**UNIVERSITY OF SWAZILAND**

**SUPPLEMENTARY EXAMINATION PAPER 2007**

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|---|---|---|
| **TITLE OF PAPER** | **:** | **MULTIVARIATE ANALYSIS** |
| **COURSE CODE** | **:** | **ST410** |
| **TIME ALLOWED** | **:** | **2 (TWO) HOURS** |
| **REQUIRMENTS** | **:** | **STATISTICAL TABLES AND CALCULATOR** |
| **INSTRUCTIONS** | **:** | **ANSWER ANY 4 (FOUR) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.** |

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**QUESTION ONE.** [7 + 8 + 10 marks]

1.1 Define Principal component analysis and discuss its all important properties.

1.2 State and discuss the four steps of the procedure for a principal component analysis.

1.3 Consider the following table:

| Component | Eigenvalue | Eigenvectors | | | | |
|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
| 1 | 2.616 | 0.452 | 0.462 | 0.451 | 0.471 | 0.398 |
| 2 | 1.532 | – 0.051 | 0.300 | 0.325 | 0.185 | –0.377 |
| 3 | 0.386 | 0.691 | 0.341 | –0.455 | –0.411 | –0.179 |
| 4 | 0.302 | – 0.420 | 0.548 | –0.606 | 0.388 | 0.069 |
| 5 | 0.165 | 0.374 | –0.530 | –0.343 | 0.652 | –0.192 |

a. How many components will you choose? Explain why.

b. List those selected components and interpret those in terms of original variables, $X_i$'s.

**QUESTION TWO.** [3 + 3 + 3 + 4 +12 marks]

2.1 Discuss the purpose of the discriminant function analysis.

2.2 State the criteria for selecting canonical discriminant functions and also state the number of functions you can select.

2.3 The following table shows the eigenvalues and corresponding eigenvectors of $\mathbf{W}^{-1}\mathbf{B}$:

| Component | Eigenvalue | Eigenvectors | | | |
|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
| 1 | 0.437 | –0.0107 | 0.0040 | 0.0119 | –0.0068 |
| 2 | 0.035 | 0.0031 | 0.0168 | –0.0046 | –0.0022 |
| 3 | 0.015 | –0.0068 | 0.0010 | 0.0000 | 0.0247 |
| 4 | 0.002 | 0.0126 | –0.0001 | 0.0112 | 0.0054 |

a. How many groups and variables were considered in this problem?

b. List all the canonical discriminant functions.

c. Assuming that the $i^{th}$ sample size, $n_i = 30$ for all $i = 1, 2, 3, 4, 5$; test whether each of these functions varies significantly from group to group.

**QUESTION THREE.** [8 + 10 + 5 + 2 marks]

3.1 Define factor analysis and compare its properties with those of principal component analysis.

3.2 Discuss the procedures of principal component factor analysis to determine the final factors.

3.3 Using the table given in Question 1.3, write the unrotated factor model.

3.4 Compute the communalities.

**QUESTION FOUR.** [15 + 10 marks]

4.1 Suppose we have two groups, with 10 subjects in each group. The means for the two variables ($X_1$ and $X_2$) measures in group A are 10 and 7.5, while the means in group B are 9 and 9.5. The respective pooled sample variances are 9 and 4 for variables $X_1$ and $X_2$, while the pooled correlation is 0.7. Perform Hotellings' $\mathbf{T}^2$ test and also perform univariate $t$ tests for each of these two variables at 5% level of significance. Compare the results.

4.2 Suppose we have three variables in each of the 3 groups, with 10 subjects per group. Let the sum of squares matrices are as follows:

$$\mathbf{B} = \begin{bmatrix} 1.68 & 1.38 & -1.26 \\ 1.38 & 1.14 & -1.08 \\ -1.26 & -1.08 & 1.26 \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} 1.24 & 0.06 & 0.56 \\ 0.06 & 1.08 & 0.18 \\ 0.56 & 0.18 & 2.74 \end{bmatrix}, \quad \& \quad \mathbf{T} = \begin{bmatrix} 2.92 & 1.44 & -0.70 \\ 1.44 & 2.22 & -0.90 \\ -0.70 & -0.90 & 4.00 \end{bmatrix}$$

Compute Wilk's $\Lambda$ statistics and use $\chi^2$ approximation to test the equality of population mean vectors.

**QUESTION FIVE.** [3 + 3 + 4 + 4 + 3 + 8 marks]

The following tables are part of the complete output running SPSS for a set of multivariate variables; not necessarily from the same set of variables. Tables 1-6 are obtained running Factor Analysis and Tables 7-9 are from Discriminant Function Analysis:

**Table 1:**

**Correlation Matrix**

| | | X1 | X2 | X3 | X4 | X5 | X6 | X7 |
|---|---|---|---|---|---|---|---|---|
| Correlatio | X1 | 1.000 | .036 | -.671 | -.400 | -.538 | -.645 | -.764 |
| | X2 | .036 | 1.000 | .445 | .405 | -.026 | -.495 | -.221 |
| | X3 | -.671 | .445 | 1.000 | .385 | .494 | .080 | .200 |
| | X4 | -.400 | .405 | .385 | 1.000 | .060 | .199 | .185 |
| | X5 | -.538 | -.026 | .494 | .060 | 1.000 | .271 | .210 |
| | X6 | -.645 | -.495 | .080 | .199 | .271 | 1.000 | .424 |
| | X7 | -.764 | -.221 | .200 | .185 | .210 | .424 | 1.000 |

**Table 2:**

| Component | Eigenvalues |
|---|---|
| 1 | 2.965 |
| 2 | 1.850 |
| 3 | 0.971 |
| 4 | 0.643 |
| 5 | 0.359 |
| 6 | 0.211 |
| 7 | 0.000 |

**Table 3:**

**Component Matrix[a]**

| | Component | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| X1 | -.986 | 5.070E-02 | -2.47E-02 | .113 | .102 | -2.99E-02 |
| X2 | -1.24E-02 | .936 | 7.822E-02 | -.109 | 3.541E-04 | .325 |
| X3 | .683 | .566 | -.258 | -3.93E-02 | -.333 | -.186 |
| X4 | .483 | .481 | .590 | .327 | .241 | -.153 |
| X5 | .621 | 4.503E-02 | -.677 | .186 | .344 | 4.014E-02 |
| X6 | .641 | -.562 | .179 | .402 | -.186 | .212 |
| X7 | .705 | -.319 | .246 | -.560 | .166 | 2.041E-02 |

Extraction Method: Principal Component Analysis.

a. 6 components extracted.

**Table 4:**

**Component Matrix[a]**

| | Component | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| X1 | -.986 | 5.070E-02 | -2.47E-02 |
| X2 | -1.24E-02 | .936 | 7.822E-02 |
| X3 | .683 | .566 | -.258 |
| X4 | .483 | .481 | .590 |
| X5 | .621 | 4.503E-02 | -.677 |
| X6 | .641 | -.562 | .179 |
| X7 | .705 | -.319 | .246 |

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

**Table 5:**

**Rotated Component Matrix[a]**

| | Component | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| X1 | -.764 | -.550 | -.301 |
| X2 | -.515 | .195 | .761 |
| X3 | 8.552E-02 | .756 | .523 |
| X4 | .295 | -2.72E-02 | .851 |
| X5 | .185 | .892 | -.127 |
| X6 | .858 | 8.626E-02 | -.124 |
| X7 | .790 | .141 | .120 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 6 iterations.

**Table 6:**

**Component Score Coefficient Matrix**

| | Component | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| X1 | -.270 | -.170 | -.103 |
| X2 | -.263 | .078 | .433 |
| X3 | -.102 | .422 | .169 |
| X4 | .192 | -.291 | .584 |
| X5 | -.113 | .658 | -.290 |
| X6 | .400 | -.097 | -.063 |
| X7 | .365 | -.099 | .083 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

**Table 7:**

**Wilks' Lambda**

| Test of Function(s) | Wilks' Lambda | Chi-square | df | Sig. |
|---|---|---|---|---|
| 1 through 2 | .147 | 39.244 | 12 | .000 |
| 2 | .656 | 8.655 | 5 | .124 |

**Table 8:**

**Standardized Canonical Discriminant Function Coefficients**

| | Function | |
|---|---|---|
| | 1 | 2 |
| X1 | .268 | 1.193 |
| X2 | .202 | -1.065 |
| X3 | .412 | .938 |
| X4 | .573 | .478 |
| X5 | .129 | .097 |
| X6 | -1.040 | -.249 |

**Table 9:**

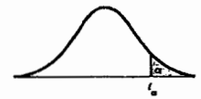
**Canonical Discriminant Function Coefficients**

| | Function | |
|---|---|---|
| | 1 | 2 |
| X1 | .019 | .083 |
| X2 | .305 | -1.603 |
| X3 | .059 | .135 |
| X4 | 1.512 | 1.263 |
| X5 | .076 | .057 |
| X6 | -.268 | -.064 |
| (Constant) | .217 | -3.732 |

Unstandardized coefficients

a. Examine Table 1 and discuss the suitability of principal component analysis. How many principal components will you have?
b. How many factors will you choose if you wish to use factor analysis method? Explain your answer.
c. How many factors will you get in your factor model from Table 3? List the last two equations of your model and compute their communalities.
d. Suppose the same data were analyzed using with a restriction on the number of factors. How many factors were chosen in Table 4? List the first two equations of your model and compute their communalities.
e. List all equations needed to compute factor scores.
f. Write all the discriminant functions and test whether each of those are significant at 5% level of significance.

**Table 5**
Percentage points of the t distributions

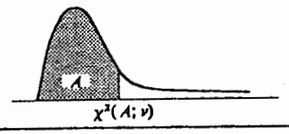



| $t_{.100}$ | $t_{.050}$ | $t_{.025}$ | $t_{.010}$ | $t_{.005}$ | d.f. |
|---|---|---|---|---|---|
| 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 |
| 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | inf. |

From "Table of Percentage Points of the *t*-Distribution." Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300. Reproduced by permission of Professor E. S. Pearson.

**TABLE A.3** Percentiles of the $\chi^2$ Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \le \chi^2(A; \nu)\} = A$



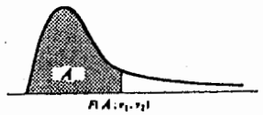

| | $A$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | .005 | .010 | .025 | .050 | .100 | .900 | .950 | .975 | .990 | .995 |
| 1 | $0.0^4393$ | $0.0^3157$ | $0.0^3982$ | $0.0^2393$ | 0.0158 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | 0.0100 | 0.0201 | 0.0506 | 0.103 | 0.211 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 |
| 6 | 0.676 | 0.872 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 7 | 0.989 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188–89.



**TABLE A.4** Percentiles of the $F$ Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \le F(A; \nu_1, \nu_2)\} = A$




$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

**TABLE A.4 *(continued)*** Percentiles of the *F* Distribution

| Den. df | A | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .50 | 1.00 | 1.50 | 1.71 | 1.82 | 1.89 | 1.94 | 1.98 | 2.00 | 2.03 |
| | .90 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 |
| | .95 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 |
| | .975 | 648 | 800 | 864 | 900 | 922 | 937 | 948 | 957 | 963 |
| | .99 | 4,052 | 5,000 | 5,403 | 5,625 | 5,764 | 5,859 | 5,928 | 5,981 | 6,022 |
| | .995 | 16,211 | 20,000 | 21,615 | 22,500 | 23,056 | 23,437 | 23,715 | 23,925 | 24,091 |
| | .999 | 405,280 | 500,000 | 540,380 | 562,500 | 576,400 | 585,940 | 592,870 | 598,140 | 602,280 |
| 2 | .50 | 0.667 | 1.00 | 1.13 | 1.21 | 1.25 | 1.28 | 1.30 | 1.32 | 1.33 |
| | .90 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 |
| | .95 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 |
| | .975 | 38.5 | 39.0 | 39.2 | 39.2 | 39.3 | 39.3 | 39.4 | 39.4 | 39.4 |
| | .99 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 |
| | .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 |
| | .999 | 998.5 | 999.0 | 999.2 | 999.2 | 999.3 | 999.3 | 999.4 | 999.4 | 999.4 |
| 3 | .50 | 0.585 | 0.881 | 1.00 | 1.06 | 1.10 | 1.13 | 1.15 | 1.16 | 1.17 |
| | .90 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 |
| | .95 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| | .975 | 17.4 | 16.0 | 15.4 | 15.1 | 14.9 | 14.7 | 14.6 | 14.5 | 14.5 |
| | .99 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 |
| | .995 | 55.6 | 49.8 | 47.5 | 46.2 | 45.4 | 44.8 | 44.4 | 44.1 | 43.9 |
| | .999 | 167.0 | 148.5 | 141.1 | 137.1 | 134.6 | 132.8 | 131.6 | 130.6 | 129.9 |
| 4 | .50 | 0.549 | 0.828 | 0.941 | 1.00 | 1.04 | 1.06 | 1.08 | 1.09 | 1.10 |
| | .90 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 |
| | .95 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| | .975 | 12.2 | 10.6 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 |
| | .99 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 |
| | .995 | 31.3 | 26.3 | 24.3 | 23.2 | 22.5 | 22.0 | 21.6 | 21.4 | 21.1 |
| | .999 | 74.1 | 61.2 | 56.2 | 53.4 | 51.7 | 50.5 | 49.7 | 49.0 | 48.5 |
| 5 | .50 | 0.528 | 0.799 | 0.907 | 0.965 | 1.00 | 1.02 | 1.04 | 1.05 | 1.06 |
| | .90 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 |
| | .95 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| | .975 | 10.0 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 |
| | .99 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 |
| | .995 | 22.8 | 18.3 | 16.5 | 15.6 | 14.9 | 14.5 | 14.2 | 14.0 | 13.8 |
| | .999 | 47.2 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 27.2 |
| 6 | .50 | 0.515 | 0.780 | 0.886 | 0.942 | 0.977 | 1.00 | 1.02 | 1.03 | 1.04 |
| | .90 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 |
| | .95 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| | .975 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 |
| | .99 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| | .995 | 18.6 | 14.5 | 12.9 | 12.0 | 11.5 | 11.1 | 10.8 | 10.6 | 10.4 |
| | .999 | 35.5 | 27.0 | 23.7 | 21.9 | 20.8 | 20.0 | 19.5 | 19.0 | 18.7 |
| 7 | .50 | 0.506 | 0.767 | 0.871 | 0.926 | 0.960 | 0.983 | 1.00 | 1.01 | 1.02 |
| | .90 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 |
| | .95 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| | .975 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 |
| | .99 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| | .995 | 16.2 | 12.4 | 10.9 | 10.1 | 9.52 | 9.16 | 8.89 | 8.68 | 8.51 |
| | .999 | 29.2 | 21.7 | 18.8 | 17.2 | 16.2 | 15.5 | 15.0 | 14.6 | 14.3 |



**TABLE A.4 *(continued)*** Percentiles of the *F* Distribution

| Den. df | A | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .50 | 2.04 | 2.07 | 2.09 | 2.12 | 2.13 | 2.15 | 2.17 | 2.18 | 2.20 |
| | .90 | 60.2 | 60.7 | 61.2 | 61.7 | 62.0 | 62.3 | 62.8 | 63.1 | 63.3 |
| | .95 | 242 | 244 | 246 | 248 | 249 | 250 | 252 | 253 | 254 |
| | .975 | 969 | 977 | 985 | 993 | 997 | 1,001 | 1,010 | 1,014 | 1,018 |
| | .99 | 6,056 | 6,106 | 6,157 | 6,209 | 6,235 | 6,261 | 6,313 | 6,339 | 6,366 |
| | .995 | 24,224 | 24,426 | 24,630 | 24,836 | 24,940 | 25,044 | 25,253 | 25,359 | 25,464 |
| | .999 | 605,620 | 610,670 | 615,760 | 620,910 | 623,500 | 626,100 | 631,340 | 633,970 | 636,620 |
| 2 | .50 | 1.34 | 1.36 | 1.38 | 1.39 | 1.40 | 1.41 | 1.43 | 1.43 | 1.44 |
| | .90 | 9.39 | 9.41 | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.48 | 9.49 |
| | .95 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 |
| | .975 | 39.4 | 39.4 | 39.4 | 39.4 | 39.5 | 39.5 | 39.5 | 39.5 | 39.5 |
| | .99 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 |
| | .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 200 |
| | .999 | 999.4 | 999.4 | 999.4 | 999.4 | 999.5 | 999.5 | 999.5 | 999.5 | 999.5 |
| 3 | .50 | 1.18 | 1.20 | 1.21 | 1.23 | 1.23 | 1.24 | 1.25 | 1.26 | 1.27 |
| | .90 | 5.23 | 5.22 | 5.20 | 5.18 | 5.18 | 5.17 | 5.15 | 5.14 | 5.13 |
| | .95 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.57 | 8.55 | 8.53 |
| | .975 | 14.4 | 14.3 | 14.3 | 14.2 | 14.1 | 14.1 | 14.0 | 13.9 | 13.9 |
| | .99 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.3 | 26.2 | 26.1 |
| | .995 | 43.7 | 43.4 | 43.1 | 42.8 | 42.6 | 42.5 | 42.1 | 42.0 | 41.8 |
| | .999 | 129.2 | 128.3 | 127.4 | 126.4 | 125.9 | 125.4 | 124.5 | 124.0 | 123.5 |
| 4 | .50 | 1.11 | 1.13 | 1.14 | 1.15 | 1.16 | 1.16 | 1.18 | 1.18 | 1.19 |
| | .90 | 3.92 | 3.90 | 3.87 | 3.84 | 3.83 | 3.82 | 3.79 | 3.78 | 3.76 |
| | .95 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.69 | 5.66 | 5.63 |
| | .975 | 8.84 | 8.75 | 8.66 | 8.56 | 8.51 | 8.46 | 8.36 | 8.31 | 8.26 |
| | .99 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.6 | 13.5 |
| | .995 | 21.0 | 20.7 | 20.4 | 20.2 | 20.0 | 19.9 | 19.6 | 19.5 | 19.3 |
| | .999 | 48.1 | 47.4 | 46.8 | 46.1 | 45.8 | 45.4 | 44.7 | 44.4 | 44.1 |
| 5 | .50 | 1.07 | 1.09 | 1.10 | 1.11 | 1.12 | 1.12 | 1.14 | 1.14 | 1.15 |
| | .90 | 3.30 | 3.27 | 3.24 | 3.21 | 3.19 | 3.17 | 3.14 | 3.12 | 3.11 |
| | .95 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.43 | 4.40 | 4.37 |
| | .975 | 6.62 | 6.52 | 6.43 | 6.33 | 6.28 | 6.23 | 6.12 | 6.07 | 6.02 |
| | .99 | 10.1 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.20 | 9.11 | 9.02 |
| | .995 | 13.6 | 13.4 | 13.1 | 12.9 | 12.8 | 12.7 | 12.4 | 12.3 | 12.1 |
| | .999 | 26.9 | 26.4 | 25.9 | 25.4 | 25.1 | 24.9 | 24.3 | 24.1 | 23.8 |
| 6 | .50 | 1.05 | 1.06 | 1.07 | 1.08 | 1.09 | 1.10 | 1.11 | 1.12 | 1.12 |
| | .90 | 2.94 | 2.90 | 2.87 | 2.84 | 2.82 | 2.80 | 2.76 | 2.74 | 2.72 |
| | .95 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.74 | 3.70 | 3.67 |
| | .975 | 5.46 | 5.37 | 5.27 | 5.17 | 5.12 | 5.07 | 4.96 | 4.90 | 4.85 |
| | .99 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.06 | 6.97 | 6.88 |
| | .995 | 10.2 | 10.0 | 9.81 | 9.59 | 9.47 | 9.36 | 9.12 | 9.00 | 8.88 |
| | .999 | 18.4 | 18.0 | 17.6 | 17.1 | 16.9 | 16.7 | 16.2 | 16.0 | 15.7 |
| 7 | .50 | 1.03 | 1.04 | 1.05 | 1.07 | 1.07 | 1.08 | 1.09 | 1.10 | 1.10 |
| | .90 | 2.70 | 2.67 | 2.63 | 2.59 | 2.58 | 2.56 | 2.51 | 2.49 | 2.47 |
| | .95 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.30 | 3.27 | 3.23 |
| | .975 | 4.76 | 4.67 | 4.57 | 4.47 | 4.42 | 4.36 | 4.25 | 4.20 | 4.14 |
| | .99 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.82 | 5.74 | 5.65 |
| | .995 | 8.38 | 8.18 | 7.97 | 7.75 | 7.65 | 7.53 | 7.31 | 7.19 | 7.08 |
| | .999 | 14.1 | 13.7 | 13.3 | 12.9 | 12.7 | 12.5 | 12.1 | 11.9 | 11.7 |

**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | .50 | 0.499 | 0.757 | 0.860 | 0.915 | 0.948 | 0.971 | 0.988 | 1.00 | 1.01 |
| | .90 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 |
| | .95 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| | .975 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 |
| | .99 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| | .995 | 14.7 | 11.0 | 9.60 | 8.81 | 8.30 | 7.95 | 7.69 | 7.50 | 7.34 |
| | .999 | 25.4 | 18.5 | 15.8 | 14.4 | 13.5 | 12.9 | 12.4 | 12.0 | 11.8 |
| 9 | .50 | 0.494 | 0.749 | 0.852 | 0.906 | 0.939 | 0.962 | 0.978 | 0.990 | 1.00 |
| | .90 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 |
| | .95 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| | .975 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 |
| | .99 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| | .995 | 13.6 | 10.1 | 8.72 | 7.96 | 7.47 | 7.13 | 6.88 | 6.69 | 6.54 |
| | .999 | 22.9 | 16.4 | 13.9 | 12.6 | 11.7 | 11.1 | 10.7 | 10.4 | 10.1 |
| 10 | .50 | 0.490 | 0.743 | 0.845 | 0.899 | 0.932 | 0.954 | 0.971 | 0.983 | 0.992 |
| | .90 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 |
| | .95 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| | .975 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 |
| | .99 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| | .995 | 12.8 | 9.43 | 8.08 | 7.34 | 6.87 | 6.54 | 6.30 | 6.12 | 5.97 |
| | .999 | 21.0 | 14.9 | 12.6 | 11.3 | 10.5 | 9.93 | 9.52 | 9.20 | 8.96 |
| 12 | .50 | 0.484 | 0.735 | 0.835 | 0.888 | 0.921 | 0.943 | 0.959 | 0.972 | 0.981 |
| | .90 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 |
| | .95 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| | .975 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 |
| | .99 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| | .995 | 11.8 | 8.51 | 7.23 | 6.52 | 6.07 | 5.76 | 5.52 | 5.35 | 5.20 |
| | .999 | 18.6 | 13.0 | 10.8 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 |
| 15 | .50 | 0.478 | 0.726 | 0.826 | 0.878 | 0.911 | 0.933 | 0.949 | 0.960 | 0.970 |
| | .90 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
| | .95 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| | .975 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 |
| | .99 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| | .995 | 10.8 | 7.70 | 6.48 | 5.80 | 5.37 | 5.07 | 4.85 | 4.67 | 4.54 |
| | .999 | 16.6 | 11.3 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 |
| 20 | .50 | 0.472 | 0.718 | 0.816 | 0.868 | 0.900 | 0.922 | 0.938 | 0.950 | 0.959 |
| | .90 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
| | .95 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| | .975 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
| | .99 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| | .995 | 9.94 | 6.99 | 5.82 | 5.17 | 4.76 | 4.47 | 4.26 | 4.09 | 3.96 |
| | .999 | 14.8 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
| 24 | .50 | 0.469 | 0.714 | 0.812 | 0.863 | 0.895 | 0.917 | 0.932 | 0.944 | 0.953 |
| | .90 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |
| | .95 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| | .975 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |
| | .99 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| | .995 | 9.55 | 6.66 | 5.52 | 4.89 | 4.49 | 4.20 | 3.99 | 3.83 | 3.69 |
| | .999 | 14.0 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |



**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | .50 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.07 | 1.08 | 1.08 | 1.09 |
| | .90 | 2.54 | 2.50 | 2.46 | 2.42 | 2.40 | 2.38 | 2.34 | 2.32 | 2.29 |
| | .95 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.01 | 2.97 | 2.93 |
| | .975 | 4.30 | 4.20 | 4.10 | 4.00 | 3.95 | 3.89 | 3.78 | 3.73 | 3.67 |
| | .99 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.03 | 4.95 | 4.86 |
| | .995 | 7.21 | 7.01 | 6.81 | 6.61 | 6.50 | 6.40 | 6.18 | 6.06 | 5.95 |
| | .999 | 11.5 | 11.2 | 10.8 | 10.5 | 10.3 | 10.1 | 9.73 | 9.53 | 9.33 |
| 9 | .50 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.05 | 1.07 | 1.07 | 1.08 |
| | .90 | 2.42 | 2.38 | 2.34 | 2.30 | 2.28 | 2.25 | 2.21 | 2.18 | 2.16 |
| | .95 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.79 | 2.75 | 2.71 |
| | .975 | 3.96 | 3.87 | 3.77 | 3.67 | 3.61 | 3.56 | 3.45 | 3.39 | 3.33 |
| | .99 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.48 | 4.40 | 4.31 |
| | .995 | 6.42 | 6.23 | 6.03 | 5.83 | 5.73 | 5.62 | 5.41 | 5.30 | 5.19 |
| | .999 | 9.89 | 9.57 | 9.24 | 8.90 | 8.72 | 8.55 | 8.19 | 8.00 | 7.81 |
| 10 | .50 | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.06 | 1.07 |
| | .90 | 2.32 | 2.28 | 2.24 | 2.20 | 2.18 | 2.16 | 2.11 | 2.08 | 2.06 |
| | .95 | 2.98 | 2.91 | 2.84 | 2.77 | 2.74 | 2.70 | 2.62 | 2.58 | 2.54 |
| | .975 | 3.72 | 3.62 | 3.52 | 3.42 | 3.37 | 3.31 | 3.20 | 3.14 | 3.08 |
| | .99 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.08 | 4.00 | 3.91 |
| | .995 | 5.85 | 5.66 | 5.47 | 5.27 | 5.17 | 5.07 | 4.86 | 4.75 | 4.64 |
| | .999 | 8.75 | 8.45 | 8.13 | 7.80 | 7.64 | 7.47 | 7.12 | 6.94 | 6.76 |
| 12 | .50 | 0.989 | 1.00 | 1.01 | 1.02 | 1.03 | 1.03 | 1.05 | 1.05 | 1.06 |
| | .90 | 2.19 | 2.15 | 2.10 | 2.06 | 2.04 | 2.01 | 1.96 | 1.93 | 1.90 |
| | .95 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.38 | 2.34 | 2.30 |
| | .975 | 3.37 | 3.28 | 3.18 | 3.07 | 3.02 | 2.96 | 2.85 | 2.79 | 2.72 |
| | .99 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.54 | 3.45 | 3.36 |
| | .995 | 5.09 | 4.91 | 4.72 | 4.53 | 4.43 | 4.33 | 4.12 | 4.01 | 3.90 |
| | .999 | 7.29 | 7.00 | 6.71 | 6.40 | 6.25 | 6.09 | 5.76 | 5.59 | 5.42 |
| 15 | .50 | 0.977 | 0.989 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 | 1.04 | 1.05 |
| | .90 | 2.06 | 2.02 | 1.97 | 1.92 | 1.90 | 1.87 | 1.82 | 1.79 | 1.76 |
| | .95 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.16 | 2.11 | 2.07 |
| | .975 | 3.06 | 2.96 | 2.86 | 2.76 | 2.70 | 2.64 | 2.52 | 2.46 | 2.40 |
| | .99 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.05 | 2.96 | 2.87 |
| | .995 | 4.42 | 4.25 | 4.07 | 3.88 | 3.79 | 3.69 | 3.48 | 3.37 | 3.26 |
| | .999 | 6.08 | 5.81 | 5.54 | 5.25 | 5.10 | 4.95 | 4.64 | 4.48 | 4.31 |
| 20 | .50 | 0.966 | 0.977 | 0.989 | 1.00 | 1.01 | 1.01 | 1.02 | 1.03 | 1.03 |
| | .90 | 1.94 | 1.89 | 1.84 | 1.79 | 1.77 | 1.74 | 1.68 | 1.64 | 1.61 |
| | .95 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.95 | 1.90 | 1.84 |
| | .975 | 2.77 | 2.68 | 2.57 | 2.46 | 2.41 | 2.35 | 2.22 | 2.16 | 2.09 |
| | .99 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.61 | 2.52 | 2.42 |
| | .995 | 3.85 | 3.68 | 3.50 | 3.32 | 3.22 | 3.12 | 2.92 | 2.81 | 2.69 |
| | .999 | 5.08 | 4.82 | 4.56 | 4.29 | 4.15 | 4.00 | 3.70 | 3.54 | 3.38 |
| 24 | .50 | 0.961 | 0.972 | 0.983 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 |
| | .90 | 1.88 | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.61 | 1.57 | 1.53 |
| | .95 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.84 | 1.79 | 1.73 |
| | .975 | 2.64 | 2.54 | 2.44 | 2.33 | 2.27 | 2.21 | 2.08 | 2.01 | 1.94 |
| | .99 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.40 | 2.31 | 2.21 |
| | .995 | 3.59 | 3.42 | 3.25 | 3.06 | 2.97 | 2.87 | 2.66 | 2.55 | 2.43 |
| | .999 | 4.64 | 4.39 | 4.14 | 3.87 | 3.74 | 3.59 | 3.29 | 3.14 | 2.97 |

**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | .50 | 0.466 | 0.709 | 0.807 | 0.858 | 0.890 | 0.912 | 0.927 | 0.939 | 0.948 |
| | .90 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
| | .95 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| | .975 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 |
| | .99 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| | .995 | 9.18 | 6.35 | 5.24 | 4.62 | 4.23 | 3.95 | 3.74 | 3.58 | 3.45 |
| | .999 | 13.3 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
| 60 | .50 | 0.461 | 0.701 | 0.798 | 0.849 | 0.880 | 0.901 | 0.917 | 0.928 | 0.937 |
| | .90 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |
| | .95 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| | .975 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 |
| | .99 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| | .995 | 8.49 | 5.80 | 4.73 | 4.14 | 3.76 | 3.49 | 3.29 | 3.13 | 3.01 |
| | .999 | 12.0 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |
| 120 | .50 | 0.458 | 0.697 | 0.793 | 0.844 | 0.875 | 0.896 | 0.912 | 0.923 | 0.932 |
| | .90 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 |
| | .95 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 |
| | .975 | 5.15 | 3.80 | 3.23 | 2.89 | 2.67 | 2.52 | 2.39 | 2.30 | 2.22 |
| | .99 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 |
| | .995 | 8.18 | 5.54 | 4.50 | 3.92 | 3.55 | 3.28 | 3.09 | 2.93 | 2.81 |
| | .999 | 11.4 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 |
| ∞ | .50 | 0.455 | 0.693 | 0.789 | 0.839 | 0.870 | 0.891 | 0.907 | 0.918 | 0.927 |
| | .90 | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 |
| | .95 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 |
| | .975 | 5.02 | 3.69 | 3.12 | 2.79 | 2.57 | 2.41 | 2.29 | 2.19 | 2.11 |
| | .99 | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 |
| | .995 | 7.88 | 5.30 | 4.28 | 3.72 | 3.35 | 3.09 | 2.90 | 2.74 | 2.62 |
| | .999 | 10.8 | 6.91 | 5.42 | 4.62 | 4.10 | 3.74 | 3.47 | 3.27 | 3.10 |



**TABLE A.4 *(concluded)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | .50 | 0.955 | 0.966 | 0.978 | 0.989 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 |
| | .90 | 1.82 | 1.77 | 1.72 | 1.67 | 1.64 | 1.61 | 1.54 | 1.50 | 1.46 |
| | .95 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.74 | 1.68 | 1.62 |
| | .975 | 2.51 | 2.41 | 2.31 | 2.20 | 2.14 | 2.07 | 1.94 | 1.87 | 1.79 |
| | .99 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.21 | 2.11 | 2.01 |
| | .995 | 3.34 | 3.18 | 3.01 | 2.82 | 2.73 | 2.63 | 2.42 | 2.30 | 2.18 |
| | .999 | 4.24 | 4.00 | 3.75 | 3.49 | 3.36 | 3.22 | 2.92 | 2.76 | 2.59 |
| 60 | .50 | 0.945 | 0.956 | 0.967 | 0.978 | 0.983 | 0.989 | 1.00 | 1.01 | 1.01 |
| | .90 | 1.71 | 1.66 | 1.60 | 1.54 | 1.51 | 1.48 | 1.40 | 1.35 | 1.29 |
| | .95 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.53 | 1.47 | 1.39 |
| | .975 | 2.27 | 2.17 | 2.06 | 1.94 | 1.88 | 1.82 | 1.67 | 1.58 | 1.48 |
| | .99 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.84 | 1.73 | 1.60 |
| | .995 | 2.90 | 2.74 | 2.57 | 2.39 | 2.29 | 2.19 | 1.96 | 1.83 | 1.69 |
| | .999 | 3.54 | 3.32 | 3.08 | 2.83 | 2.69 | 2.55 | 2.25 | 2.08 | 1.89 |
| 120 | .50 | 0.939 | 0.950 | 0.961 | 0.972 | 0.978 | 0.983 | 0.994 | 1.00 | 1.01 |
| | .90 | 1.65 | 1.60 | 1.55 | 1.48 | 1.45 | 1.41 | 1.32 | 1.26 | 1.19 |
| | .95 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.43 | 1.35 | 1.25 |
| | .975 | 2.16 | 2.05 | 1.95 | 1.82 | 1.76 | 1.69 | 1.53 | 1.43 | 1.31 |
| | .99 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.66 | 1.53 | 1.38 |
| | .995 | 2.71 | 2.54 | 2.37 | 2.19 | 2.09 | 1.98 | 1.75 | 1.61 | 1.43 |
| | .999 | 3.24 | 3.02 | 2.78 | 2.53 | 2.40 | 2.26 | 1.95 | 1.77 | 1.54 |
| ∞ | .50 | 0.934 | 0.945 | 0.956 | 0.967 | 0.972 | 0.978 | 0.989 | 0.994 | 1.00 |
| | .90 | 1.60 | 1.55 | 1.49 | 1.42 | 1.38 | 1.34 | 1.24 | 1.17 | 1.00 |
| | .95 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.32 | 1.22 | 1.00 |
| | .975 | 2.05 | 1.94 | 1.83 | 1.71 | 1.64 | 1.57 | 1.39 | 1.27 | 1.00 |
| | .99 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.47 | 1.32 | 1.00 |
| | .995 | 2.52 | 2.36 | 2.19 | 2.00 | 1.90 | 1.79 | 1.53 | 1.36 | 1.00 |
| | .999 | 2.96 | 2.74 | 2.51 | 2.27 | 2.13 | 1.99 | 1.66 | 1.45 | 1.00 |

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