UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2008/9

COURSE TITLE:	**INTRODUCTION TO DISTRIBUTION THEORY**
COURSE CODE:	**ST 301**
TIME ALLOWED:	**TWO (2) HOURS**
INSTRUCTION:	**ANSWER ANY FOUR QUESTIONS**
SPECIAL REQUIREMENTS:	**STATISTICAL TABLES**

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Question 1

(a) Let Y have probability density function given by $f(y) = \begin{cases} \frac{y+1}{2}, -1 \le y \le 1 \\ o, elsewhere \end{cases}$

Use the distribution function technique to find the density function for $U = y^2$

(7 marks)

(b) The waiting time Y until delivery of a new component for an industrial operation is uniformly distributed over the interval from 1 to 5 days. The cost of this delay is given by $U = 2y^2 + 3$. Use the distribution function technique to find the probability density function of U.

(8 marks)

Question 2

A random sample of size n = 100 is taken from an infinite population with mean $\mu = 75$ and the variance $\sigma^2 = 256$.

(a) Based on Chebyshev's theorem, with what probability can we assert that the value we obtain for $\bar{X}$ will fall between 67 and 83? **(7 marks)**

(b) Based on the Central Limit Theorem, with what probability can we assert that the value we obtain for $\bar{X}$ will fall between 67 and 83? **(8 marks)**

Question 3

(a) If Y has an Exponential distribution and P(Y>2) = 0.0821, what is

(i) $\theta = E(Y)$?

(ii) $P(Y \le 1.7)$? **(6 marks)**

(b) Suppose that the magnitude of earthquakes recorded in a region of North America can be modelled as having a Gamma distribution with $\alpha = 0.8$ and $\beta = 2.4$ as measured on the Richter scale.

(i) What is the mean magnitude of earthquakes striking the region?

(ii) What is the probability that the magnitude an earthquake striking the region will exceed 3.0 on the Richter scale? **(9 marks)**

Question 4

(a) A geological study indicates that an exploratory oil well should strike oil with probability 0.2.

(i) What is the probability that the first strike comes on the third well drilled?

(3 marks)

(ii) What is the probability that the third strike comes on the seventh well drilled?

(3 marks)

(b) In California, a growing number of individuals pursuing teaching credentials are choosing paid internship over traditional student teaching programs. A group of eight candidates for three local teaching positions consisted of five who had enrolled in paid internships and three who enrolled in traditional student teaching programs. All eight candidates appear to be equally qualified, so three are randomly selected to fill the open positions. Let Y be the number of internship trained candidates who are hired.

(i) What distribution does Y have and why? **(2 marks)**

(ii) Find the probability that two or more internship trained candidates are hired.

(7 marks)

Question 5

A random variable X has a **Weibull distribution** if and only if its probability density is given by:

$$f(x) = kx^{\beta-1}e^{-\alpha x^{\beta}} \quad \textbf{for } x > 0;\ 0,\ \textbf{elsewhere}$$

where $\alpha > 0;\ \beta > 0$ *and* $k = \alpha\beta$

(a) Suppose that the service life in hours of a semiconductor is a random variable having a Weibull distribution with $\alpha = 0.005$ and $\beta = 0.2500$. What is the probability that such a semiconductor will still be in operating condition after 4, 000 hours?

(8 marks)

(b) Suppose that P, the price of a certain commodity (in dollars), and S, its total sales (in 10,000 units), are random variables whose joint probability distribution can be approximated closely with the joint probability density:

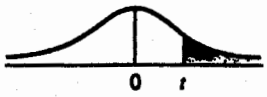
$$f(p,s) = \begin{cases} 5pe^{-ps} & for 0.20 < p < 0.40; s > 0 \\ o & elsewhere \end{cases}$$

Find the joint probability density of the two random variables V=SP and W=P, then find the marginal density of V. **(7 marks)**

END OF EXAM!!

Table VIII The t Distribution Table[†]

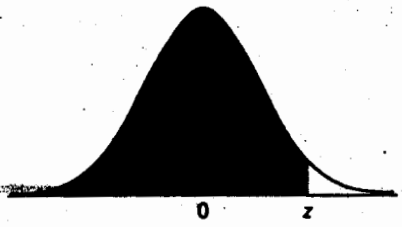
The entries in the table give the critical values of t for the specified number of degrees of freedom and areas in the right tail.



	Area in the Right Tail under the t Distribution Curve					
df	.10	.05	.025	.01	.005	.001
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
31	1.309	1.696	2.040	2.453	2.744	3.375
32	1.309	1.694	2.037	2.449	2.738	3.365
33	1.308	1.692	2.035	2.445	2.733	3.356
34	1.307	1.691	2.032	2.441	2.728	3.348
35	1.306	1.690	2.030	2.438	2.724	3.340
36	1.306	1.688	2.028	2.434	2.719	3.333
37	1.305	1.687	2.026	2.431	2.715	3.326
38	1.304	1.686	2.024	2.429	2.712	3.319
39	1.304	1.685	2.023	2.426	2.708	3.313
40	1.303	1.684	2.021	2.423	2.704	3.307
∞	1.282	1.645	1.960	2.326	2.576	3.090

[†]This table is an abbreviated version of Table VIII that appears in Appendix C. This table goes up to 40 degrees of freedom. For degrees of freedom from 41 to 70, use Table VIII of Appendix C.

Table B
(concluded)
2. Areas under the standardized normal curve, from $-\infty$ to z



z	00	01	02	03	04	05	06	07	08	09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9991	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998