

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2007/8

COURSE TITLE: INTRODUCTION TO DISTRIBUTION THEORY

COURSE CODE: ST 301

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS

SPECIAL REQUIREMENTS: STATISTICAL TABLES

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Question 1

(a) If a company employs 'k' salespersons, its gross sales in thousands of emalangeneni may be regarded as a random variable having a gamma distribution with $\alpha = 80\sqrt{k}$ and $\beta = 2$. If the sales cost is E8, 000 per salesperson, how many salesperson should the company employ to maximise expected profit?

(10 marks)

(b) Show that if $\alpha > 1$ and $\beta > 1$, the beta density $f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$, for $0 < x < 1$; has a

relative maximum at $x = \frac{\alpha - 1}{\alpha + \beta - 2}$.

(10 marks)**Question 2**

(a) Let X has the probability density function given by:

(10 marks)

$$f(x) = 3(1-x)^2, \text{ for } 0 < x < 1.$$

If $Y = (1-x)^3$, find the probability density of Y using the change of variable (transformation) technique.

(b) Let X has a gamma distribution with $\alpha = 2$ and $\beta = 2$. Determine the probability density function of $Y = x^2$ using either the distribution function technique or the change of variable (transformation) technique.

(10 marks)

$$\text{Where } f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & \text{for } x > 0. \\ 0, & \text{elsewhere} \end{cases}$$

Question 3

(a) Suppose that the time spent by a randomly selected student who uses a terminal connected to a local time-sharing computer facility as a gamma distribution with mean 20 minutes and variance 80 min^2 .

(i) What are the values of α and β ? (5 marks)

(ii) Given that $f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$ for $x > 0$ and 0, otherwise,

What is the probability that the student uses the terminal for at most 24 minutes? (5 marks)

(b) Suppose a random sample of $n=25$ observations is selected from an infinite population that is normally distributed with mean of 106 and standard deviation equal to 12. Given the mean and standard deviation of the sampling distribution of the sample mean:

(i) Find the probability that \bar{x} exceeds 110. (5 marks)

(ii) Find the probability that the sample mean deviates from the population mean $\mu = 106$ by no more than 4. (5 marks)

Question 4

(a) The telephone lines serving an airline reservation office are all busy about 60% of the time.

(i) If you happen to be calling this office, what is the probability that you will complete your call on the first try? (5 marks)

(ii) If you and your friend must both complete calls to this office, what is the probability that it takes a total of four tries for both of you to get through? (5 marks)

(b) If 90% of the calls that come into the airline reservation office mentioned in part (a) are from people who want to purchase an airline ticket, what is the probability that among 20 calls received by this office, two of them are for other reasons other than buying a ticket? (5 marks)

(c) If the probability is 0.20 that a certain bank will refuse a loan application, use the normal approximation to determine (to three decimals) the probability that the bank will refuse at least 40 of the 225 loan applications. (5 marks)

Question 5

Suppose that the length of time a light bulb will work is a random variable Y , with a density function given by:

$$f(y) = 1000e^{-1000y} \text{ for } y > 0 \text{ or } 0, \text{ otherwise}$$

- (a) Obtain the moment generating function. **(5 marks)**
- (b) If the moment generating function of a random variable X is given by $M_x(t) = 3t^3 + 2t^2 + t + 1$.
Find the mean and variance of X . **(8 marks)**
- (c) Suppose that W is a linear combination of two random variables Y and Z , such that $W = 3Y - 2Z$.
If $\mu_y = 3, \mu_z = 1, \sigma_y^2 = 3, \sigma_z^2 = 1$ and $\sigma_{yz} = 2$, find $E(W)$ and $V(W)$. **(7 marks)**

END OF EXAM!!

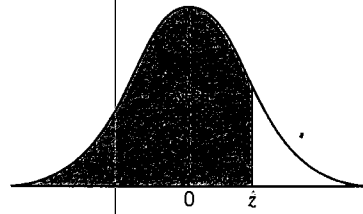


TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

