UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION 2007

COURSE TITLE : OPERATIONS RESEARCH I

COURSE CODE : ST 307

TIME ALLOWED : TWO (2) HOURS

INSTRUCTION : ANSWER ANY FOUR (4) QUESTIONS.
EACH QUESTION CARRIES 25 MARKS.

REQUIREMENTS : CALCULATOR

THIS PAPER MUST NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

Question 1

(a) Define the following terms in the context of linear programming:

(i) Augmented solution;
(ii) Basic solution;
(iii) Basic feasible solution.

(b) A farmer is raising pigs for market, and he wishes to determine the quantities of the available types of feed that should be given to each pig to meet certain nutritional requirements at a *minimum cost*. The number of units of each type of basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional ingredient	*Kilogram of corn*	*Kilogram of tankage*	*Kilogram of alfalfa*	*Minimum daily requirement*
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (¢)	21	18	15	

(i) Formulate the linear programming model for this problem.
(ii) Rewrite this model in an equivalent way to fit our standard form discussed in class.

Question 2

(a) Consider the linear programming problem

$$\text{Maximize } Z = 10X_1 + 20X_2 ,$$

subject to

$$\begin{aligned} -X_1 + 2X_2 &\le 10 \\ X_1 + X_2 &\le 8 \\ 5X_1 + 3X_2 &\le 30 \end{aligned}$$

and

$$X_1 \ge 0,\ X_2 \ge 0.$$

(i) Solve the problem graphically.
(ii) List all of the corner-point basic feasible solutions, and demonstrate that the optimal solution obtained in (a) above is, indeed, optimal. [You may use the property that the optimal solution is the one with a value of

Z better than the corresponding values for all of its adjacent corner point feasible solutions.]

(b) Consider the problem in (a) above.

(i) Construct the initial simplex tableau, introducing slack variables and so forth as needed for applying the simplex method.

(ii) Solve the problem by simplex method.

Question 3

Consider the *dual* problem:

Minimize $y_o = 4y_1 + 12y_2 + 18y_3$,
Subject to

$$\begin{aligned} y_1 \quad\quad + 3y_3 &\geq 3 \\ 2y_2 + 2y_3 &\geq 5 \end{aligned}$$

And

$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0.$$

(a) Find the *primal* problem.
(b) Find the optimal solution of the primal problem using the simplex method.
(c) Use the information in the final simplex tableau in (b) above to find the optimal solution for the *dual* problem.

(d) Verify the *dual theorem* using the optimal solutions in (b) and (c) above.

Question 4

Consider the assignment problem having the following cost table:

		Assignment 1	2	3	4
Assignee	*A*	9	6	5	6
	B	6	8	9	5
	C	8	7	6	8
	D	7	7	8	5

(a) Reformulate this problem as an equivalent *transportation problem* by constructing the appropriate cost and requirements table.

(b) Use the *northwest corner rule* to obtain an initial basic feasible solution for the problem formulated in (a).

(c) Use the transportation simplex method to obtain an optimal solution for the problem formulated in (a).

Question 5

A new type of camera film has been developed. It is packaged in sets of 5 sheets, each sheet providing an instantaneous snapshot. Since this is a new process, the manufacturer has attached an additional sheet to the package, so that the store may test 1 before the package of five is sold. In promoting the film, an offer to refund the entire purchase price of the film if 1 of the 5 is defective has been made. This refund must be paid by the camera store, and the selling price has been fixed at $1 if this guarantee is to be valid. The camera store may sell the film for 50 cents if the above guarantee is replaced by one that pays 10 cents for each defective sheet. The cost of the film to the camera store is 20 cents and is not returnable. The store may take three actions:

a_1: scrap the film,
a_2: sell the film for $1,
a_3: sell the film for 50 cents.

(a) If the six states of nature correspond to 0,1,2,3,4,5 defective sheets in the package, complete the following loss table: [5 marks]

a \ θ	0	1	2	3	4	5
a_1	0.20					
a_2	-0.80		0.20			
a_3	-0.30	-0.20		0.00		

Hint:

	$a=a_1$, $\theta=1$	$a=a_2$, $\theta=1$	$a=a_3$, $\theta=1$	$a=a_1$, $\theta=2$	$A=a_2$, $\theta=2$	$a=a_3$, $\theta=2$
Buy	0.20	0.20	0.20	0.20	0.20	0.20
Sell		-1.00	-0.50		-1.00	-0.50
Refund		1.00	0.10		1.00	0.20
Loss	0.20	0.20	-0.20	0.20	0.20	-0.10

etc.

(b) The store has accumulated the following information on sales of 60 such packages:

Quality of attached sheet	*Defectives in box*					
	0	1	2	3	4	5
Good	10	8	6	4	2	0
Bad	0	2	4	6	8	10
Total	10	10	10	10	10	10

These data indicated that each state of nature is equally likely, so that this prior can be assumed.

What is the Bayes' procedure (before testing the attached sheet) for package of film? [5 marks]

(c) (i) What is the optimal expected loss for a package of film if the attached sheet is tested? [5 marks]

(ii) What is the Bayes' action if the sheet is good? [10 marks]

				Coefficient of										
Iteration	Basic Variable	Eq. no.	Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Right side
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											

			Coefficient of											
Iteration	Basic Variable	Eq. no.	Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Right side
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											

Iteration	Basic Variable	Eq. no.	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Right side
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											

Iteration	*Destination* 1	2	3	4	5	6	*Supply*	u_i
1								
2								
3								
Source 4								
5								
6								
Demand								
v_j								

Iteration	*Destination* 1	2	3	4	5	6	*Supply*	u_i
Source 1								
2								
3								
4								
5								
6								
Demand								
v_j								

Iteration		Destination 1	2	3	4	5	6	Supply	u_i
Source	1								
	2								
	3								
	4								
	5								
	6								
Demand									
v_j									

Iteration	Destination 1	2	3	4	5	6	Supply	u_i
Source 1								
2								
3								
4								
5								
6								
Demand								
v_j								

Iteration	Destination 1	2	3	4	5	6	Supply	u_i
Source 1								
2								
3								
4								
5								
6								
Demand								
v_j								

Iteration	Destination 1	2	3	4	5	6	Supply	u_i
Source 1								
2								
3								
4								
5								
6								
Demand								
v_j								

Iteration	Destination 1	2	3	4	5	6	Supply	u_i
Source 1								
2								
3								
4								
5								
6								
Demand								
v_j								