UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION 2008

COURSE TITLE	:	OPERATIONS RESEARCH I
COURSE CODE	:	ST 307
TIME ALLOWED	:	TWO (2) HOURS
INSTRUCTION	:	ANSWER ANY FOUR (4) QUESTIONS. EACH QUESTION CARRIES 25 MARKS.
REQUIREMENTS	:	CALCULATOR

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Question 1

(a) Define the following terms in the context of linear programming:

(i) Boundary equation;
(ii) Boundary;
(iii) Corner-point feasible solution.

(b) A certain corporation has three branch plants with excess production capacity. All three plants have the capability for producing a certain product, and management has decided to use some of the excess production capacity as follows. This product can be made in three sizes – large, medium, and small – that yield a net unit profit of $140, $120, and $100, respectively. Plants 1, 2, and 3 have the excess manpower and equipment capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. However, the amount of available in-process storage space also imposes a limitation on the production rates. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that 900, 1200, and 750 units of the large, medium, and small sizes, respectively, can be sold per day.

To maintain a uniform workload among the plants and to retain some flexibility, management has decided that the additional production assigned to each plant must use the same percentage of the excess manpower and equipment capacity.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

Formulate the linear programming model for this problem.

Question 2

(a) Consider the following linear programming problem:

Maximize $Z = 2X_1 + X_2$,

subject to

$$
\begin{array}{rcrcr}
 & & X_2 & \leq & 10 \\
2X_1 & + & 5X_2 & \leq & 60 \\
X_1 & + & X_2 & \leq & 18 \\
3X_1 & + & X_2 & \leq & 44
\end{array}
$$

and

$X_1 \geq 0,\ X_2 \geq 0.$

(i) Construct the initial simplex tableau, introducing slack variables and so forth as needed for applying the simplex method.

(ii) Solve the problem by simplex method.

(b) Consider the problem in (a) above.

(i) Find the *dual* problem.

(ii) Use the final simplex tableau in (a) (ii) to find the optimal solution to the *dual* problem.

(iii) Verify the *dual theorem* using the optimal solutions in (b) and (c) above.

Question 3

Consider the following problem:

Maximize $Z = -5X_1 + 5X_2 + 13X_3$,

Subject to

$$
\begin{array}{rcrcrcr}
-X_1 & + & X_2 & + & 3X_3 & \leq & 20 \\
12X_1 & + & 4X_2 & + & 10X_3 & \leq & 90
\end{array}
$$

and

$X_j \geq 0\ (j = 1,2,3).$

Letting X_4 and X_5 be the slack variables for the respective constraints, the simplex method yields the following *final* set of equations:

$$
\begin{array}{llrcrcrcrcrcl}
(0) & Z & & & & & 2X_3 & + & 5X_4 & & & = & 100 \\
(1) & & -X_1 & + & X_2 & + & 3X_3 & + & X_4 & & & = & 20 \\
(2) & & 16X_1 & & & - & 2X_3 & - & 4X_4 & + & X_5 & = & 10
\end{array}
$$

You are now to conduct sensitivity analysis by *independently* investigating each of the nine changes in the original model indicated below. For each change, use the sensitivity analysis procedure to convert this set of equations (in tabular form) to the proper form for identifying and evaluating the current basic solution, and then test this solution for *feasibility* and for *optimality*.

(a) Change the right-hand side of constraint 1 to $b_1 = 30$.

(b) Change the right-hand side of constraint 2 to $b_1 = 70$.

(c) Change the right-hand sides to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$.

(d) Change the coefficient of X_3 in the objective function to $c_3 = 8$.

(e) Change the coefficients of X_1 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$.

(f) Change the coefficients of X_2 to $\begin{bmatrix} c_2 \\ a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$.

(g) Introduce a new variable X_6 with coefficients $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$.

(h) Introduce a new constraint $2X_1 + 3X_2 + 5X_3 \le 50$. (Denote its slack variable by X_6.)

(i) Change constraint 2 to $10X_1 + 5X_2 + 10X_3 \le 100$.

Question 4

For each of the following payoff tables, use the approach described in the lecture to formulate the problem of finding the optimal mixed strategies according to the minimax criterion as a *linear programming* problem.

(a)

		II: 1	2	3
I	1	5	3	2
	2	3	-2	3
	3	2	6	-1

(b)

		II: 1	2	3	4	5
I	1	-3	-6	5	-2	3
	2	-1	4	-4	1	-2
	3	0	-2	-5	-3	1
	4	2	-3	0	2	-4

Question 5

Assume that there are two weighted coins. Coin 1 has a probability of 0.25 of turning up heads, and coin 2 has a probability of 0.6 of turning up heads. A coin is tossed once. The decision maker must decide which coin was tossed. The probability that coin 1 was tossed is 0.6, and the probability that coin 2 was tossed is 0.4. The loss matrix is as follows:

action: a	θ: 1 *(Coin 1 tossed)*	2 *(Coin 2 tossed)*
a_1: say coin 1 tossed	0	1
a_2: say coin 2 tossed	1	0

(a) What is the Bayes' procedure (action) before the coin is tossed?
(b) What is the minimum expected loss (Perfect Information)?
(c) What is the Bayes' procedure if the outcome is:
  (i) heads?
  (ii) tails?

FORMULAE:

Linear Programming:

- $\Delta y_o^* = \sum \Delta b_i \, y_i^*$,
- $\Delta b_k^* = \sum \Delta b_i \, s_{ki}^*$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$
- $\Delta(z_j^* - c_j) = -\Delta c_j + \sum \Delta a_{ij} \, y_i^*$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$
- $\Delta a_{kj}^* = \sum \Delta a_{ij} \, s_{ki}^*$.

Decision Analysis:

$$l(a) = E[l(a,\theta)] = \begin{cases} \sum_{all \cdot k} l(a,k) P_\theta(k), \cdot if \cdot \theta \cdot is \cdot discrete \\ \int_{-\infty}^{\infty} \cdot l(a,y) P_\theta(y) dy, \cdot if \cdot \theta \cdot is \cdot continuous. \end{cases}$$

SIMPLEX METHOD DUMMY TABLES

Iteration	Basic Variable	Eq. no.	Coefficient of Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Right side
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
	Z	0	1											
		1	0											
		2	0											
		3	0											
		4	0											
		5	0											
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		4	0											
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		4	0											
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