UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2007

TITLE OF PAPER: TOPICS IN STATISTICS

COURSE CODE : ST 405

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : THIS PAPER HAS FIVE QUESTIONS.
ANSWER ANY FOUR (4) QUESTIONS.
EACH QUESTION CARRIES 15 MARKS.

REQUIREMENTS: Scientific calculator and statistical table

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QUESTION ONE

(a) Find the spectral density functions of the following moving average processes:

(i) $X_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2}$

(ii) $X_t = \varepsilon_t + 0.5\varepsilon_{t-1} + 0.3\varepsilon_{t-2}$

(b) Generate the forecasts for t=21,22 and 23 from the transfer function model

$$Y_t = 10 + \frac{(2+B)X_{t-1}}{(1-0.6B)} + \frac{\varepsilon_t}{(1-0.8B)}$$

$$Given: Y_{20} = 17; Y_{19} = 15; \varepsilon_{20} = 1; X_{20} = 4; X_{19} = 3; X_{18} = 2$$

QUESTION TWO

(a) The general linear process is given as

$X_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}\ (\psi_0 = 1)$, where (ε_t) is a white noise process with variance σ^2. Show that the canonical factorisation of the spectral density function $f(\omega) = \frac{1}{2\pi}\sum_{-\infty}^{\infty}\gamma_k \ell^{-i\omega k}$ is $f_c(\omega) = \frac{\sigma^2}{2\pi}\left|\psi\left(\ell^{-i\omega}\right)\right|^2$, where

$\psi(z) = \sum_{j=0}^{\infty}\psi_j Z^j$

(b) Define the spectral distribution function $F(\omega)$ of a process $\{X_t\}$. Show that $F^*(\omega) = F(\omega)\sigma_X^{-2}$, the normalised spectral distribution function has the same properties as the probability function.

QUESTION THREE

(a) Define periodogram and show that it is an asymptotically unbiased estimator of the spectral function $f(\omega) = \frac{1}{2\pi}\sum_{-\infty}^{\infty}\gamma_k \ell^{-i\omega k}$, $\omega \in [-\pi, \pi]$.

(b) Describe the Weiner's approach for obtaining an M- step ahead predictors for a zero mean stationary Process $\{X_t\}$.

QUESTION FOUR

(a) (i) What are the characteristics of the autocorrelation functions when we have: an alternating series; randomised series and stationary series.

(ii) Distinguish between strict stationary and m-th order stationary of a time series.

(b) (i) Briefly describe four major objectives of analysing time series.

(ii) Explain what is meant by deseasonalised series

QUESTION FIVE

(a) The table below shows calculations for an unrealistically short series z_t for which the Model (0, 1 ,1) of the form $W_t = \nabla Z_t = (1 - \theta B)a_t$ with $\theta = 0.5$ is being entertained with unknown starting value a_o. Complete the entries in the table.

t	0	1	2	3	4	5	6	7
z_t	40	42	47	47	52	51	57	59
$W_t = \nabla Z_t$				0		-1		2
a_t			$4{+}0.25\,a_o$.				$8{+}0.02\,a_o$.	$-2{-}0.01\,a_o$.

(b) Sixteen successive observations on a given time series are as follows: 1.6,0.8,1.2,0.5,0.9,1.1,1.1,0.6,1.5,0.8,0.9,1.2,0.5,1.3,0.8,1.2. Calculate the autocorrelation coefficient of lag one.