

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2007

TITLE OF PAPER : NON-PARAMETRIC METHODS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER QUESTION ONE AND ANY
3(THREE) QUESTIONS. ALL QUESTIONS
CARRY MARKS AS INDICATED WITHIN THE
PARENTHESIS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

ANSWER QUESTION ONE & ANY THREE QUESTIONS:

For all questions, clearly state the name of the test, the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, and the decision & conclusions.

QUESTION ONE.

[40 marks]

The average heights (in cm) of Year 1 students in UNISWA for the last 21 years were as follows:

163.5 164.2 163.7 162.9 163.7 163.2 164.5
 165.0 165.3 164.8 165.3 164.2 165.8 165.8
 165.0 164.2 164.2 164.8 165.8 164.8 164.5

- It is estimated that the yearly intakes in Year 1 of any undergraduate university are tall at least once in four years. The students are considered reasonably tall if their average height exceeds 165cm. Test the claim using the above data. Use $\alpha = 0.05$ and calculate the P-value.
- Also test whether the above averages indicate an increasing trend in height. Use $\alpha = 0.05$ and calculate the P-value.
- Comment on whether there exists any link between the results of these two tests. Explain.

QUESTION TWO.

[20 marks]

As a rural grocery store receives eggs from the neighbouring farmers it “candles” the eggs to detect any eggs that are not fresh. Eight crates of eggs, 144 eggs per crate, were candled with the following numbers of eggs rejected from each crate: 4, 0, 2, 0, 2, 0, 2, 0. Previous records have indicated that the number of rejected eggs per crate follows the Poisson distribution with mean 1.5. Use an appropriate test to test the hypothesis that these eight crates came from the same distribution function. Use $\alpha = 0.05$.

QUESTION THREE.

[20 marks]

Three different types of radios, manufactured by the same company, all carry 1-year guarantees. A record is kept of how many radios needed to be replaced, were repairable, or were not returned under warranty.

	Type		
	A	B	C
Replaced	12	3	6
Repaired	10	8	7
Not Returned	82	96	58

Does there seem to be a significant difference among the reliabilities of the different radio types? If so, which ones seem to be different?

QUESTION FOUR.

[20 marks]

The following table gives the scores of a group of fifteen students in mathematics and art:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Math	22	37	36	38	42	58	58	60	62	65	66	56	66	67	62
Art	53	68	42	49	51	65	51	71	55	74	68	64	67	73	65

Use Wilcoxon's signed-rank test to determine if the locations of the distributions of scores for these students differ significantly for the two subjects. Indicate the appropriate conclusion with $\alpha = 0.01$ and also calculate the P-value.

QUESTION FIVE.

[20 marks]

A political scientist wished to examine the relationship of the voter image of a conservative political candidate and the distance in km between the residences of the voter and the candidate. Each of the twelve voters rated the candidate on a scale of 1 to 20. The data are shown in the table below:

Voter	Rating	Distance
1	12	75
2	12	75
3	12	75
4	12	75
5	12	75
6	12	75
7	12	75
8	12	75
9	12	75
10	12	75
11	12	75
12	12	75

Do these data provide sufficient evidence to indicate a negative correlation between rating and distance? Use either Kendall's Tau or Spearman's Rho for the above test.

TABLE A1 Normal Distribution*

p	Selected values									
	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00										
0.01	-2.3263	-2.2904	-2.2571	-2.2262	-2.1973	-2.1701	-2.1444	-2.1201	-2.0969	-2.0749
0.02	-2.0537	-2.0335	-2.0141	-1.9954	-1.9774	-1.9600	-1.9431	-1.9268	-1.9110	-1.8957
0.03	-1.8808	-1.8663	-1.8522	-1.8384	-1.8250	-1.8119	-1.7991	-1.7866	-1.7744	-1.7624
0.04	-1.7507	-1.7392	-1.7279	-1.7169	-1.7060	-1.6954	-1.6849	-1.6747	-1.6646	-1.6546
0.05	-1.6449	-1.6352	-1.6258	-1.6164	-1.6072	-1.5982	-1.5893	-1.5805	-1.5718	-1.5632
0.06	-1.5548	-1.5464	-1.5382	-1.5301	-1.5220	-1.5141	-1.5063	-1.4985	-1.4909	-1.4833
0.07	-1.4758	-1.4684	-1.4611	-1.4538	-1.4466	-1.4395	-1.4325	-1.4255	-1.4187	-1.4118
0.08	-1.4051	-1.3984	-1.3917	-1.3852	-1.3787	-1.3722	-1.3658	-1.3595	-1.3532	-1.3469
0.09	-1.3408	-1.3346	-1.3285	-1.3225	-1.3165	-1.3106	-1.3047	-1.2988	-1.2930	-1.2873
0.10	-1.2816	-1.2759	-1.2702	-1.2646	-1.2591	-1.2536	-1.2481	-1.2426	-1.2372	-1.2319
0.11	-1.2265	-1.2212	-1.2160	-1.2107	-1.2055	-1.2004	-1.1952	-1.1901	-1.1850	-1.1800
0.12	-1.1750	-1.1700	-1.1650	-1.1601	-1.1552	-1.1503	-1.1455	-1.1407	-1.1359	-1.1311
0.13	-1.1264	-1.1217	-1.1170	-1.1123	-1.1077	-1.1031	-1.0985	-1.0939	-1.0893	-1.0848
0.14	-1.0803	-1.0758	-1.0714	-1.0669	-1.0625	-1.0581	-1.0537	-1.0494	-1.0450	-1.0407
0.15	-1.0364	-1.0322	-1.0279	-1.0237	-1.0194	-1.0152	-1.0110	-1.0069	-1.0027	-0.9986
0.16	-0.9945	-0.9904	-0.9863	-0.9822	-0.9782	-0.9741	-0.9701	-0.9661	-0.9621	-0.9581
0.17	-0.9542	-0.9502	-0.9463	-0.9424	-0.9385	-0.9346	-0.9307	-0.9269	-0.9230	-0.9192
0.18	-0.9154	-0.9116	-0.9078	-0.9040	-0.9002	-0.8965	-0.8927	-0.8890	-0.8853	-0.8816
0.19	-0.8779	-0.8742	-0.8705	-0.8669	-0.8633	-0.8596	-0.8560	-0.8524	-0.8488	-0.8452
0.20	-0.8416	-0.8381	-0.8345	-0.8310	-0.8274	-0.8239	-0.8204	-0.8169	-0.8134	-0.8099
0.21	-0.8064	-0.8030	-0.7995	-0.7961	-0.7926	-0.7892	-0.7858	-0.7824	-0.7790	-0.7756
0.22	-0.7722	-0.7688	-0.7655	-0.7621	-0.7588	-0.7554	-0.7521	-0.7488	-0.7454	-0.7421
0.23	-0.7388	-0.7356	-0.7323	-0.7290	-0.7257	-0.7225	-0.7192	-0.7160	-0.7128	-0.7095
0.24	-0.7063	-0.7031	-0.6999	-0.6967	-0.6935	-0.6903	-0.6871	-0.6840	-0.6808	-0.6776

TABLE A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.25	-0.6745	-0.6713	-0.6682	-0.6651	-0.6620	-0.6588	-0.6557	-0.6526	-0.6495	-0.6464
0.26	-0.6433	-0.6403	-0.6372	-0.6341	-0.6311	-0.6280	-0.6250	-0.6219	-0.6189	-0.6158
0.27	-0.6128	-0.6098	-0.6068	-0.6038	-0.6008	-0.5978	-0.5948	-0.5918	-0.5888	-0.5858
0.28	-0.5828	-0.5799	-0.5769	-0.5740	-0.5710	-0.5681	-0.5651	-0.5622	-0.5592	-0.5563
0.29	-0.5534	-0.5505	-0.5476	-0.5446	-0.5417	-0.5388	-0.5359	-0.5330	-0.5302	-0.5273
0.30	-0.5244	-0.5215	-0.5187	-0.5158	-0.5129	-0.5101	-0.5072	-0.5044	-0.5015	-0.4987
0.31	-0.4959	-0.4930	-0.4902	-0.4874	-0.4845	-0.4817	-0.4789	-0.4761	-0.4733	-0.4705
0.32	-0.4677	-0.4649	-0.4621	-0.4593	-0.4565	-0.4538	-0.4510	-0.4482	-0.4454	-0.4427
0.33	-0.4399	-0.4372	-0.4344	-0.4316	-0.4289	-0.4261	-0.4234	-0.4207	-0.4179	-0.4152
0.34	-0.4125	-0.4097	-0.4070	-0.4043	-0.4016	-0.3989	-0.3961	-0.3934	-0.3907	-0.3880
0.35	-0.3853	-0.3826	-0.3799	-0.3772	-0.3745	-0.3719	-0.3692	-0.3665	-0.3638	-0.3611
0.36	-0.3585	-0.3558	-0.3531	-0.3505	-0.3478	-0.3451	-0.3425	-0.3398	-0.3372	-0.3345
0.37	-0.3319	-0.3292	-0.3266	-0.3239	-0.3213	-0.3186	-0.3160	-0.3134	-0.3107	-0.3081
0.38	-0.3055	-0.3029	-0.3002	-0.2976	-0.2950	-0.2924	-0.2898	-0.2871	-0.2845	-0.2819
0.39	-0.2793	-0.2767	-0.2741	-0.2715	-0.2689	-0.2663	-0.2637	-0.2611	-0.2585	-0.2559
0.40	-0.2533	-0.2508	-0.2482	-0.2456	-0.2430	-0.2404	-0.2378	-0.2353	-0.2327	-0.2301
0.41	-0.2275	-0.2250	-0.2224	-0.2198	-0.2173	-0.2147	-0.2121	-0.2096	-0.2070	-0.2045
0.42	-0.2019	-0.1993	-0.1968	-0.1942	-0.1917	-0.1891	-0.1866	-0.1840	-0.1815	-0.1789
0.43	-0.1764	-0.1738	-0.1713	-0.1687	-0.1662	-0.1637	-0.1611	-0.1586	-0.1560	-0.1535
0.44	-0.1510	-0.1484	-0.1459	-0.1434	-0.1408	-0.1383	-0.1358	-0.1332	-0.1307	-0.1282
0.45	-0.1257	-0.1231	-0.1206	-0.1181	-0.1156	-0.1130	-0.1105	-0.1080	-0.1055	-0.1030
0.46	-0.1004	-0.0979	-0.0954	-0.0929	-0.0904	-0.0878	-0.0853	-0.0828	-0.0803	-0.0778
0.47	-0.0753	-0.0728	-0.0702	-0.0677	-0.0652	-0.0627	-0.0602	-0.0577	-0.0552	-0.0527
0.48	-0.0502	-0.0476	-0.0451	-0.0426	-0.0401	-0.0376	-0.0351	-0.0326	-0.0301	-0.0276
0.49	-0.0251	-0.0226	-0.0201	-0.0175	-0.0150	-0.0125	-0.0100	-0.0075	-0.0050	-0.0025
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

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*The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

TABLE A3 (Continued)

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
1	0	0.5000	0.4500	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500
1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	0.2500	0.2025	0.1600	0.1225	0.0900	0.0625	0.0400	0.0225	0.0100	0.0025
2	1	0.7500	0.6975	0.6400	0.5775	0.5100	0.4375	0.3600	0.2775	0.1900	0.0975
2	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.1250	0.0911	0.0640	0.0429	0.0270	0.0156	0.0080	0.0034	0.0010	0.0001
3	1	0.5000	0.4252	0.3520	0.2818	0.2160	0.1562	0.1040	0.0608	0.0280	0.0072
3	2	0.8750	0.8336	0.7840	0.7254	0.6570	0.5781	0.4880	0.3859	0.2710	0.1426
3	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.0625	0.0410	0.0256	0.0150	0.0081	0.0039	0.0016	0.0005	0.0001	0.0000
4	1	0.3125	0.2415	0.1792	0.1265	0.0837	0.0508	0.0272	0.0120	0.0037	0.0005
4	2	0.6875	0.6090	0.5248	0.4370	0.3483	0.2617	0.1808	0.1095	0.0523	0.0140
4	3	0.9375	0.9085	0.8704	0.8215	0.7599	0.6836	0.5904	0.4780	0.3439	0.1855
4	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.0312	0.0185	0.0102	0.0053	0.0024	0.0010	0.0003	0.0001	0.0000	0.0000
5	1	0.1875	0.1312	0.0870	0.0540	0.0308	0.0156	0.0067	0.0022	0.0005	0.0000
5	2	0.5000	0.4069	0.3174	0.2352	0.1631	0.1035	0.0579	0.0266	0.0086	0.0012
5	3	0.8125	0.7438	0.6630	0.5716	0.4718	0.3672	0.2627	0.1648	0.0815	0.0226
5	4	0.9688	0.9497	0.9222	0.8840	0.8319	0.7627	0.6723	0.5563	0.4095	0.2262
5	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	0.0156	0.0083	0.0041	0.0018	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
6	1	0.1094	0.0692	0.0410	0.0223	0.0109	0.0046	0.0016	0.0004	0.0001	0.0000
6	2	0.3438	0.2553	0.1792	0.1174	0.0705	0.0376	0.0170	0.0059	0.0013	0.0001
6	3	0.6562	0.5585	0.4557	0.3529	0.2557	0.1694	0.0989	0.0473	0.0158	0.0022
6	4	0.8906	0.8364	0.7667	0.6809	0.5789	0.4661	0.3446	0.2235	0.1143	0.0328
6	5	0.9844	0.9723	0.9533	0.9246	0.8824	0.8220	0.7379	0.6229	0.4686	0.2649
6	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	0.0078	0.0037	0.0016	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
7	1	0.0625	0.0357	0.0188	0.0090	0.0038	0.0013	0.0004	0.0001	0.0000	0.0000
7	2	0.2266	0.1529	0.0963	0.0556	0.0288	0.0129	0.0047	0.0012	0.0002	0.0000
7	3	0.5000	0.3917	0.2898	0.1998	0.1260	0.0706	0.0333	0.0121	0.0027	0.0002
7	4	0.7734	0.6836	0.5801	0.4677	0.3529	0.2436	0.1480	0.0738	0.0257	0.0038
7	5	0.9375	0.8976	0.8414	0.7662	0.6706	0.5551	0.4233	0.2834	0.1497	0.0444
7	6	0.9922	0.9848	0.9720	0.9510	0.9176	0.8665	0.7903	0.6794	0.5217	0.3017
7	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

n	y	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084
8	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632
8	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201
8	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770
8	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396
8	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115
8	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819
8	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9993	0.9983
8	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046
9	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385
9	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495
9	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614
9	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214
9	5	1.0000	0.9999	0.9994	0.9900	0.9747	0.9464	0.9006	0.8342	0.7502
9	6	1.0000	1.0000	1.0000	0.9997	0.9969	0.9907	0.9888	0.9750	0.9502
9	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909
9	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9992
9	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025
10	1	0.9139	0.7361	0.5443	0.3758	0.2443	0.1493	0.0860	0.0464	0.0233
10	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996
10	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660
10	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044
10	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384
10	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980
10	7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9996	0.9984	0.9952	0.9877
10	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9983
10	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995
10	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014
11	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139
11	2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652
11	3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911
11	4	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971
11	5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331
11	6	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262
11	7	1.0000	1.0000	1.0000	0.9998	0.9998	0.9957	0.9878	0.9707	0.9390
11	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852
11	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9978
11	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
11	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000
	11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002
	14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020
	15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	0.9998	0.9992	0.9997	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* Y has the binomial distribution with parameters n and p. The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$ for p ranging from 0.05 to 0.95. For n larger than 20, the rth quantile y_r of a binomial random variable may be approximated using $y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the rth quantile of a standard normal random variable, obtained from Table A1.

TABLE A4 Exact Confidence Intervals for the Binomial Parameter p

n	Y	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
1	0	0.000	0.950	0.000	0.975	0.000	0.995
	1	0.050	1.000	0.025	1.000	0.005	1.000
2	0	0.000	0.776	0.000	0.842	0.000	0.929
	1	0.025	0.975	0.013	0.987	0.003	0.997
	2	0.224	1.000	0.158	1.000	0.071	1.000
3	0	0.000	0.632	0.000	0.708	0.000	0.829
	1	0.017	0.865	0.008	0.906	0.002	0.959
	2	0.135	0.983	0.094	0.992	0.041	0.998
	3	0.368	1.000	0.292	1.000	0.171	1.000
4	0	0.000	0.527	0.000	0.602	0.000	0.734
	1	0.013	0.751	0.006	0.806	0.001	0.889
	2	0.098	0.902	0.068	0.932	0.029	0.971
	3	0.249	0.987	0.194	0.994	0.111	0.999
	4	0.473	1.000	0.398	1.000	0.266	1.000
5	0	0.000	0.451	0.000	0.522	0.000	0.653
	1	0.010	0.657	0.005	0.716	0.001	0.815
	2	0.076	0.811	0.053	0.853	0.023	0.917
	3	0.189	0.924	0.147	0.947	0.083	0.977
	4	0.343	0.990	0.284	0.995	0.185	0.999
	5	0.549	1.000	0.478	1.000	0.347	1.000
6	0	0.000	0.393	0.000	0.459	0.000	0.586
	1	0.009	0.582	0.004	0.641	0.001	0.746
	2	0.063	0.729	0.043	0.777	0.019	0.856
	3	0.153	0.847	0.118	0.882	0.066	0.934
	4	0.271	0.937	0.223	0.957	0.144	0.981
	5	0.418	0.991	0.359	0.996	0.254	0.999
	6	0.607	1.000	0.541	1.000	0.414	1.000
7	0	0.000	0.348	0.000	0.410	0.000	0.531
	1	0.007	0.521	0.004	0.579	0.001	0.685
	2	0.053	0.659	0.037	0.710	0.016	0.797
	3	0.129	0.775	0.099	0.816	0.055	0.882
	4	0.225	0.871	0.184	0.901	0.118	0.945
	5	0.341	0.947	0.290	0.963	0.203	0.984
	6	0.479	0.993	0.421	0.996	0.315	0.999
	7	0.652	1.000	0.590	1.000	0.469	1.000

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes*

Sample Sizes	W _{0.10}	W _{0.05}	W _{0.01}
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

Source: Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.

*The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A10 Quantiles of Spearman's ρ^r

n	$\rho = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000	0.9000	0.9000	0.9429	0.9643
5	0.7000	0.8000	0.8286	0.8857	0.8929	0.9286
6	0.6000	0.7714	0.7500	0.8571	0.8571	0.9286
7	0.5357	0.6786	0.7143	0.8095	0.8095	0.9286
8	0.5000	0.6190	0.6833	0.9000	0.8167	0.9000
9	0.4667	0.5833	0.6833	0.7667	0.7667	0.8667
10	0.4424	0.5515	0.6364	0.7333	0.7333	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p \approx \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

Source: Adapted from Glasser and Winer (1961), with corrections, with permission from the Biometric Trustees.

The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$\rho = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.5000)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.4555)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4110)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3287)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A11 (Continued)

n	p = 0.900	0.950	0.975	0.990	0.995
38	103 (0.1465)	131 (0.1863)	155 (0.2205)	183 (0.2603)	203 (0.2888)
39	107 (0.1444)	137 (0.1849)	161 (0.2173)	191 (0.2578)	211 (0.2848)
40	110 (0.1372)	142 (0.1821)	168 (0.2154)	198 (0.2538)	220 (0.2821)
41	114 (0.1390)	146 (0.1780)	174 (0.2122)	206 (0.2512)	228 (0.2780)
42	119 (0.1382)	151 (0.1754)	181 (0.2102)	213 (0.2474)	235 (0.2729)
43	123 (0.1362)	157 (0.1739)	187 (0.2071)	221 (0.2447)	245 (0.2713)
44	128 (0.1353)	162 (0.1712)	194 (0.2051)	228 (0.2410)	252 (0.2664)
45	132 (0.1333)	168 (0.1697)	200 (0.2020)	236 (0.2383)	262 (0.2646)
46	135 (0.1304)	173 (0.1671)	207 (0.2000)	245 (0.2367)	271 (0.2618)
47	141 (0.1304)	179 (0.1656)	213 (0.1970)	253 (0.2340)	279 (0.2581)
48	144 (0.1277)	186 (0.1649)	220 (0.1950)	260 (0.2305)	288 (0.2553)
49	150 (0.1276)	190 (0.1616)	228 (0.1939)	268 (0.2279)	296 (0.2517)
50	153 (0.1249)	197 (0.1608)	233 (0.1902)	277 (0.2261)	305 (0.2490)
51	159 (0.1247)	203 (0.1592)	241 (0.1890)	285 (0.2235)	315 (0.2471)
52	162 (0.1222)	208 (0.1569)	248 (0.1870)	294 (0.2217)	324 (0.2443)
53	168 (0.1219)	214 (0.1553)	256 (0.1858)	302 (0.2192)	334 (0.2424)
54	173 (0.1209)	221 (0.1544)	263 (0.1838)	311 (0.2173)	343 (0.2397)
55	177 (0.1192)	227 (0.1529)	269 (0.1811)	319 (0.2148)	353 (0.2377)
56	182 (0.1182)	232 (0.1506)	276 (0.1792)	328 (0.2130)	362 (0.2351)
57	186 (0.1165)	240 (0.1504)	284 (0.1779)	336 (0.2105)	372 (0.2331)
58	191 (0.1155)	245 (0.1482)	291 (0.1760)	345 (0.2087)	381 (0.2305)
59	197 (0.1151)	251 (0.1467)	299 (0.1748)	355 (0.2075)	391 (0.2285)
60	202 (0.1141)	258 (0.1458)	306 (0.1729)	364 (0.2056)	402 (0.2271)

For n greater than 60, approximate quantiles of T may be obtained from

$$w_p \approx z_p \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

where z_p is from the standard normal distribution given by Table A1. Approximate quantiles of τ may be obtained from

$$w_p \approx z_p \frac{\sqrt{2(n+5)}}{3\sqrt{n(n-1)}}$$

Critical regions correspond to values of T greater than (or less than) but not including the appropriate quantile. Note that the median of T is 0. Quantiles for τ are obtained by dividing the quantiles of T by $n(n-1)^{1/2}$.

Source: Adapted from Table 1, Best (1974), with permission from the author.

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

n	W _{sig}										n(n+1) 2
	0	1	2	3	4	5	6	7	8	9	
4	0	0	0	0	1	3	4	5	6	7	10
5	0	0	0	1	3	4	6	8	9	10	15
6	0	0	1	3	4	6	9	11	12	14	21
7	0	1	3	4	6	9	12	14	16	18	28
8	1	2	4	6	9	12	15	18	20	22	36
9	2	4	6	9	12	15	19	22	25	27	45
10	4	6	9	12	15	19	22	27	30	33	55
11	6	8	11	14	18	22	27	32	36	39	66
12	8	10	14	18	22	27	33	38	42	45	78
13	10	13	18	22	27	33	38	44	48	52	91
14	13	16	22	26	32	39	44	51	55	60	105
15	16	20	26	31	37	45	51	58	63	68	120
16	20	24	30	36	43	51	58	65	71	76	136
17	24	28	35	42	49	58	65	73	80	85	153
18	28	33	41	48	56	66	74	82	89	95	171
19	33	38	47	54	63	74	83	91	98	105	190
20	38	44	53	61	70	81	91	100	108	115	210
21	44	50	59	68	78	88	99	108	119	126	231
22	49	56	67	76	87	97	107	117	128	138	253
23	55	63	74	84	95	105	115	125	135	145	276
24	62	70	82	92	102	112	122	132	141	150	300
25	69	77	90	101	114	125	135	145	155	165	325
26	76	85	99	111	125	142	155	165	175	185	351
27	84	94	108	120	135	154	167	178	189	199	378
28	92	102	117	131	146	166	180	192	203	213	406
29	101	111	127	141	158	178	193	206	217	227	435
30	110	121	138	152	170	191	207	220	232	242	465
31	119	131	148	164	182	205	221	235	248	258	496
32	129	141	160	176	195	219	236	250	264	274	528
33	139	152	171	188	208	233	251	266	280	290	561
34	149	163	183	201	222	248	266	282	297	307	595
35	160	175	196	214	236	263	283	299	315	325	630
36	172	187	209	228	251	279	299	317	333	343	666
37	184	199	222	242	266	295	316	335	351	361	703
38	196	212	236	257	282	312	334	353	370	380	741
39	208	225	250	272	298	329	352	372	390	400	780
40	221	239	265	287	314	347	371	391	410	420	820
41	235	253	280	303	331	365	390	411	430	440	861
42	248	267	295	320	349	384	409	431	451	460	903

TABLE A12 (Continued)

	$n(n+1)/2$									
	$W_{0.05}$	$W_{0.10}$	$W_{0.15}$	$W_{0.20}$	$W_{0.25}$	$W_{0.30}$	$W_{0.35}$	$W_{0.40}$	$W_{0.45}$	$W_{0.50}$
43	263	282	311	337	366	403	429	452	473	496
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n+1)/4] + z_p \sqrt{n(n+1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

Source: Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

*The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^+ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T^+ < w_p) \leq p$ and $P(T^+ > w_p) \leq 1-p$ if H_0 is true. Critical regions correspond to values of T^+ less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic*

n	One-Sided Test					Two-Sided Test				
	$p = 0.90$	$p = 0.95$	$p = 0.975$	$p = 0.99$	$p = 0.995$	$p = 0.80$	$p = 0.90$	$p = 0.95$	$p = 0.98$	$p = 0.99$
1	0.900	0.950	0.975	0.990	0.995	0.900	0.950	0.975	0.990	0.995
2	0.684	0.776	0.842	0.900	0.929	0.776	0.842	0.900	0.929	0.950
3	0.565	0.636	0.708	0.785	0.829	0.565	0.624	0.689	0.734	0.785
4	0.447	0.509	0.563	0.627	0.669	0.447	0.509	0.563	0.627	0.669
5	0.410	0.468	0.519	0.577	0.617	0.410	0.468	0.519	0.577	0.617
6	0.381	0.436	0.483	0.538	0.576	0.381	0.436	0.483	0.538	0.576
7	0.358	0.410	0.454	0.507	0.542	0.358	0.410	0.454	0.507	0.542
8	0.339	0.387	0.430	0.480	0.513	0.339	0.387	0.430	0.480	0.513
9	0.323	0.369	0.409	0.457	0.489	0.323	0.369	0.409	0.457	0.489
10	0.308	0.352	0.391	0.437	0.468	0.308	0.352	0.391	0.437	0.468
11	0.296	0.338	0.375	0.419	0.449	0.296	0.338	0.375	0.419	0.449
12	0.285	0.325	0.361	0.404	0.432	0.285	0.325	0.361	0.404	0.432
13	0.275	0.314	0.349	0.390	0.418	0.275	0.314	0.349	0.390	0.418
14	0.266	0.304	0.338	0.377	0.404	0.266	0.304	0.338	0.377	0.404
15	0.258	0.295	0.327	0.366	0.392	0.258	0.295	0.327	0.366	0.392
16	0.250	0.286	0.318	0.355	0.381	0.250	0.286	0.318	0.355	0.381
17	0.244	0.279	0.309	0.346	0.371	0.244	0.279	0.309	0.346	0.371
18	0.237	0.271	0.301	0.337	0.361	0.237	0.271	0.301	0.337	0.361
19	0.232	0.265	0.294	0.329	0.352	0.232	0.265	0.294	0.329	0.352
20										

Approximation for $n > 40$

n	$\sqrt{1.07}/\sqrt{n}$	$\sqrt{1.22}/\sqrt{n}$	$\sqrt{1.36}/\sqrt{n}$	$\sqrt{1.52}/\sqrt{n}$	$\sqrt{1.63}/\sqrt{n}$
40	0.165	0.189	0.210	0.235	0.252
50	0.147	0.167	0.185	0.207	0.222
60	0.130	0.147	0.163	0.182	0.195
70	0.118	0.133	0.148	0.165	0.177
80	0.110	0.124	0.138	0.153	0.164
90	0.105	0.118	0.131	0.146	0.156
100	0.100	0.112	0.125	0.140	0.149

Source: Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

*The entries in this table are selected quantiles w_p of the Kolmogorov test statistic T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1-\alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-sided test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.

Table 2. Table of e^{-x}

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.00	1.000000	2.60	.074274	5.10	.006097	7.60	.000501
0.10	.904837	2.70	.067206	5.20	.005517	7.70	.000453
0.20	.818731	2.80	.060810	5.30	.004992	7.80	.000410
0.30	.740818	2.90	.055023	5.40	.004517	7.90	.000371
0.40	.670320	3.00	.049787	5.50	.004087	8.00	.000336
0.50	.606531	3.10	.045049	5.60	.003698	8.10	.000304
0.60	.548812	3.20	.040762	5.70	.003346	8.20	.000275
0.70	.496585	3.30	.036883	5.80	.003028	8.30	.000249
0.80	.449329	3.40	.033373	5.90	.002739	8.40	.000225
0.90	.406570	3.50	.030197	6.00	.002479	8.50	.000204
1.00	.367879	3.60	.027324	6.10	.002243	8.60	.000184
1.10	.332871	3.70	.024724	6.20	.002029	8.70	.000167
1.20	.301194	3.80	.022371	6.30	.001836	8.80	.000151
1.30	.272532	3.90	.020242	6.40	.001661	8.90	.000136
1.40	.246597	4.00	.018316	6.50	.001503	9.00	.000123
1.50	.223130	4.10	.016573	6.60	.001360	9.10	.000112
1.60	.201897	4.20	.014996	6.70	.001231	9.20	.000101
1.70	.182684	4.30	.013569	6.80	.001114	9.30	.000091
1.80	.165299	4.40	.012277	6.90	.001008	9.40	.000083
1.90	.149569	4.50	.011109	7.00	.000912	9.50	.000075
2.00	.135335	4.60	.010052	7.10	.000825	9.60	.000068
2.10	.122456	4.70	.009095	7.20	.000747	9.70	.000061
2.20	.110803	4.80	.008230	7.30	.000676	9.80	.000056
2.30	.100259	4.90	.007447	7.40	.000611	9.90	.000050
2.40	.090718	5.00	.006738	7.50	.000553	10.00	.000045
2.50	.082085						

Table 3. Poisson Probabilities

$$P(Y \leq a) = \sum_{y=0}^a \frac{e^{-\lambda} \lambda^y}{y!}$$

λ	0	1	2	3	4	5	6	7	8	9
0.02	0.980	1.000	1.000							
0.04	0.961	0.999	1.000							
0.06	0.942	0.998	1.000							
0.08	0.923	0.997	1.000							
0.10	0.905	0.995	1.000							
0.15	0.861	0.990	0.999	1.000						
0.20	0.819	0.982	0.999	1.000						
0.25	0.779	0.974	0.998	1.000						
0.30	0.741	0.963	0.996	1.000						
0.35	0.705	0.951	0.994	1.000	1.000					
0.40	0.670	0.938	0.992	0.999	1.000	1.000				
0.45	0.638	0.925	0.989	0.999	1.000	1.000	1.000			
0.50	0.607	0.910	0.986	0.998	1.000	1.000	1.000	1.000		
0.55	0.577	0.894	0.982	0.988	1.000	1.000	1.000	1.000	1.000	
0.60	0.549	0.878	0.977	0.997	1.000	1.000	1.000	1.000	1.000	1.000
0.65	0.522	0.861	0.972	0.996	0.999	1.000	1.000	1.000	1.000	1.000
0.70	0.497	0.844	0.966	0.994	0.999	1.000	1.000	1.000	1.000	1.000
0.75	0.472	0.827	0.959	0.993	0.999	1.000	1.000	1.000	1.000	1.000
0.80	0.449	0.809	0.953	0.991	0.999	1.000	1.000	1.000	1.000	1.000
0.85	0.427	0.791	0.945	0.989	0.998	1.000	1.000	1.000	1.000	1.000
0.90	0.407	0.772	0.937	0.987	0.998	1.000	1.000	1.000	1.000	1.000
0.95	0.387	0.754	0.929	0.981	0.997	1.000	1.000	1.000	1.000	1.000
1.00	0.368	0.736	0.920	0.981	0.996	0.999	1.000	1.000	1.000	1.000
1.1	0.333	0.699	0.900	0.974	0.995	0.999	1.000	1.000	1.000	1.000
1.2	0.301	0.663	0.879	0.966	0.992	0.998	1.000	1.000	1.000	1.000
1.3	0.273	0.627	0.857	0.957	0.989	0.998	1.000	1.000	1.000	1.000
1.4	0.247	0.592	0.833	0.946	0.986	0.997	0.999	1.000	1.000	1.000
1.5	0.223	0.558	0.809	0.934	0.981	0.996	0.999	1.000	1.000	1.000
1.6	0.202	0.525	0.783	0.921	0.976	0.994	0.999	1.000	1.000	1.000
1.7	0.183	0.493	0.757	0.907	0.970	0.992	0.998	1.000	1.000	1.000
1.8	0.165	0.463	0.731	0.891	0.964	0.990	0.997	0.999	1.000	1.000
1.9	0.150	0.434	0.704	0.875	0.956	0.987	0.997	0.999	1.000	1.000
2.0	0.135	0.406	0.677	0.857	0.947	0.983	0.995	0.999	1.000	1.000

