 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6366
2	98·5	99·0	99·2	99·2	99·3	99·3	99·4	99·4	99·4	99·4	99·5	99·5
3	34·1	30·8	29·5	28·7	28·2	27·9	27·7	27·5	27·2	27·1	26·6	26·1
4	21·2	18·0	16·7	16·0	15·5	15·2	15·0	14·8	14·5	14·4	13·9	13·5
5	16·26	13·27	12·06	11·39	10·97	10·67	10·46	10·29	10·05	9·89	9·47	9·02
6	13·74	10·92	9·78	9·15	8·75	8·47	8·26	8·10	7·87	7·72	7·31	6·88
7	12·25	9·55	8·45	7·85	7·46	7·19	6·99	6·84	6·62	6·47	6·07	5·65
8	11·26	8·65	7·59	7·01	6·63	6·37	6·18	6·03	5·81	5·67	5·28	4·86
9	10·56	8·02	6·99	6·42	6·06	5·80	5·61	5·47	5·26	5·11	4·73	4·31
10	10·04	7·56	6·55	5·99	5·64	5·39	5·20	5·06	4·85	4·71	4·33	3·91
11	9·65	7·21	6·22	5·67	5·32	5·07	4·89	4·74	4·54	4·40	4·02	3·60
12	9·33	6·93	5·95	5·41	5·06	4·82	4·64	4·50	4·30	4·16	3·78	3·36
13	9·07	6·70	5·74	5·21	4·86	4·62	4·44	4·30	4·10	3·96	3·59	3·17
14	8·86	6·51	5·56	5·04	4·70	4·46	4·28	4·14	3·94	3·80	3·43	3·00
15	8·68	6·36	5·42	4·89	4·56	4·32	4·14	4·00	3·80	3·67	3·29	2·87
16	8·53	6·23	5·29	4·77	4·44	4·20	4·03	3·89	3·69	3·55	3·18	2·75
17	8·40	6·11	5·18	4·67	4·34	4·10	3·93	3·79	3·59	3·46	3·08	2·65
18	8·29	6·01	5·09	4·58	4·25	4·01	3·84	3·71	3·51	3·37	3·00	2·57
19	8·18	5·93	5·01	4·50	4·17	3·94	3·77	3·63	3·43	3·30	2·92	2·49
20	8·10	5·85	4·94	4·43	4·10	3·87	3·70	3·56	3·37	3·23	2·86	2·42
21	8·02	5·78	4·87	4·37	4·04	3·81	3·64	3·51	3·31	3·17	2·80	2·36
22	7·95	5·72	4·82	4·31	3·99	3·76	3·59	3·45	3·26	3·12	2·75	2·31
23	7·88	5·66	4·76	4·26	3·94	3·71	3·54	3·41	3·21	3·07	2·70	2·26
24	7·82	5·61	4·72	4·22	3·90	3·67	3·50	3·36	3·17	3·03	2·66	2·21
25	7·77	5·57	4·68	4·18	3·86	3·63	3·46	3·32	3·13	2·99	2·62	2·17
26	7·72	5·53	4·64	4·14	3·82	3·59	3·42	3·29	3·09	2·96	2·58	2·13
27	7·68	5·49	4·60	4·11	3·78	3·56	3·39	3·26	3·06	2·93	2·55	2·10
28	7·64	5·45	4·57	4·07	3·75	3·53	3·36	3·23	3·03	2·90	2·52	2·06
29	7·60	5·42	4·54	4·04	3·73	3·50	3·33	3·20	3·00	2·87	2·49	2·03
30	7·56	5·39	4·51	4·02	3·70	3·47	3·30	3·17	2·98	2·84	2·47	2·01
32	7·50	5·34	4·46	3·97	3·65	3·43	3·26	3·13	2·93	2·80	2·42	1·96
34	7·45	5·29	4·42	3·93	3·61	3·39	3·22	3·09	2·90	2·76	2·38	1·91
36	7·40	5·25	4·38	3·89	3·58	3·35	3·18	3·05	2·86	2·72	2·35	1·87
38	7·35	5·21	4·34	3·86	3·54	3·32	3·15	3·02	2·83	2·69	2·32	1·84
40	7·31	5·18	4·31	3·83	3·51	3·29	3·12	2·99	2·80	2·66	2·29	1·80
60	7·08	4·98	4·13	3·65	3·34	3·12	2·95	2·82	2·63	2·50	2·12	1·60
120	6·85	4·79	3·95	3·48	3·17	2·96	2·79	2·66	2·47	2·34	1·95	1·38
∞	6·63	4·61	3·78	3·32	3·02	2·80	2·64	2·51	2·32	2·18	1·79	1·00

The function tabulated in Table 7 is F_P defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1+\frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\nu_1^{\frac{1}{2}\nu_1}\nu_2^{\frac{1}{2}\nu_2}\int_{F_P}^{\infty}\frac{F^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

with $P=5$, $2\frac{1}{2}$, 1 and 0·1. If F is the ratio of a mean square on ν_1 degrees of freedom to an independent mean square on ν_2 degrees of freedom, and if the mean squares have equal expectations, then $P/100$ is the probability that $F \geqslant F_P$. The lower percentage points, that is the value F_P' such that $P/100$ is the probability that $F \leqslant F_P'$ may be found by interchanging ν_1 and ν_2 and using the reciprocal of the tabulated value.

Linear interpolation will usually be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, though occasionally a slight improvement may be effected by using harmonic interpolation. Otherwise, except

TABLE 7 (*d*). 0·1 PER CENT POINTS OF THE *F*-DISTRIBUTION

$\nu_1=$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2=1$*	4053	5000	5404	5625	5764	5859	5929	5981	6056	6107	6235	6366*
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.5	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.2	128.3	125.9	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.05	47.41	45.77	44.05
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.92	26.42	25.14	23.79
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	15.75
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	11.70
8	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.34
9	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	9.87	9.57	8.72	7.81
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.74	8.44	7.64	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	7.92	7.63	6.85	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.25	5.42
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.80	6.52	5.78	4.97
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.40	6.13	5.41	4.60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.08	5.81	5.10	4.31
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.81	5.55	4.85	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.58	5.32	4.63	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.39	5.13	4.45	3.67
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.22	4.97	4.29	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.08	4.82	4.15	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	4.95	4.70	4.03	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.83	4.58	3.92	3.15
23	14.19	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.73	4.48	3.82	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.64	4.39	3.74	2.97
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.56	4.31	3.66	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.48	4.24	3.59	2.82
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.41	4.17	3.52	2.75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.35	4.11	3.46	2.69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.29	4.05	3.41	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.24	4.00	3.36	2.59
32	13.12	8.64	6.94	6.01	5.43	5.02	4.72	4.48	4.14	3.91	3.27	2.50
34	12.97	8.52	6.83	5.92	5.34	4.93	4.63	4.40	4.06	3.83	3.19	2.42
36	12.83	8.42	6.74	5.84	5.26	4.86	4.56	4.33	3.99	3.76	3.12	2.35
38	12.71	8.33	6.66	5.76	5.19	4.79	4.49	4.26	3.93	3.70	3.06	2.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	3.87	3.64	3.01	2.23
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.54	3.32	2.69	1.89
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.24	3.02	2.40	1.54
∞	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	2.96	2.74	2.13	1.00

when ν_1 and ν_2 are both large, interpolation should be linear in $\nu_1 F_P$ or $\nu_2 F_P$ (this is equivalent to harmonic interpolation). When ν_1 and ν_2 are both large the percentage points may be found from the formula

$$1.1513 \log F_P = \tfrac{1}{2} \ln F_P = \frac{x_P\sqrt{h+\lambda}}{h} - \left(\frac{1}{\nu_1-1} - \frac{1}{\nu_2-1}\right)\left(\lambda + \frac{5}{6}\right)$$

where x_P is the P-per cent point of the normal distribution (Table 2), $\lambda = \frac{1}{6}(x_P^2-3)$ and $\frac{2}{h} = \frac{1}{\nu_1-1} + \frac{1}{\nu_2-1}$.
For the values of P given in Table 7, x_P and λ are as follows:

P	5	$2\frac{1}{2}$	1	0.1
x_P	+1.6449	1.9600	2.3263	3.0902
λ	−0.0491	+0.1402	0.4020	1.0916

* Entries for $\nu_2 = 1$ must be multiplied by 100.

Table B-5 (*Continued*)

	k'=1		k'=2		k'=3		k'=4		k'=5		k'=6		k'=7		k'=8		k'=9		k'=10	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.610	1.400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.700	1.356	0.467	1.896	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	—	—	—	—	—	—	—	—	—	—	—	—
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	—	—	—	—	—	—	—	—	—	—
11	0.927	1.324	0.758	1.604	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005	—	—	—	—	—	—	—	—
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149	—	—	—	—	—	—
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.445	2.390	0.328	2.692	0.230	2.985	0.147	3.266	—	—	—	—
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360	—	—
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.459	2.396	0.456	2.589	0.369	2.783	0.290	2.974
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	1.944	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.080	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.131	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.970	2.098	0.912	2.180
39	1.435	1.540	1.382	1.597	1.328	1.658	1.273	1.722	1.218	1.789	1.161	1.859	1.104	1.932	1.047	2.007	0.990	2.085	0.932	2.164
40	1.442	1.544	1.391	1.600	1.338	1.659	1.285	1.721	1.230	1.786	1.175	1.854	1.120	1.924	1.064	1.997	1.008	2.072	0.945	2.149
45	1.475	1.566	1.430	1.615	1.383	1.666	1.336	1.720	1.287	1.776	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.002	1.038	2.088
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771	1.291	1.822	1.246	1.875	1.201	1.930	1.156	1.986	1.110	2.044
55	1.528	1.601	1.490	1.641	1.452	1.681	1.414	1.724	1.374	1.768	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.170	2.010
60	1.549	1.616	1.514	1.652	1.480	1.689	1.444	1.727	1.408	1.767	1.372	1.808	1.335	1.850	1.298	1.894	1.260	1.939	1.222	1.984
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767	1.404	1.805	1.370	1.843	1.336	1.882	1.301	1.923	1.266	1.964
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.910	1.305	1.948
75	1.598	1.652	1.571	1.680	1.543	1.709	1.515	1.739	1.487	1.770	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935
80	1.611	1.662	1.586	1.688	1.560	1.715	1.534	1.743	1.507	1.772	1.480	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925
85	1.624	1.671	1.600	1.696	1.575	1.721	1.550	1.747	1.525	1.774	1.500	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.916
90	1.635	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.420	1.909
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778	1.535	1.802	1.512	1.827	1.489	1.852	1.465	1.877	1.442	1.903
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.780	1.550	1.803	1.528	1.826	1.506	1.850	1.484	1.874	1.462	1.898
150	1.720	1.746	1.706	1.760	1.693	1.774	1.679	1.788	1.665	1.802	1.651	1.817	1.637	1.832	1.622	1.847	1.608	1.862	1.594	1.877
200	1.758	1.778	1.748	1.789	1.738	1.799	1.728	1.810	1.718	1.820	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.665	1.874

Table B-5 (*Continued*)

	k'=11		k'=12		k'=13		k'=14		k'=15		k'=16		k'=17		k'=18		k'=19		k'=20	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.098	3.503	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.138	3.378	0.087	3.557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.177	3.265	0.123	3.441	0.078	3.603	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.220	3.159	0.160	3.335	0.111	3.496	0.070	3.642	—	—	—	—	—	—	—	—	—	—	—	—
20	0.263	3.063	0.200	3.234	0.145	3.395	0.100	3.542	0.063	3.676	—	—	—	—	—	—	—	—	—	—
21	0.307	2.976	0.240	3.141	0.182	3.300	0.132	3.448	0.091	3.583	0.058	3.705	—	—	—	—	—	—	—	—
22	0.349	2.897	0.281	3.057	0.220	3.211	0.166	3.358	0.120	3.495	0.083	3.619	0.052	3.731	—	—	—	—	—	—
23	0.391	2.826	0.322	2.979	0.259	3.128	0.202	3.272	0.153	3.409	0.110	3.535	0.076	3.650	0.048	3.753	—	—	—	—
24	0.431	2.761	0.362	2.908	0.297	3.053	0.239	3.193	0.186	3.327	0.141	3.454	0.101	3.572	0.070	3.678	0.044	3.773	—	—
25	0.470	2.702	0.400	2.844	0.335	2.983	0.275	3.119	0.221	3.251	0.172	3.376	0.130	3.494	0.094	3.604	0.065	3.702	0.041	3.790
26	0.508	2.649	0.438	2.784	0.373	2.919	0.312	3.051	0.256	3.179	0.205	3.303	0.160	3.420	0.120	3.531	0.087	3.632	0.060	3.724
27	0.544	2.600	0.475	2.730	0.409	2.859	0.348	2.987	0.291	3.112	0.238	3.233	0.191	3.349	0.149	3.460	0.112	3.563	0.081	3.658
28	0.578	2.555	0.510	2.680	0.445	2.805	0.383	2.928	0.325	3.050	0.271	3.168	0.222	3.283	0.178	3.392	0.138	3.495	0.104	3.592
29	0.612	2.515	0.544	2.634	0.479	2.755	0.418	2.874	0.359	2.992	0.305	3.107	0.254	3.219	0.208	3.327	0.166	3.431	0.129	3.528
30	0.643	2.477	0.577	2.592	0.512	2.708	0.451	2.823	0.392	2.937	0.337	3.050	0.286	3.160	0.238	3.266	0.195	3.368	0.156	3.465
31	0.674	2.443	0.608	2.553	0.545	2.665	0.484	2.776	0.425	2.987	0.370	2.996	0.317	3.103	0.269	3.208	0.224	3.309	0.183	3.406
32	0.703	2.411	0.638	2.517	0.576	2.625	0.515	2.733	0.457	2.840	0.401	2.946	0.349	3.050	0.299	3.153	0.253	3.252	0.211	3.348
33	0.731	2.382	0.668	2.484	0.606	2.588	0.546	2.692	0.488	2.796	0.432	2.899	0.379	3.000	0.329	3.100	0.283	3.198	0.239	3.293
34	0.758	2.355	0.695	2.454	0.634	2.554	0.575	2.654	0.518	2.754	0.462	2.854	0.409	2.954	0.359	3.051	0.312	3.147	0.267	3.240
35	0.783	2.330	0.722	2.425	0.662	2.521	0.604	2.619	0.547	2.716	0.492	2.813	0.439	2.910	0.388	3.005	0.340	3.099	0.295	3.190
36	0.808	2.306	0.748	2.398	0.689	2.492	0.631	2.586	0.575	2.680	0.520	2.774	0.467	2.868	0.417	2.961	0.369	3.053	0.323	3.142
37	0.831	2.285	0.772	2.374	0.714	2.464	0.657	2.555	0.602	2.646	0.548	2.738	0.495	2.829	0.445	2.920	0.397	3.009	0.351	3.097
38	0.854	2.265	0.796	2.351	0.739	2.438	0.683	2.526	0.628	2.614	0.575	2.703	0.522	2.792	0.472	2.880	0.424	2.968	0.378	3.054
39	0.875	2.246	0.819	2.329	0.763	2.413	0.707	2.499	0.653	2.585	0.600	2.671	0.549	2.757	0.499	2.843	0.451	2.929	0.404	3.013
40	0.896	2.228	0.840	2.309	0.785	2.391	0.731	2.473	0.678	2.557	0.626	2.641	0.575	2.724	0.525	2.808	0.477	2.892	0.430	2.974
45	0.988	2.156	0.938	2.225	0.887	2.296	0.838	2.367	0.788	2.439	0.740	2.512	0.692	2.586	0.644	2.659	0.598	2.733	0.553	2.807
50	1.064	2.103	1.019	2.163	0.973	2.225	0.927	2.287	0.882	2.350	0.836	2.414	0.792	2.479	0.747	2.544	0.703	2.610	0.660	2.675
55	1.129	2.062	1.087	2.116	1.045	2.170	1.003	2.225	0.961	2.281	0.919	2.338	0.877	2.396	0.836	2.454	0.795	2.512	0.754	2.571
60	1.184	2.031	1.145	2.079	1.106	2.127	1.068	2.177	1.029	2.227	0.990	2.278	0.951	2.330	0.913	2.382	0.874	2.434	0.836	2.487
65	1.231	2.006	1.195	2.049	1.160	2.093	1.124	2.138	1.088	2.183	1.052	2.229	1.016	2.276	0.980	2.323	0.944	2.371	0.908	2.419
70	1.272	1.986	1.239	2.026	1.206	2.066	1.172	2.106	1.139	2.148	1.105	2.189	1.072	2.232	1.038	2.275	1.005	2.318	0.971	2.362
75	1.308	1.970	1.277	2.006	1.247	2.043	1.215	2.080	1.184	2.118	1.153	2.156	1.121	2.195	1.090	2.235	1.058	2.275	1.027	2.315
80	1.340	1.957	1.311	1.991	1.283	2.024	1.253	2.059	1.224	2.093	1.195	2.129	1.165	2.165	1.136	2.201	1.106	2.238	1.076	2.275
85	1.369	1.946	1.342	1.977	1.315	2.009	1.287	2.040	1.260	2.073	1.232	2.105	1.205	2.139	1.177	2.172	1.149	2.206	1.121	2.241
90	1.395	1.937	1.369	1.966	1.344	1.995	1.318	2.025	1.292	2.055	1.266	2.085	1.240	2.116	1.213	2.148	1.187	2.179	1.160	2.211
95	1.418	1.929	1.394	1.956	1.370	1.984	1.345	2.012	1.321	2.040	1.296	2.068	1.271	2.097	1.247	2.126	1.222	2.156	1.197	2.186
100	1.434	1.923	1.416	1.948	1.393	1.974	1.371	2.000	1.347	2.026	1.324	2.053	1.301	2.080	1.277	2.108	1.253	2.135	1.229	2.164
150	1.579	1.892	1.564	1.908	1.550	1.924	1.535	1.940	1.519	1.956	1.504	1.972	1.489	1.989	1.474	2.006	1.458	2.023	1.443	2.040
200	1.654	1.885	1.643	1.896	1.632	1.908	1.621	1.919	1.610	1.931	1.599	1.943	1.588	1.955	1.576	1.967	1.565	1.979	1.554	1.991

[a] k' is the number of regressors excluding the intercept.

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Table B-5 Durbin-Watson statistic (Savin-White tables) Durbin-Watson statistic: 1 percent significance points of dL and dU[a]

n	k'=1		k'=2		k'=3		k'=4		k'=5		k'=6		k'=7		k'=8		k'=9		k'=10	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.390	1.142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.435	1.036	0.294	1.676	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.497	1.003	0.345	1.489	0.229	2.102	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.554	0.998	0.408	1.389	0.279	1.875	0.183	2.433	—	—	—	—	—	—	—	—	—	—	—	—
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690	—	—	—	—	—	—	—	—	—	—
11	0.653	1.010	0.519	1.297	0.396	1.640	0.286	2.030	0.193	2.453	0.124	2.892	—	—	—	—	—	—	—	—
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280	0.164	2.665	0.105	3.053	—	—	—	—	—	—
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.826	0.294	2.150	0.211	2.490	0.140	2.838	0.090	3.182	—	—	—	—
14	0.776	1.054	0.660	1.254	0.547	1.490	0.441	1.757	0.343	2.049	0.257	2.354	0.183	2.667	0.122	2.981	0.078	3.287	—	—
15	0.811	1.070	0.700	1.252	0.591	1.464	0.488	1.704	0.391	1.967	0.303	2.244	0.226	2.530	0.161	2.817	0.107	3.101	0.068	3.374
16	0.844	1.086	0.737	1.252	0.633	1.446	0.532	1.663	0.437	1.900	0.349	2.153	0.269	2.416	0.200	2.681	0.142	2.944	0.094	3.201
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.630	0.480	1.847	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053
18	0.902	1.118	0.805	1.259	0.708	1.422	0.613	1.604	0.522	1.803	0.435	2.015	0.355	2.238	0.282	2.467	0.216	2.697	0.160	2.925
19	0.928	1.132	0.835	1.265	0.742	1.415	0.650	1.584	0.561	1.767	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.598	1.737	0.515	1.918	0.436	2.110	0.362	2.308	0.294	2.510	0.232	2.714
21	0.975	1.161	0.890	1.277	0.803	1.408	0.718	1.554	0.633	1.712	0.552	1.881	0.474	2.059	0.400	2.244	0.331	2.434	0.268	2.625
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691	0.587	1.849	0.510	2.015	0.437	2.188	0.368	2.367	0.304	2.548
23	1.018	1.187	0.938	1.291	0.858	1.407	0.777	1.534	0.698	1.673	0.620	1.821	0.545	1.977	0.473	2.140	0.404	2.308	0.340	2.479
24	1.037	1.199	0.960	1.298	0.882	1.407	0.805	1.528	0.728	1.658	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417
25	1.055	1.211	0.981	1.305	0.906	1.409	0.831	1.523	0.756	1.645	0.682	1.766	0.610	1.915	0.540	2.059	0.473	2.209	0.409	2.362
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635	0.711	1.759	0.640	1.889	0.572	2.026	0.505	2.168	0.441	2.313
27	1.089	1.233	1.019	1.319	0.949	1.413	0.878	1.515	0.808	1.626	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269
28	1.104	1.244	1.037	1.325	0.969	1.415	0.900	1.513	0.832	1.618	0.764	1.729	0.696	1.847	0.630	1.970	0.566	2.098	0.504	2.229
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611	0.788	1.718	0.723	1.830	0.658	1.947	0.595	2.068	0.533	2.193
30	1.133	1.263	1.070	1.339	1.006	1.421	0.941	1.511	0.877	1.606	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.160
31	1.147	1.273	1.085	1.345	1.023	1.425	0.960	1.510	0.897	1.601	0.834	1.698	0.772	1.800	0.710	1.906	0.649	2.017	0.589	2.131
32	1.160	1.282	1.100	1.352	1.040	1.428	0.979	1.510	0.917	1.597	0.856	1.690	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104
33	1.172	1.291	1.114	1.358	1.055	1.432	0.996	1.510	0.936	1.594	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.080
34	1.184	1.299	1.128	1.364	1.070	1.435	1.012	1.511	0.954	1.591	0.896	1.677	0.837	1.766	0.779	1.860	0.722	1.957	0.665	2.057
35	1.195	1.307	1.140	1.370	1.085	1.439	1.028	1.512	0.971	1.589	0.914	1.671	0.857	1.757	0.800	1.847	0.744	1.940	0.689	2.037
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586	0.950	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585	0.966	1.658	0.913	1.735	0.860	1.816	0.807	1.899	0.754	1.985
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584	0.982	1.655	0.930	1.729	0.878	1.807	0.826	1.887	0.774	1.970
40	1.246	1.344	1.198	1.398	1.148	1.457	1.098	1.518	1.048	1.584	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.789	1.956
45	1.288	1.376	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902
50	1.324	1.403	1.285	1.446	1.245	1.491	1.205	1.538	1.164	1.587	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864
55	1.356	1.427	1.320	1.466	1.284	1.506	1.247	1.548	1.209	1.592	1.172	1.638	1.134	1.685	1.095	1.734	1.057	1.785	1.018	1.837
60	1.383	1.449	1.350	1.484	1.317	1.520	1.283	1.558	1.249	1.598	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817
65	1.407	1.468	1.377	1.500	1.346	1.534	1.315	1.568	1.283	1.604	1.251	1.642	1.218	1.680	1.186	1.720	1.153	1.761	1.120	1.802
70	1.429	1.485	1.400	1.515	1.372	1.546	1.343	1.578	1.313	1.611	1.283	1.645	1.253	1.680	1.223	1.716	1.192	1.754	1.162	1.792
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.340	1.617	1.313	1.646	1.284	1.682	1.256	1.716	1.227	1.746	1.199	1.785
80	1.466	1.515	1.441	1.541	1.416	1.568	1.390	1.595	1.364	1.624	1.338	1.653	1.312	1.683	1.285	1.714	1.259	1.745	1.232	1.777
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.630	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773
90	1.496	1.540	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636	1.383	1.661	1.360	1.687	1.336	1.714	1.312	1.741	1.288	1.769
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.642	1.403	1.666	1.381	1.690	1.358	1.715	1.336	1.741	1.313	1.767
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647	1.421	1.670	1.400	1.693	1.378	1.717	1.357	1.741	1.335	1.765
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693	1.543	1.708	1.530	1.722	1.515	1.737	1.501	1.752	1.486	1.767
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725	1.613	1.735	1.603	1.746	1.592	1.757	1.582	1.768	1.571	1.779

Table B-5 (*Continued*)

n	k'=11		k'=12		k'=13		k'=14		k'=15		k'=16		k'=17		k'=18		k'=19		k'=20	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.060	3.446	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.084	3.286	0.053	3.506	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.113	3.146	0.075	3.358	0.047	3.557	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.145	3.023	0.102	3.227	0.067	3.420	0.043	3.601	—	—	—	—	—	—	—	—	—	—	—	—
20	0.178	2.914	0.131	3.109	0.092	3.297	0.061	3.474	0.038	3.639	—	—	—	—	—	—	—	—	—	—
21	0.212	2.817	0.162	3.004	0.119	3.185	0.084	3.358	0.055	3.521	0.035	3.671	—	—	—	—	—	—	—	—
22	0.246	2.729	0.194	2.909	0.148	3.084	0.109	3.252	0.077	3.412	0.050	3.562	0.032	3.700	—	—	—	—	—	—
23	0.281	2.651	0.227	2.822	0.178	2.991	0.136	3.155	0.100	3.311	0.070	3.459	0.046	3.597	0.029	3.725	—	—	—	—
24	0.315	2.580	0.260	2.744	0.209	2.906	0.165	3.065	0.125	3.218	0.092	3.363	0.065	3.501	0.043	3.629	0.027	3.747	—	—
25	0.348	2.517	0.292	2.674	0.240	2.829	0.194	2.982	0.152	3.131	0.116	3.274	0.085	3.410	0.060	3.538	0.039	3.657	0.025	3.766
26	0.381	2.460	0.324	2.610	0.272	2.758	0.224	2.906	0.180	3.050	0.141	3.191	0.107	3.325	0.079	3.452	0.055	3.572	0.036	3.682
27	0.413	2.409	0.356	2.552	0.303	2.694	0.253	2.836	0.208	2.976	0.167	3.113	0.131	3.245	0.100	3.371	0.073	3.490	0.051	3.602
28	0.444	2.363	0.387	2.499	0.333	2.635	0.283	2.772	0.237	2.907	0.194	3.040	0.156	3.169	0.122	3.294	0.093	3.412	0.068	3.524
29	0.474	2.321	0.417	2.451	0.363	2.582	0.313	2.713	0.266	2.843	0.222	2.972	0.182	3.098	0.146	3.220	0.114	3.338	0.087	3.450
30	0.503	2.283	0.447	2.407	0.393	2.533	0.342	2.659	0.294	2.785	0.249	2.909	0.208	3.032	0.171	3.152	0.137	3.267	0.107	3.379
31	0.531	2.248	0.475	2.367	0.422	2.487	0.371	2.609	0.322	2.730	0.277	2.851	0.234	2.970	0.196	3.087	0.160	3.201	0.128	3.311
32	0.558	2.216	0.503	2.330	0.450	2.446	0.399	2.563	0.350	2.680	0.304	2.797	0.261	2.912	0.221	3.026	0.184	3.137	0.151	3.246
33	0.585	2.187	0.530	2.296	0.477	2.408	0.426	2.520	0.377	2.633	0.331	2.746	0.287	2.858	0.246	2.969	0.209	3.078	0.174	3.184
34	0.610	2.160	0.556	2.266	0.503	2.373	0.452	2.481	0.404	2.590	0.357	2.699	0.313	2.808	0.272	2.915	0.233	3.022	0.197	3.126
35	0.634	2.136	0.581	2.237	0.529	2.340	0.478	2.444	0.430	2.550	0.383	2.655	0.339	2.761	0.297	2.865	0.257	2.969	0.221	3.071
36	0.658	2.113	0.605	2.210	0.554	2.310	0.504	2.410	0.455	2.512	0.409	2.614	0.364	2.717	0.322	2.818	0.282	2.919	0.244	3.019
37	0.680	2.092	0.628	2.186	0.578	2.282	0.528	2.379	0.480	2.477	0.434	2.576	0.389	2.675	0.347	2.774	0.306	2.872	0.268	2.969
38	0.702	2.073	0.651	2.164	0.601	2.256	0.552	2.350	0.504	2.445	0.458	2.540	0.414	2.637	0.371	2.733	0.330	2.828	0.291	2.923
39	0.723	2.055	0.673	2.143	0.623	2.232	0.575	2.323	0.528	2.414	0.482	2.507	0.438	2.600	0.395	2.694	0.354	2.787	0.315	2.879
40	0.744	2.039	0.694	2.123	0.645	2.210	0.597	2.297	0.551	2.386	0.505	2.476	0.461	2.566	0.418	2.657	0.377	2.748	0.338	2.838
45	0.835	1.972	0.790	2.044	0.744	2.118	0.700	2.193	0.655	2.269	0.612	2.346	0.570	2.424	0.528	2.503	0.488	2.582	0.448	2.661
50	0.913	1.925	0.871	1.987	0.829	2.051	0.787	2.116	0.746	2.182	0.705	2.250	0.665	2.318	0.625	2.387	0.586	2.456	0.548	2.526
55	0.979	1.891	0.940	1.945	0.902	2.002	0.863	2.059	0.825	2.117	0.786	2.176	0.748	2.237	0.711	2.298	0.674	2.359	0.637	2.421
60	1.037	1.865	1.001	1.914	0.965	1.964	0.929	2.015	0.893	2.067	0.857	2.120	0.822	2.173	0.786	2.227	0.751	2.283	0.716	2.338
65	1.087	1.845	1.053	1.889	1.020	1.934	0.986	1.980	0.953	2.027	0.919	2.075	0.886	2.123	0.852	2.172	0.819	2.221	0.786	2.272
70	1.131	1.831	1.099	1.870	1.068	1.911	1.037	1.953	1.005	1.995	0.974	2.038	0.943	2.082	0.911	2.127	0.880	2.172	0.849	2.217
75	1.170	1.819	1.141	1.856	1.111	1.893	1.082	1.931	1.052	1.970	1.023	2.009	0.993	2.049	0.964	2.090	0.934	2.131	0.905	2.172
80	1.205	1.810	1.777	1.844	1.150	1.878	1.122	1.913	1.094	1.949	1.066	1.984	1.039	2.022	1.011	2.057	0.983	2.097	0.955	2.135
85	1.236	1.803	1.210	1.834	1.184	1.866	1.158	1.898	1.132	1.931	1.106	1.965	1.080	1.999	1.053	2.033	1.027	2.068	1.000	2.104
90	1.264	1.798	1.240	1.827	1.215	1.856	1.191	1.886	1.166	1.917	1.141	1.948	1.116	1.979	1.091	2.012	1.066	2.044	1.041	2.077
95	1.290	1.793	1.267	1.821	1.244	1.848	1.221	1.876	1.197	1.905	1.174	1.934	1.150	1.963	1.126	1.993	1.102	2.023	1.079	2.054
100	1.314	1.790	1.292	1.816	1.270	1.841	1.248	1.868	1.225	1.895	1.203	1.922	1.181	1.949	1.158	1.977	1.136	2.006	1.113	2.034
150	1.473	1.783	1.458	1.799	1.444	1.814	1.429	1.830	1.414	1.847	1.400	1.863	1.385	1.880	1.370	1.897	1.355	1.913	1.340	1.931
200	1.561	1.791	1.550	1.801	1.539	1.813	1.528	1.824	1.518	1.836	1.507	1.847	1.495	1.860	1.484	1.871	1.474	1.883	1.462	1.896

[a] k' is the number of regressors excluding the intercept.