UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION 2009

COURSE TITLE   :   OPERATIONS RESEARCH I

COURSE CODE   :   ST 307

TIME ALLOWED   :   TWO (2) HOURS

INSTRUCTION   :   ANSWER ANY FOUR (4) QUESTIONS.
EACH QUESTION CARRIES 25 MARKS.

REQUIREMENTS :   CALCULATOR

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Question 1

(a)   Briefly discuss the four (4) assumptions of linear programming.

[8 marks]

(b)   A farmer is raising pigs for market, and he wishes to determine the quantities of the available types of feed that should be given to each pig to meet certain nutritional requirements at a *minimum cost*. The number of units of each type of basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional ingredient	Kilogram of corn	Kilogram of tankage	Kilogram of alfalfa	Minimum daily requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (¢)	21	18	15	

(i)   Formulate the linear programming model for this problem.   [9 marks]
(ii)   Rewrite this model in an equivalent way to fit our standard form discussed in class.   [8 marks]

Question 2

Consider the problem

-    Maximize $Z = 2X_1 + 3X_2$,

subject to

$$X_1 + 2X_2 \leq 10 \quad \text{(resource 1)}$$
$$3X_1 + X_2 \leq 15 \quad \text{(resource 2)}$$
$$X_2 \leq 4 \quad \text{(resource 3)}$$

And

$$X_1 \geq 0, X_2 \geq 0.$$

(a)   Solve the problem graphically.   [7 marks]
(b)   Solve the problem by the simplex method.   [12marks]

(c)   Identify the shadow prices for the three resources from the final tableau for the simplex method. Demonstrate graphically that these are the correct shadow prices.   [6 marks]

Question 3

A company has three plants producing a certain product that is to be shipped to four distribution centres. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance from each plant to the respective distributing centres is given below in miles:

<div align="center">

Distribution Centre

</div>

		1	2	3	4
	1	800	1,300	400	700
Plant	2	1,100	1,400	600	1,000
	3	600	1,200	800	900

The freight cost for each shipment is $100 plus 50 cents per mile.

The company wishes to determine how much should be shipped from each plant to each of the distribution centers to minimize the total shipping costs.

(a)   Formulate this problem as a transformation problem by constructing the appropriate cost and requirements table.   [10 marks]

(b)   Use the northwest corner rule to obtain an initial basic feasible solution.   [5 marks]

(c)   Use the transportation simplex method to obtain the optimal solution.   [10 marks]

Question 4

(a)   Find the saddle point for the game having the following payoff table

[10 marks]

		II			
		1	2	3	4
	1	3	-3	-1	-7
I	2	1	-1	5	3
	3	-7	-5	-3	7

(b)   Consider the game having the following payoff table:

		II			
		1	2	3	4
	1	5	0	3	1
I	2	2	4	3	2
	3	3	2	0	4

Use the approach described in the lecture to formulate the problem of finding the optimal mixed strategies according to the minimax criterion as a *linear programming* problem.   [15 marks]

Question 5

An oil company owns some land that is purported to contain oil. The company classifies such land into four categories by the total number of barrels that are expected to be obtained from the well, i.e., a 500,000-barrel well, a 200,000-barrel well, a 50,000-barrel well, or dry well. The company is faced with deciding whether to drill for oil, to unconditionally lease the land to an independent oil driller, or to conditionally lease the land at a rate depending upon the oil strike. Suppose the loss function is given by the table below.

	State of nature (θ)			
	θ_1:500,000-	θ_2:200,000-	θ_3:50,000-	θ_4:
	Barrel well	Barrel well	Barrel well	Dry well
Action (a)				
a_1: drill for oil	– 650,000	– 200,000	25,000	75,000
a_2: unconditionally lease	– 45,000	– 45,000	– 45,000	– 45,000
a_3: conditionally lease	– 250,000	– 100,000	0	0

Further, suppose that the prior distribution given below applies:

$$Pr\{\theta = \theta_1\} = P_\theta(1) = 0.10$$
$$Pr\{\theta = \theta_2\} = P_\theta(2) = 0.15$$
$$Pr\{\theta = \theta_3\} = P_\theta(3) = 0.25$$
$$Pr\{\theta = \theta_4\} = P_\theta(4) = 0.50$$

a.   What is the Bayes' procedure before obtaining the seismic readings?

[6 marks]

b.   What is the optimal expected loss with Perfect Information (PI)

[3 marks]

Suppose the company obtains seismic soundings at a cost of $12,000. This information leads to four possible seismic classifications, denoted by (1), (2), (3), and (4) as follows:

(1) denotes that there is definitely a closed geological structure to the site (a very favourable outcome if the presence of oil is desired);

(2) denotes that there is probably a closed structure to the site; the classification;

(3) denotes that there is a nonclosed structure (a relatively unfavourable report); and

(4) denotes that there is no structure to the site (an unfavourable condition).

Based upon past examination of similar geological areas (100 such examinations) the data presented in the table below are obtained.