

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2009

TITLE OF PAPER : LINEAR STATISTICAL METHODS

COURSE CODE : ST204

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[4 + 6 + 10 + 5 marks]

- 1.1 Define the Functional Relation and Statistical Relation between two variables.
- 1.2 Define Simple Linear Regression Model. Is it a Functional Relation model? Explain.
- 1.3 Find the point estimators of β_0 and β_1 using Method of Least Squares.
- 1.4 State the estimated (fitted) regression function and discuss the important properties of the Fitted Regression Line.

QUESTION TWO.

[2 + 2 + 2 + 4 + 1 + 5 + 5 + 4 marks]

A study is conducted with a group of dieters to see if the number of grams of fat each consumes per day is related to cholesterol level. The following output was obtained from running the model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ using SPSS:

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2964.587	1	2964.587	3.407	.114
	Residual	5221.413	6	870.235		
	Total	8186.000	7			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	110.115	59.108		1.863	.112
	x	12.784	6.926	.602	1.846	.114

- 2.1 Identify the dependent variable and independent variable.
- 2.2 State the fitted regression line.
- 2.3 What would be expected cholesterol level of a dieter who decides to consume 10 grams of fat every day?
- 2.4 Perform the F-test and clearly state the conclusion.
- 2.5 What is the estimated value of σ^2 ?
- 2.6 Test $\beta_1 = 15$ against $\beta_1 < 15$ at $\alpha = 0.01$.
- 2.7 Test $\beta_0 = 100$ against $\beta_0 \neq 100$ at $\alpha = 0.05$.
- 2.8 Compute coefficient of correlation, r and explain the nature and strength of the relationship between dependent and independent variables.

QUESTION THREE.

[4 + 6 + 3 + 10 + 2 marks]

- 3.1 Discuss the distinction between the *fixed effects models* and *random effects models* in single-factor studies.
- 3.2 State the *Cell Means Model* and *Factor Effects Model* for single factor studies and compare these two models.
- 3.3 A researcher wishes to see whether there is any difference in the weight gains of athletes following one of three special diets. Athletes are randomly assigned to three groups and placed on the diet for six weeks. The weight gains (in kg) are shown below:

Diet A	Diet B	Diet C
3	10	8
6	12	3
7	11	2
4	14	5
	8	
	6	

- a. Identify the dependent variable, factor studied and factor levels.
- b. Complete the computation of the ANOVA table and conduct the F test. Clearly state all the steps in the test including the conclusion.
- c. Which diet would you prefer for the athletes? Explain.

QUESTION FOUR.

[6 + 6 + 2 + 6 + 5 marks]

- 4.1 We have two descriptive measures that are frequently used in practice to describe the degree of linear association between the dependent (Y) and independent (X) variables. Define these two measures and discuss the relationship between these two measures.
- 4.2 An educator wants to see how the number of absences a student in her class has affected the student's final grade. The data obtained from a sample follow.

No. of absences	10	12	2	1	8	5
Final Mark	70	65	96	94	75	82

- a. Fit the regression line, $Y_i = \beta_0 + \beta_1 X_i$.
- b. Interpret the estimated values of β_0 and β_1 .
- c. Construct a 99% confidence interval for β_1 .
- d. Compute the coefficient of determination and interpret the value.

QUESTION FIVE.

[8 + 4 + 1 + 2 + 1 + 1 + 8 marks]

- 5.1 State the Factor Effects Model for two-factor studies with equal sample sizes and its important features.
- 5.2 A company wishes to test the effectiveness of its advertising. A product is selected, and two types of ads are written; one is serious and one is humorous. Also the ads run on both medium of advertising; television and radio. Sixteen potential customers are selected and assigned randomly to one of the four groups. After seeing or listening to the ad, each customer is asked to rate its effectiveness on a scale of 1 to 20 and the data was analyzed using SPSS. The following ANOVA table is a part of the output from that analysis:

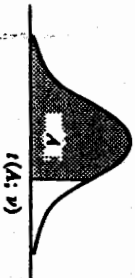
ANOVA TABLE

Source of Variation	Sum of Squares	df	Mean Square	F
Between treatments	186.189			
Factor A	10.563			
Factor B	175.563			
A X B	0.063			
Within treatments	66.250			
Total	252.439			

- Complete the ANOVA Table
- The size of the sample used in this experiment?
- What are the treatments in this experiment?
- Which one is the Factor A?
- Which one is the Factor B?
- Using 5% level of significance, describe only the conclusions (based on F-test) in terms of effectiveness of the types of ad, effectiveness of the medium of advertising and effectiveness of their interaction.

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$

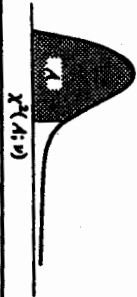


ν	A									
	.60	.70	.80	.85	.90	.95	.975	.975	.975	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706			
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303			
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182			
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776			
5	0.267	0.559	0.920	1.156	1.476	2.013	2.571			
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447			
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365			
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306			
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262			
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228			
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201			
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179			
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160			
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145			
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131			
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120			
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110			
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101			
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093			
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086			
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080			
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074			
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069			
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064			
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060			
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056			
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052			
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048			
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045			
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042			
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021			
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000			
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980			
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960			

TABLE A.2 (continued) Percentiles of the *t* Distribution

ν	A									
	.98	.985	.99	.9925	.995	.9975	.9975	.9975	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590			
2	4.849	5.643	6.965	8.073	9.923	14.089	31.598			
3	3.482	3.896	4.541	5.087	5.841	7.453	12.974			
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610			
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869			
6	2.612	2.829	3.143	3.372	3.707	4.317	5.939			
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408			
8	2.449	2.634	2.896	3.135	3.355	3.833	5.041			
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781			
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587			
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437			
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318			
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221			
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140			
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073			
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015			
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965			
18	2.214	2.356	2.552	2.690	2.878	3.197	3.922			
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883			
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849			
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819			
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792			
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768			
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745			
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725			
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707			
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690			
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674			
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659			
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646			
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551			
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460			
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373			
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291			

TABLE A.3 Percentiles of the χ^2 Distribution
Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.04963	0.0793	0.10138	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	75.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution
Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

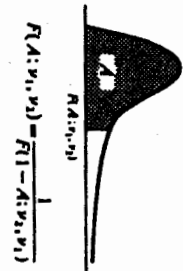


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df									
	1	2	3	4	5	6	7	8	9	
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.3
.95	161	200	216	225	230	234	237	239	241	242
.975	648	800	864	900	922	937	948	957	963	966
.99	4,032	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022	6,052
.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091	24,210
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	605,620
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.38
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.4	39.4	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.9
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.98	3.95	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.3	49.7	49.0	48.5	48.5
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	27.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	18.7
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df									
	10	12	15	20	24	30	60	120	∞	
1	.50	2.04	2.07	2.09	2.12	2.13	2.17	2.18	2.20	
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	
.95	742	744	746	748	749	750	752	753	754	
.975	969	977	985	993	997	1,001	1,010	1,014	1,018	
.99	6,036	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,364	
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620	
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.44	
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	
.995	199	199	199	199	199	199	199	199	200	
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.27	
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13	
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53	
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9	
.99	27.2	27.1	27.1	26.7	26.6	26.5	26.3	26.2	26.1	
.995	43.7	43.4	43.1	42.8	42.5	42.5	42.1	42.0	41.8	
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	
4	.50	1.11	1.13	1.14	1.15	1.16	1.18	1.18	1.19	
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.78	3.78	3.76	
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63	
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	
.995	21.0	20.7	20.4	20.2	20.0	20.0	19.9	19.6	19.5	
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	
5	.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.15	
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11	
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37	
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72	
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67	
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88	
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	
7	.50	1.03	1.04	1.05	1.07	1.07	1.09	1.09	1.10	
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47	
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23	
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14	
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	.50	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8
	.90	0.484	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.95	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.975	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.99	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
10	.50	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.33
	.90	0.480	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.95	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.975	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.99	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
12	.50	12.8	9.43	8.08	7.54	7.19	6.87	6.54	6.12	5.97
	.90	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.95	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.975	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.99	6.35	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
15	.50	11.8	8.51	7.23	6.52	6.06	5.76	5.52	5.35	5.20
	.90	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.95	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.975	4.34	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.99	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.12	3.12
20	.50	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
	.90	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.95	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.975	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.99	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
24	.50	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
	.90	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.95	2.93	2.54	2.33	2.23	2.19	2.10	2.04	1.98	1.94
	.975	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.99	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df												
	10	12	15	20	24	30	40	60	120	∞			
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09			
	.90	2.94	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29			
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93			
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.67	3.63			
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.86	4.80			
9	.50	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33			
	.90	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08			
	.95	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16			
	.975	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71			
	.99	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33			
10	.50	10.0	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07			
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06			
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54			
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08			
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91			
12	.50	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81			
	.90	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07			
	.95	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06			
	.975	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54			
	.99	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08			
15	.50	8.75	8.46	8.13	7.80	7.64	7.47	7.12	6.94	6.76			
	.90	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06			
	.95	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90			
	.975	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30			
	.99	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72			
20	.50	8.08	7.81	7.52	7.19	7.03	6.85	6.49	6.30	6.12			
	.90	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.03	1.04			
	.95	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76			
	.975	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07			
	.99	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40			
24	.50	7.59	7.32	7.04	6.71	6.54	6.35	5.99	5.79	5.62			
	.90	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.02	1.03			
	.95	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61			
	.975	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84			
	.99	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09			

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
30	0.935	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02		
.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46		
.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62		
.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79		
.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01		
.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18		
.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59		
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01		
.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29		
.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39		
.975	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48		
.99	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60		
.995	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69		
.999	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89		
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01		
.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19		
.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25		
.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31		
.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38		
.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43		
.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54		
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00		
.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00		
.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00		
.975	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00		
.99	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00		
.995	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00		
.999	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00		

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