

UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2008/9

**COURSE TITLE: DESIGN AND ANALYSIS OF EXPERIMENTS**

**COURSE CODE: ST 404**

**TIME ALLOWED: TWO (2) HOURS**

**INSTRUCTION: ANSWER QUESTIONS 1 & 2 AND ANY TWO QUESTIONS**

**SPECIAL REQUIREMENTS: STATISTICAL TABLES**

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**Question 1**

A farmer is considering adding diet supplements to the food currently given to his dairy cows in an attempt to increase milk yields. In order to assist his decision, he sets up an experiment using a random sample of 32 of his cows who are assigned randomly to one of the following 4 diet regimes: standard diet, standard diet plus supplement A, standard diet plus supplement B, standard diet plus supplement C. At the end of the three month study period, the average daily milk yield (in pints) is recorded for each cow with the following results:

| Standard diet | Standard diet plus supplement A | Standard diet plus supplement B | Standard diet plus supplement C |
|---------------|---------------------------------|---------------------------------|---------------------------------|
| 16            | 24                              | 25                              | 28                              |
| 18            | 20                              | 21                              | 27                              |
| 19            | 29                              | 23                              | 25                              |
| 21            | 25                              | 23                              | 27                              |
| 15            | 27                              | 24                              | 30                              |
| 21            | 26                              | 22                              | 35                              |
| 20            | 23                              | 26                              | 30                              |
| 17            | 21                              | 20                              | 32                              |

- (i) Carryout an exhaustive analysis of this data and write a report on your findings for the farmer  
(15 marks)

**Question 2**

(a) Use the Fisher LSD method to make comparisons between the pairs of means. What conclusions can you draw?

(7 marks)

(b) Suppose that the standard diet is a control, use  $\alpha = 0.05$  to compare all the other means with the control.

(8 marks)

**Question 3**

Suppose that a manufacturer of electronic golf carts is interested in the resistance to wear of four brands of ball bearings. He decides to test the bearings under actual conditions of use, rather than in laboratory simulation. Four golf carts are used in the experiment. Wear is measured by decrease in weight of the ball bearing after 50 hours of use. One ball bearing is required for each wheel of a golf cart; 16 ball bearings (4 of each brand) are used in the experiment. Suppose that the data were obtained using the Latin Square Design with rows representing wheel positions (in the order right front, left front, right rear, left rear), columns representing golf carts and treatments representing brands of ball bearings. Is there any evidence that the brands of ball bearing differ in resistance to wear? Use  $\alpha = 0.05$ .

| Wheel Position | Golf Carts |        |        |        |
|----------------|------------|--------|--------|--------|
|                | I          | II     | III    | IV     |
| 1              | B = 3      | C = 1  | D = -2 | A = 1  |
| 2              | A = 2      | B = -1 | C = -3 | D = -3 |
| 3              | D = 0      | A = 4  | B = -2 | C = -4 |
| 4              | C = 1      | D = -1 | A = 1  | B = -5 |

**(15 marks)****Question 4**

A market analyst is interested in whether average rates of return differ for bonds having different ratings (AA, A, BBB). Rates of return for the preceding year are recorded for random samples of 10 bonds from each rating class, as given below. Is there evidence that bonds having different ratings have different average rates of return?

**(10 marks)**

| Bond Rating |     |     |
|-------------|-----|-----|
| AA          | A   | BBB |
| 6.2         | 5.4 | 5.2 |
| 7.8         | 5.8 | 6.3 |
| 9.5         | 6.9 | 5.9 |
| 8.9         | 7.5 | 7.5 |
| 7.9         | 8.4 | 6.4 |
| 8.7         | 9.1 | 5.8 |
| 7.4         | 7.8 | 7.2 |
| 6.9         | 5.6 | 6.5 |
| 9.1         | 6.5 | 5.6 |
| 9.4         | 6.7 | 5.5 |

b) Use Tukey's test to compare pairs of treatment means. Use  $\alpha = 0.05$ .

**(5 marks)**

**Question 5**

An industrial engineer employed by a beverage bottler is interested in the effects of two different types of bottle (glass and plastic) on the time taken to deliver 12-bottle cases of the product. Two workers are used to perform the task consisting of moving 40 cases of the product 50 feet on a standard type of hand truck. Two replicates of a  $2^2$  factorial design are performed, and the times observed are listed in the following table:

| Bottle Type | Worker |      |
|-------------|--------|------|
|             | 1      | 2    |
| Glass       | 5.12   | 6.65 |
|             | 4.98   | 6.24 |
| Plastic     | 4.95   | 5.28 |
|             | 4.43   | 4.91 |

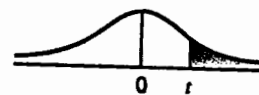
Estimate the effects of bottle type (B) and worker (W) and the interaction (BW) from the following experiment. List the ANOVA table and interpret the results.

**(15 marks)**

**END OF EXAM**

**Table VIII The *t* Distribution Table†**

The entries in the table give the critical values of *t* for the specified number of degrees of freedom and areas in the right tail.



| <i>df</i> | Area in the Right Tail under the <i>t</i> Distribution Curve |       |        |        |        |         |
|-----------|--|-------|--------|--------|--------|---------|
|           | .10  | .05   | .025   | .01    | .005   | .001    |
| 1         | 3.078  | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 |
| 2         | 1.886  | 2.920 | 4.303  | 6.965  | 9.925  | 22.327  |
| 3         | 1.638  | 2.353 | 3.182  | 4.541  | 5.841  | 10.215  |
| 4         | 1.533  | 2.132 | 2.776  | 3.747  | 4.604  | 7.173   |
| 5         | 1.476  | 2.015 | 2.571  | 3.365  | 4.032  | 5.893   |
| 6         | 1.440  | 1.943 | 2.447  | 3.143  | 3.707  | 5.208   |
| 7         | 1.415  | 1.895 | 2.365  | 2.998  | 3.499  | 4.785   |
| 8         | 1.397  | 1.860 | 2.306  | 2.896  | 3.355  | 4.501   |
| 9         | 1.383  | 1.833 | 2.262  | 2.821  | 3.250  | 4.297   |
| 10        | 1.372  | 1.812 | 2.228  | 2.764  | 3.169  | 4.144   |
| 11        | 1.363  | 1.796 | 2.201  | 2.718  | 3.106  | 4.025   |
| 12        | 1.356  | 1.782 | 2.179  | 2.681  | 3.055  | 3.930   |
| 13        | 1.350  | 1.771 | 2.160  | 2.650  | 3.012  | 3.852   |
| 14        | 1.345  | 1.761 | 2.145  | 2.624  | 2.977  | 3.787   |
| 15        | 1.341  | 1.753 | 2.131  | 2.602  | 2.947  | 3.733   |
| 16        | 1.337  | 1.746 | 2.120  | 2.583  | 2.921  | 3.686   |
| 17        | 1.333  | 1.740 | 2.110  | 2.567  | 2.898  | 3.646   |
| 18        | 1.330  | 1.734 | 2.101  | 2.552  | 2.878  | 3.610   |
| 19        | 1.328  | 1.729 | 2.093  | 2.539  | 2.861  | 3.579   |
| 20        | 1.325  | 1.725 | 2.086  | 2.528  | 2.845  | 3.552   |
| 21        | 1.323  | 1.721 | 2.080  | 2.518  | 2.831  | 3.527   |
| 22        | 1.321  | 1.717 | 2.074  | 2.508  | 2.819  | 3.505   |
| 23        | 1.319  | 1.714 | 2.069  | 2.500  | 2.807  | 3.485   |
| 24        | 1.318  | 1.711 | 2.064  | 2.492  | 2.797  | 3.467   |
| 25        | 1.316  | 1.708 | 2.060  | 2.485  | 2.787  | 3.450   |
| 26        | 1.315  | 1.706 | 2.056  | 2.479  | 2.779  | 3.435   |
| 27        | 1.314  | 1.703 | 2.052  | 2.473  | 2.771  | 3.421   |
| 28        | 1.313  | 1.701 | 2.048  | 2.467  | 2.763  | 3.408   |
| 29        | 1.311  | 1.699 | 2.045  | 2.462  | 2.756  | 3.396   |
| 30        | 1.310  | 1.697 | 2.042  | 2.457  | 2.750  | 3.385   |
| 31        | 1.309  | 1.696 | 2.040  | 2.453  | 2.744  | 3.375   |
| 32        | 1.309  | 1.694 | 2.037  | 2.449  | 2.738  | 3.365   |
| 33        | 1.308  | 1.692 | 2.035  | 2.445  | 2.733  | 3.356   |
| 34        | 1.307  | 1.691 | 2.032  | 2.441  | 2.728  | 3.348   |
| 35        | 1.306  | 1.690 | 2.030  | 2.438  | 2.724  | 3.340   |
| 36        | 1.306  | 1.688 | 2.028  | 2.434  | 2.719  | 3.333   |
| 37        | 1.305  | 1.687 | 2.026  | 2.431  | 2.715  | 3.326   |
| 38        | 1.304  | 1.686 | 2.024  | 2.429  | 2.712  | 3.319   |
| 39        | 1.304  | 1.685 | 2.023  | 2.426  | 2.708  | 3.313   |
| 40        | 1.303  | 1.684 | 2.021  | 2.423  | 2.704  | 3.307   |
| ∞         | 1.282  | 1.645 | 1.960  | 2.326  | 2.576  | 3.090   |

†This table is an abbreviated version of Table VIII that appears in Appendix C. This table goes up to 40 degrees of freedom. For degrees of freedom from 41 to 70, use Table VIII of Appendix C.

VII. Percentage Points of the Studentized Range Statistic<sup>a</sup>  
 $q_{\alpha}(p, f)$

| $f$      | $p$  |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |  |
|----------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
|          | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    | 18    | 19    | 20    |  |
| 1        | 90   | 135  | 164  | 186  | 202  | 216  | 227  | 237  | 246   | 253   | 260   | 266   | 272   | 272   | 282   | 286   | 290   | 294   | 298   |  |
| 2        | 14.0 | 19.0 | 22.3 | 24.7 | 26.6 | 28.2 | 29.5 | 30.7 | 31.7  | 32.6  | 33.4  | 34.4  | 34.8  | 35.4  | 36.0  | 36.5  | 37.0  | 37.5  | 37.9  |  |
| 3        | 8.26 | 10.6 | 12.2 | 13.3 | 14.2 | 15.0 | 15.6 | 16.2 | 16.7  | 17.1  | 17.5  | 17.9  | 18.2  | 18.5  | 18.8  | 19.1  | 19.3  | 19.5  | 19.8  |  |
| 4        | 6.51 | 8.12 | 9.17 | 9.96 | 10.6 | 11.1 | 11.5 | 11.9 | 12.3  | 12.6  | 12.8  | 13.1  | 13.3  | 13.5  | 13.7  | 13.9  | 14.1  | 14.2  | 14.4  |  |
| 5        | 5.70 | 6.97 | 7.80 | 8.42 | 8.91 | 9.32 | 9.67 | 9.97 | 10.24 | 10.48 | 10.70 | 10.89 | 11.08 | 11.24 | 11.40 | 11.55 | 11.68 | 11.81 | 11.93 |  |
| 6        | 5.24 | 6.33 | 7.03 | 7.56 | 7.97 | 8.32 | 8.61 | 8.87 | 9.10  | 9.30  | 9.49  | 9.65  | 9.81  | 9.95  | 10.08 | 10.21 | 10.32 | 10.43 | 10.54 |  |
| 7        | 4.95 | 5.92 | 6.54 | 7.01 | 7.37 | 7.68 | 7.94 | 8.17 | 8.37  | 8.55  | 8.71  | 8.86  | 9.00  | 9.12  | 9.24  | 9.35  | 9.46  | 9.55  | 9.65  |  |
| 8        | 4.74 | 5.63 | 6.20 | 6.63 | 6.96 | 7.24 | 7.47 | 7.68 | 7.87  | 8.03  | 8.18  | 8.31  | 8.44  | 8.55  | 8.66  | 8.76  | 8.85  | 8.94  | 9.03  |  |
| 9        | 4.60 | 5.43 | 5.96 | 6.35 | 6.66 | 6.91 | 7.13 | 7.32 | 7.49  | 7.65  | 7.78  | 7.91  | 8.03  | 8.13  | 8.23  | 8.32  | 8.41  | 8.49  | 8.57  |  |
| 10       | 4.48 | 5.27 | 5.77 | 6.14 | 6.43 | 6.67 | 6.87 | 7.05 | 7.21  | 7.36  | 7.48  | 7.60  | 7.71  | 7.81  | 7.91  | 7.99  | 8.07  | 8.15  | 8.22  |  |
| 11       | 4.39 | 5.14 | 5.62 | 5.97 | 6.25 | 6.48 | 6.67 | 6.84 | 6.99  | 7.13  | 7.25  | 7.36  | 7.46  | 7.56  | 7.65  | 7.73  | 7.81  | 7.88  | 7.95  |  |
| 12       | 4.32 | 5.04 | 5.50 | 5.84 | 6.10 | 6.32 | 6.51 | 6.67 | 6.81  | 6.94  | 7.06  | 7.17  | 7.26  | 7.36  | 7.44  | 7.52  | 7.59  | 7.66  | 7.73  |  |
| 13       | 4.26 | 4.96 | 5.40 | 5.73 | 5.98 | 6.19 | 6.37 | 6.53 | 6.67  | 6.79  | 6.90  | 7.01  | 7.10  | 7.19  | 7.27  | 7.34  | 7.42  | 7.48  | 7.55  |  |
| 14       | 4.21 | 4.89 | 5.32 | 5.63 | 5.88 | 6.08 | 6.26 | 6.41 | 6.54  | 6.66  | 6.77  | 6.87  | 6.96  | 7.05  | 7.12  | 7.20  | 7.27  | 7.33  | 7.39  |  |
| 15       | 4.17 | 4.83 | 5.25 | 5.56 | 5.80 | 5.99 | 6.16 | 6.31 | 6.44  | 6.55  | 6.66  | 6.76  | 6.84  | 6.93  | 7.00  | 7.07  | 7.14  | 7.20  | 7.26  |  |
| 16       | 4.13 | 4.78 | 5.19 | 5.49 | 5.72 | 5.92 | 6.08 | 6.22 | 6.35  | 6.46  | 6.56  | 6.66  | 6.74  | 6.82  | 6.90  | 6.97  | 7.03  | 7.09  | 7.15  |  |
| 17       | 4.10 | 4.74 | 5.14 | 5.43 | 5.66 | 5.85 | 6.01 | 6.15 | 6.27  | 6.38  | 6.48  | 6.57  | 6.66  | 6.73  | 6.80  | 6.87  | 6.94  | 7.00  | 7.05  |  |
| 18       | 4.07 | 4.70 | 5.09 | 5.38 | 5.60 | 5.79 | 5.94 | 6.08 | 6.20  | 6.31  | 6.41  | 6.50  | 6.58  | 6.65  | 6.72  | 6.79  | 6.85  | 6.91  | 6.96  |  |
| 19       | 4.05 | 4.67 | 5.05 | 5.33 | 5.55 | 5.73 | 5.89 | 6.02 | 6.14  | 6.25  | 6.34  | 6.43  | 6.51  | 6.58  | 6.65  | 6.72  | 6.78  | 6.84  | 6.89  |  |
| 20       | 4.02 | 4.64 | 5.02 | 5.29 | 5.51 | 5.69 | 5.84 | 5.97 | 6.09  | 6.19  | 6.29  | 6.37  | 6.45  | 6.52  | 6.59  | 6.65  | 6.71  | 6.76  | 6.82  |  |
| 24       | 3.96 | 4.54 | 4.91 | 5.17 | 5.37 | 5.54 | 5.69 | 5.81 | 5.92  | 6.02  | 6.11  | 6.19  | 6.26  | 6.33  | 6.39  | 6.45  | 6.51  | 6.56  | 6.61  |  |
| 30       | 3.89 | 4.45 | 4.80 | 5.05 | 5.24 | 5.40 | 5.54 | 5.65 | 5.76  | 5.85  | 5.93  | 6.01  | 6.08  | 6.14  | 6.20  | 6.26  | 6.31  | 6.36  | 6.41  |  |
| 40       | 3.82 | 4.37 | 4.70 | 4.93 | 5.11 | 5.27 | 5.39 | 5.50 | 5.60  | 5.69  | 5.77  | 5.84  | 5.90  | 5.96  | 6.02  | 6.07  | 6.12  | 6.17  | 6.21  |  |
| 60       | 3.76 | 4.28 | 4.60 | 4.82 | 4.99 | 5.13 | 5.25 | 5.36 | 5.45  | 5.53  | 5.60  | 5.67  | 5.73  | 5.79  | 5.84  | 5.89  | 5.93  | 5.98  | 6.02  |  |
| 120      | 3.70 | 4.20 | 4.50 | 4.71 | 4.87 | 5.01 | 5.12 | 5.21 | 5.30  | 5.38  | 5.44  | 5.51  | 5.56  | 5.61  | 5.66  | 5.71  | 5.75  | 5.79  | 5.83  |  |
| $\infty$ | 3.64 | 4.12 | 4.40 | 4.60 | 4.76 | 4.88 | 4.99 | 5.08 | 5.16  | 5.23  | 5.29  | 5.35  | 5.40  | 5.45  | 5.49  | 5.54  | 5.57  | 5.61  | 5.65  |  |

<sup>a</sup> $f$  = degrees of freedom.  
 From J. M. May, "Extended and Corrected Tables of the Upper Percentage Points of the Studentized Range," *Blawertika*, Vol. 39, pp. 192-193, 1952. Reproduced by permission of the trustees of *Blawertika*.

VII. Percentage Points of the Studentized Range Statistic (continued)

| $f$      | $p$  |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |  |
|----------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
|          | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    | 17    | 18    | 19    | 20    |  |
| 1        | 18.1 | 26.7 | 32.8 | 37.2  | 40.5  | 43.1  | 45.4  | 47.3  | 49.1  | 50.6  | 51.9  | 53.2  | 54.3  | 55.4  | 56.3  | 57.2  | 58.0  | 58.8  | 59.6  |  |
| 2        | 6.09 | 8.28 | 9.80 | 10.89 | 11.73 | 12.43 | 13.03 | 13.54 | 13.99 | 14.39 | 14.75 | 15.08 | 15.38 | 15.65 | 15.91 | 16.14 | 16.36 | 16.57 | 16.77 |  |
| 3        | 4.50 | 5.88 | 6.83 | 7.51  | 8.04  | 8.47  | 8.85  | 9.18  | 9.46  | 9.72  | 9.95  | 10.16 | 10.35 | 10.52 | 10.69 | 10.84 | 10.98 | 11.12 | 11.24 |  |
| 4        | 3.93 | 5.00 | 5.76 | 6.31  | 6.73  | 7.06  | 7.35  | 7.60  | 7.83  | 8.03  | 8.21  | 8.37  | 8.52  | 8.67  | 8.80  | 8.92  | 9.03  | 9.14  | 9.24  |  |
| 5        | 3.64 | 4.60 | 5.22 | 5.67  | 6.03  | 6.33  | 6.58  | 6.80  | 6.99  | 7.17  | 7.32  | 7.47  | 7.60  | 7.72  | 7.83  | 7.93  | 8.03  | 8.12  | 8.21  |  |
| 6        | 3.46 | 4.34 | 4.90 | 5.31  | 5.63  | 5.89  | 6.12  | 6.32  | 6.49  | 6.65  | 6.79  | 6.92  | 7.04  | 7.14  | 7.24  | 7.34  | 7.43  | 7.51  | 7.59  |  |
| 7        | 3.34 | 4.16 | 4.68 | 5.06  | 5.35  | 5.59  | 5.80  | 5.99  | 6.15  | 6.29  | 6.42  | 6.54  | 6.65  | 6.75  | 6.84  | 6.93  | 7.01  | 7.08  | 7.16  |  |
| 8        | 3.26 | 4.04 | 4.53 | 4.89  | 5.17  | 5.40  | 5.60  | 5.77  | 5.92  | 6.05  | 6.18  | 6.29  | 6.39  | 6.48  | 6.57  | 6.65  | 6.73  | 6.80  | 6.87  |  |
| 9        | 3.20 | 3.95 | 4.42 | 4.76  | 5.02  | 5.24  | 5.43  | 5.60  | 5.74  | 5.87  | 5.98  | 6.09  | 6.19  | 6.28  | 6.36  | 6.44  | 6.51  | 6.58  | 6.65  |  |
| 10       | 3.15 | 3.88 | 4.33 | 4.66  | 4.91  | 5.12  | 5.30  | 5.46  | 5.60  | 5.72  | 5.83  | 5.93  | 6.03  | 6.12  | 6.20  | 6.27  | 6.34  | 6.41  | 6.47  |  |
| 11       | 3.11 | 3.82 | 4.26 | 4.58  | 4.82  | 5.03  | 5.20  | 5.35  | 5.49  | 5.61  | 5.71  | 5.81  | 5.90  | 5.98  | 6.06  | 6.14  | 6.20  | 6.27  | 6.33  |  |
| 12       | 3.08 | 3.77 | 4.20 | 4.51  | 4.75  | 4.95  | 5.12  | 5.27  | 5.40  | 5.51  | 5.61  | 5.71  | 5.80  | 5.88  | 5.95  | 6.02  | 6.09  | 6.15  | 6.21  |  |
| 13       | 3.06 | 3.73 | 4.15 | 4.46  | 4.69  | 4.88  | 5.05  | 5.19  | 5.32  | 5.43  | 5.53  | 5.63  | 5.71  | 5.79  | 5.86  | 5.93  | 6.00  | 6.06  | 6.11  |  |
| 14       | 3.03 | 3.70 | 4.11 | 4.41  | 4.64  | 4.83  | 4.99  | 5.13  | 5.25  | 5.36  | 5.46  | 5.56  | 5.64  | 5.72  | 5.79  | 5.86  | 5.92  | 5.98  | 6.03  |  |
| 15       | 3.01 | 3.67 | 4.08 | 4.37  | 4.59  | 4.78  | 4.94  | 5.08  | 5.20  | 5.31  | 5.40  | 5.49  | 5.57  | 5.65  | 5.72  | 5.79  | 5.85  | 5.91  | 5.96  |  |
| 16       | 3.00 | 3.65 | 4.05 | 4.34  | 4.56  | 4.74  | 4.90  | 5.03  | 5.15  | 5.26  | 5.35  | 5.44  | 5.52  | 5.59  | 5.66  | 5.73  | 5.79  | 5.84  | 5.90  |  |
| 17       | 2.98 | 3.62 | 4.02 | 4.31  | 4.52  | 4.70  | 4.86  | 4.99  | 5.11  | 5.21  | 5.31  | 5.39  | 5.47  | 5.55  | 5.61  | 5.68  | 5.74  | 5.79  | 5.84  |  |
| 18       | 2.97 | 3.61 | 4.00 | 4.28  | 4.49  | 4.67  | 4.83  | 4.96  | 5.07  | 5.17  | 5.27  | 5.35  | 5.43  | 5.50  | 5.57  | 5.63  | 5.69  | 5.74  | 5.79  |  |
| 19       | 2.96 | 3.59 | 3.98 | 4.26  | 4.47  | 4.64  | 4.79  | 4.92  | 5.04  | 5.14  | 5.23  | 5.32  | 5.39  | 5.46  | 5.53  | 5.59  | 5.65  | 5.70  | 5.75  |  |
| 20       | 2.95 | 3.58 | 3.96 | 4.24  | 4.45  | 4.62  | 4.77  | 4.90  | 5.01  | 5.11  | 5.20  | 5.28  | 5.36  | 5.43  | 5.50  | 5.56  | 5.61  | 5.66  | 5.71  |  |
| 24       | 2.92 | 3.53 | 3.90 | 4.17  | 4.37  | 4.54  | 4.68  | 4.81  | 4.92  | 5.01  | 5.10  | 5.18  | 5.25  | 5.32  | 5.38  | 5.44  | 5.50  | 5.55  | 5.59  |  |
| 30       | 2.89 | 3.48 | 3.84 | 4.11  | 4.30  | 4.46  | 4.60  | 4.72  | 4.83  | 4.92  | 5.00  | 5.08  | 5.15  | 5.21  | 5.27  | 5.33  | 5.38  | 5.43  | 5.48  |  |
| 40       | 2.86 | 3.44 | 3.79 | 4.04  | 4.23  | 4.39  | 4.52  | 4.63  | 4.74  | 4.82  | 4.90  | 4.98  | 5.05  | 5.11  | 5.17  | 5.22  | 5.27  | 5.32  | 5.36  |  |
| 60       | 2.83 | 3.40 | 3.74 | 3.98  | 4.16  | 4.31  | 4.44  | 4.55  | 4.65  | 4.73  | 4.81  | 4.88  | 4.94  | 5.00  | 5.06  | 5.11  | 5.15  | 5.20  | 5.24  |  |
| 120      | 2.80 | 3.36 | 3.69 | 3.92  | 4.10  | 4.24  | 4.36  | 4.47  | 4.56  | 4.64  | 4.71  | 4.78  | 4.84  | 4.90  | 4.95  | 5.00  | 5.04  | 5.09  | 5.13  |  |
| $\infty$ | 2.77 | 3.32 | 3.63 | 3.86  | 4.03  | 4.17  | 4.29  | 4.39  | 4.47  | 4.55  | 4.62  | 4.68  | 4.74  | 4.80  | 4.84  | 4.98  | 4.93  | 4.97  | 5.01  |  |

IV. Percentage Points of the F Distribution (continued)

$F_{0.05, p_1, p_2}$

| $p_1$    | Degrees of Freedom for the Numerator ( $p_2$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |
|----------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|          | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |
| 2        | 161.4  | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3    |
| 3        | 18.51  | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50    |
| 4        | 10.13  | 9.55  | 9.28  | 9.12  | 9.01  | 8.94  | 8.89  | 8.85  | 8.81  | 8.79  | 8.74  | 8.70  | 8.66  | 8.64  | 8.62  | 8.59  | 8.57  | 8.55  | 8.53     |
| 5        | 7.71   | 6.94  | 6.59  | 6.39  | 6.26  | 6.16  | 6.09  | 6.04  | 6.00  | 5.96  | 5.91  | 5.86  | 5.80  | 5.77  | 5.75  | 5.72  | 5.69  | 5.66  | 5.63     |
| 6        | 6.61   | 5.79  | 5.41  | 5.19  | 5.05  | 4.95  | 4.88  | 4.82  | 4.77  | 4.74  | 4.68  | 4.62  | 4.56  | 4.53  | 4.50  | 4.46  | 4.43  | 4.40  | 4.36     |
| 7        | 5.99   | 5.14  | 4.76  | 4.53  | 4.39  | 4.28  | 4.21  | 4.15  | 4.10  | 4.06  | 4.00  | 3.94  | 3.87  | 3.84  | 3.81  | 3.77  | 3.74  | 3.70  | 3.67     |
| 8        | 5.59   | 4.74  | 4.35  | 4.12  | 3.97  | 3.87  | 3.79  | 3.73  | 3.68  | 3.64  | 3.57  | 3.51  | 3.44  | 3.41  | 3.38  | 3.34  | 3.30  | 3.27  | 3.23     |
| 9        | 5.32   | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  | 3.35  | 3.28  | 3.22  | 3.15  | 3.12  | 3.08  | 3.04  | 3.01  | 2.97  | 2.93     |
| 10       | 5.12   | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  | 3.14  | 3.07  | 3.01  | 2.94  | 2.90  | 2.86  | 2.83  | 2.79  | 2.75  | 2.71     |
| 11       | 4.96   | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  | 2.98  | 2.91  | 2.85  | 2.77  | 2.74  | 2.70  | 2.66  | 2.62  | 2.58  | 2.54     |
| 12       | 4.84   | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  | 2.85  | 2.79  | 2.72  | 2.65  | 2.61  | 2.57  | 2.53  | 2.49  | 2.45  | 2.40     |
| 13       | 4.75   | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  | 2.75  | 2.69  | 2.62  | 2.54  | 2.51  | 2.47  | 2.43  | 2.38  | 2.34  | 2.30     |
| 14       | 4.67   | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  | 2.67  | 2.60  | 2.53  | 2.46  | 2.42  | 2.38  | 2.34  | 2.30  | 2.25  | 2.21     |
| 15       | 4.60   | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  | 2.60  | 2.53  | 2.46  | 2.39  | 2.35  | 2.31  | 2.27  | 2.22  | 2.18  | 2.13     |
| 16       | 4.54   | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  | 2.54  | 2.48  | 2.40  | 2.33  | 2.29  | 2.25  | 2.20  | 2.16  | 2.11  | 2.07     |
| 17       | 4.49   | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  | 2.49  | 2.42  | 2.35  | 2.28  | 2.24  | 2.20  | 2.15  | 2.11  | 2.06  | 2.01     |
| 18       | 4.45   | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  | 2.45  | 2.38  | 2.31  | 2.23  | 2.19  | 2.15  | 2.10  | 2.06  | 2.01  | 1.96     |
| 19       | 4.41   | 3.55  | 3.16  | 2.93  | 2.77  | 2.66  | 2.58  | 2.51  | 2.46  | 2.41  | 2.34  | 2.27  | 2.19  | 2.15  | 2.11  | 2.06  | 2.02  | 1.97  | 1.92     |
| 20       | 4.38   | 3.52  | 3.13  | 2.90  | 2.74  | 2.63  | 2.54  | 2.48  | 2.42  | 2.38  | 2.31  | 2.23  | 2.16  | 2.11  | 2.07  | 2.03  | 1.98  | 1.93  | 1.88     |
| 21       | 4.35   | 3.49  | 3.10  | 2.87  | 2.71  | 2.60  | 2.51  | 2.45  | 2.39  | 2.35  | 2.28  | 2.20  | 2.12  | 2.08  | 2.04  | 1.99  | 1.95  | 1.90  | 1.84     |
| 22       | 4.32   | 3.47  | 3.07  | 2.84  | 2.68  | 2.57  | 2.49  | 2.42  | 2.37  | 2.32  | 2.25  | 2.18  | 2.10  | 2.05  | 2.01  | 1.96  | 1.92  | 1.87  | 1.81     |
| 23       | 4.30   | 3.44  | 3.05  | 2.82  | 2.66  | 2.55  | 2.46  | 2.40  | 2.34  | 2.30  | 2.23  | 2.15  | 2.07  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.78     |
| 24       | 4.28   | 3.42  | 3.03  | 2.80  | 2.64  | 2.53  | 2.44  | 2.37  | 2.32  | 2.27  | 2.20  | 2.13  | 2.05  | 2.01  | 1.96  | 1.91  | 1.86  | 1.81  | 1.76     |
| 25       | 4.26   | 3.40  | 3.01  | 2.78  | 2.62  | 2.51  | 2.42  | 2.36  | 2.30  | 2.25  | 2.18  | 2.11  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.79  | 1.73     |
| 26       | 4.24   | 3.39  | 2.99  | 2.76  | 2.60  | 2.49  | 2.40  | 2.34  | 2.28  | 2.24  | 2.16  | 2.09  | 2.01  | 1.96  | 1.92  | 1.87  | 1.82  | 1.77  | 1.71     |
| 27       | 4.23   | 3.37  | 2.98  | 2.74  | 2.59  | 2.47  | 2.39  | 2.32  | 2.27  | 2.22  | 2.15  | 2.07  | 1.99  | 1.95  | 1.90  | 1.85  | 1.80  | 1.75  | 1.69     |
| 28       | 4.21   | 3.35  | 2.96  | 2.73  | 2.57  | 2.46  | 2.37  | 2.31  | 2.25  | 2.20  | 2.13  | 2.06  | 1.97  | 1.93  | 1.88  | 1.84  | 1.79  | 1.73  | 1.67     |
| 29       | 4.20   | 3.34  | 2.95  | 2.71  | 2.56  | 2.45  | 2.36  | 2.29  | 2.24  | 2.19  | 2.12  | 2.04  | 1.96  | 1.91  | 1.87  | 1.82  | 1.77  | 1.71  | 1.65     |
| 30       | 4.18   | 3.33  | 2.93  | 2.70  | 2.55  | 2.43  | 2.35  | 2.28  | 2.22  | 2.18  | 2.10  | 2.03  | 1.94  | 1.90  | 1.85  | 1.81  | 1.75  | 1.70  | 1.64     |
| 40       | 4.17   | 3.32  | 2.92  | 2.69  | 2.53  | 2.42  | 2.33  | 2.27  | 2.21  | 2.16  | 2.09  | 2.01  | 1.93  | 1.89  | 1.84  | 1.79  | 1.74  | 1.68  | 1.62     |
| 60       | 4.08   | 3.23  | 2.84  | 2.61  | 2.45  | 2.34  | 2.25  | 2.18  | 2.12  | 2.08  | 2.00  | 1.92  | 1.84  | 1.79  | 1.74  | 1.69  | 1.64  | 1.58  | 1.51     |
| 120      | 4.00   | 3.15  | 2.76  | 2.53  | 2.37  | 2.25  | 2.17  | 2.10  | 2.04  | 1.99  | 1.92  | 1.84  | 1.75  | 1.70  | 1.65  | 1.59  | 1.53  | 1.47  | 1.39     |
| $\infty$ | 3.92   | 3.07  | 2.68  | 2.45  | 2.29  | 2.17  | 2.09  | 2.02  | 1.96  | 1.91  | 1.83  | 1.75  | 1.66  | 1.61  | 1.55  | 1.55  | 1.43  | 1.35  | 1.25     |
|          | 3.84   | 3.00  | 2.60  | 2.37  | 2.21  | 2.10  | 2.01  | 1.94  | 1.88  | 1.83  | 1.75  | 1.67  | 1.57  | 1.52  | 1.46  | 1.39  | 1.32  | 1.22  | 1.00     |

Degrees of Freedom for the Denominator ( $p_2$ )



IV. Percentage Points of the F Distribution (continued)

| p <sub>1</sub> | Degrees of Freedom for the Numerator (p <sub>1</sub> ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |  | ∞ |
|----------------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|---|
|                | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | ∞     |  |   |
| 1              | 39.86  | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 60.71 | 61.22 | 61.74 | 62.00 | 62.26 | 62.53 | 62.79 | 63.06 | 63.33 |  |   |
| 2              | 8.53   | 9.00  | 9.16  | 9.24  | 9.29  | 9.33  | 9.35  | 9.37  | 9.38  | 9.39  | 9.41  | 9.42  | 9.44  | 9.45  | 9.46  | 9.47  | 9.47  | 9.48  | 9.49  |  |   |
| 3              | 5.54   | 5.46  | 5.39  | 5.34  | 5.31  | 5.28  | 5.27  | 5.25  | 5.24  | 5.23  | 5.22  | 5.20  | 5.18  | 5.18  | 5.17  | 5.16  | 5.15  | 5.14  | 5.13  |  |   |
| 4              | 4.54   | 4.32  | 4.19  | 4.11  | 4.05  | 4.01  | 3.98  | 3.95  | 3.94  | 3.92  | 3.90  | 3.87  | 3.84  | 3.83  | 3.82  | 3.80  | 3.79  | 3.78  | 3.76  |  |   |
| 5              | 4.06   | 3.78  | 3.62  | 3.52  | 3.45  | 3.40  | 3.37  | 3.34  | 3.32  | 3.30  | 3.27  | 3.24  | 3.21  | 3.19  | 3.17  | 3.16  | 3.14  | 3.12  | 3.10  |  |   |
| 6              | 3.78   | 3.46  | 3.29  | 3.18  | 3.11  | 3.05  | 3.01  | 2.98  | 2.96  | 2.94  | 2.90  | 2.87  | 2.84  | 2.82  | 2.80  | 2.78  | 2.76  | 2.74  | 2.72  |  |   |
| 7              | 3.59   | 3.26  | 3.07  | 2.96  | 2.88  | 2.83  | 2.78  | 2.75  | 2.72  | 2.70  | 2.67  | 2.63  | 2.59  | 2.58  | 2.56  | 2.54  | 2.51  | 2.49  | 2.47  |  |   |
| 8              | 3.46   | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  | 2.54  | 2.50  | 2.46  | 2.42  | 2.40  | 2.38  | 2.36  | 2.34  | 2.32  | 2.29  |  |   |
| 9              | 3.36   | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  | 2.42  | 2.38  | 2.34  | 2.30  | 2.28  | 2.25  | 2.23  | 2.21  | 2.18  | 2.16  |  |   |
| 10             | 3.29   | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  | 2.32  | 2.28  | 2.24  | 2.20  | 2.18  | 2.16  | 2.13  | 2.11  | 2.08  | 2.06  |  |   |
| 11             | 3.23   | 2.86  | 2.66  | 2.54  | 2.45  | 2.39  | 2.34  | 2.30  | 2.27  | 2.25  | 2.21  | 2.17  | 2.12  | 2.10  | 2.08  | 2.05  | 2.03  | 2.00  | 1.97  |  |   |
| 12             | 3.18   | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  | 2.19  | 2.15  | 2.10  | 2.06  | 2.04  | 2.01  | 1.99  | 1.96  | 1.93  | 1.90  |  |   |
| 13             | 3.14   | 2.76  | 2.56  | 2.43  | 2.35  | 2.28  | 2.23  | 2.20  | 2.16  | 2.14  | 2.10  | 2.05  | 2.01  | 1.98  | 1.96  | 1.93  | 1.90  | 1.88  | 1.85  |  |   |
| 14             | 3.10   | 2.73  | 2.52  | 2.39  | 2.31  | 2.24  | 2.19  | 2.15  | 2.12  | 2.10  | 2.05  | 2.01  | 1.96  | 1.94  | 1.91  | 1.89  | 1.86  | 1.83  | 1.80  |  |   |
| 15             | 3.07   | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  | 2.06  | 2.02  | 1.97  | 1.92  | 1.90  | 1.87  | 1.85  | 1.82  | 1.79  | 1.76  |  |   |
| 16             | 3.05   | 2.67  | 2.46  | 2.33  | 2.24  | 2.18  | 2.13  | 2.09  | 2.06  | 2.03  | 1.99  | 1.94  | 1.89  | 1.87  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72  |  |   |
| 17             | 3.03   | 2.64  | 2.44  | 2.31  | 2.22  | 2.15  | 2.10  | 2.06  | 2.03  | 2.00  | 1.96  | 1.91  | 1.86  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72  | 1.69  |  |   |
| 18             | 3.01   | 2.62  | 2.42  | 2.29  | 2.20  | 2.13  | 2.08  | 2.04  | 2.00  | 1.98  | 1.93  | 1.89  | 1.84  | 1.81  | 1.78  | 1.75  | 1.72  | 1.69  | 1.66  |  |   |
| 19             | 2.99   | 2.61  | 2.40  | 2.27  | 2.18  | 2.11  | 2.06  | 2.02  | 1.98  | 1.96  | 1.91  | 1.86  | 1.81  | 1.79  | 1.76  | 1.73  | 1.70  | 1.67  | 1.63  |  |   |
| 20             | 2.97   | 2.59  | 2.38  | 2.25  | 2.16  | 2.09  | 2.04  | 2.00  | 1.96  | 1.94  | 1.89  | 1.84  | 1.79  | 1.77  | 1.74  | 1.71  | 1.68  | 1.64  | 1.61  |  |   |
| 21             | 2.96   | 2.57  | 2.36  | 2.23  | 2.14  | 2.08  | 2.02  | 1.98  | 1.95  | 1.92  | 1.87  | 1.83  | 1.78  | 1.75  | 1.72  | 1.69  | 1.66  | 1.62  | 1.59  |  |   |
| 22             | 2.95   | 2.56  | 2.35  | 2.22  | 2.13  | 2.06  | 2.01  | 1.97  | 1.93  | 1.90  | 1.86  | 1.81  | 1.76  | 1.73  | 1.70  | 1.67  | 1.64  | 1.60  | 1.57  |  |   |
| 23             | 2.94   | 2.55  | 2.34  | 2.21  | 2.11  | 2.05  | 1.99  | 1.96  | 1.92  | 1.89  | 1.84  | 1.80  | 1.74  | 1.72  | 1.69  | 1.66  | 1.62  | 1.59  | 1.55  |  |   |
| 24             | 2.93   | 2.54  | 2.33  | 2.20  | 2.10  | 2.04  | 1.98  | 1.94  | 1.91  | 1.88  | 1.83  | 1.78  | 1.73  | 1.70  | 1.67  | 1.64  | 1.61  | 1.57  | 1.53  |  |   |
| 25             | 2.92   | 2.53  | 2.32  | 2.18  | 2.09  | 2.02  | 1.97  | 1.93  | 1.89  | 1.87  | 1.82  | 1.77  | 1.72  | 1.69  | 1.66  | 1.63  | 1.59  | 1.56  | 1.52  |  |   |
| 26             | 2.91   | 2.52  | 2.31  | 2.17  | 2.08  | 2.01  | 1.96  | 1.92  | 1.88  | 1.86  | 1.81  | 1.76  | 1.71  | 1.68  | 1.65  | 1.61  | 1.58  | 1.54  | 1.50  |  |   |
| 27             | 2.90   | 2.51  | 2.30  | 2.17  | 2.07  | 2.00  | 1.95  | 1.91  | 1.87  | 1.85  | 1.80  | 1.75  | 1.70  | 1.67  | 1.64  | 1.60  | 1.57  | 1.53  | 1.49  |  |   |
| 28             | 2.89   | 2.50  | 2.29  | 2.16  | 2.06  | 2.00  | 1.94  | 1.90  | 1.87  | 1.84  | 1.79  | 1.74  | 1.69  | 1.66  | 1.63  | 1.59  | 1.56  | 1.52  | 1.48  |  |   |
| 29             | 2.89   | 2.50  | 2.28  | 2.15  | 2.06  | 1.99  | 1.93  | 1.89  | 1.86  | 1.83  | 1.78  | 1.73  | 1.68  | 1.65  | 1.62  | 1.58  | 1.55  | 1.51  | 1.47  |  |   |
| 30             | 2.88   | 2.49  | 2.28  | 2.14  | 2.03  | 1.98  | 1.93  | 1.88  | 1.85  | 1.82  | 1.77  | 1.72  | 1.67  | 1.64  | 1.61  | 1.57  | 1.54  | 1.50  | 1.46  |  |   |
| 40             | 2.84   | 2.44  | 2.23  | 2.09  | 2.00  | 1.93  | 1.87  | 1.83  | 1.79  | 1.76  | 1.71  | 1.66  | 1.61  | 1.57  | 1.54  | 1.51  | 1.47  | 1.42  | 1.38  |  |   |
| 60             | 2.79   | 2.39  | 2.18  | 2.04  | 1.95  | 1.87  | 1.82  | 1.77  | 1.74  | 1.71  | 1.66  | 1.60  | 1.54  | 1.51  | 1.48  | 1.44  | 1.40  | 1.35  | 1.29  |  |   |
| 120            | 2.75   | 2.35  | 2.13  | 1.99  | 1.90  | 1.82  | 1.77  | 1.72  | 1.68  | 1.65  | 1.60  | 1.55  | 1.48  | 1.45  | 1.41  | 1.37  | 1.32  | 1.26  | 1.19  |  |   |
| ∞              | 2.71   | 2.30  | 2.08  | 1.94  | 1.85  | 1.77  | 1.72  | 1.67  | 1.63  | 1.60  | 1.55  | 1.49  | 1.42  | 1.38  | 1.34  | 1.30  | 1.24  | 1.17  | 1.00  |  |   |

Degrees of Freedom for the Denominator (p<sub>2</sub>)

IV. Percentage Points of the F Distribution\*

$F_{0.25; \nu_1, \nu_2}$

| $\nu_1$  | Degrees of Freedom for the Numerator ( $\nu_1$ ) |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |  | $\infty$ |
|----------|--|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|--|----------|
|          | 1  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 12   | 15   | 20   | 24   | 30   | 40   | 60   | 120  |      |  |          |
| 1        | 5.83   | 7.50 | 8.20 | 8.58 | 8.82 | 8.98 | 9.10 | 9.19 | 9.26 | 9.32 | 9.41 | 9.49 | 9.58 | 9.63 | 9.67 | 9.71 | 9.76 | 9.80 | 9.85 |  |          |
| 2        | 2.57   | 3.00 | 3.15 | 3.23 | 3.28 | 3.31 | 3.34 | 3.35 | 3.37 | 3.38 | 3.39 | 3.41 | 3.43 | 3.43 | 3.44 | 3.45 | 3.46 | 3.47 | 3.48 |  |          |
| 3        | 2.02   | 2.28 | 2.36 | 2.39 | 2.41 | 2.42 | 2.43 | 2.44 | 2.44 | 2.44 | 2.45 | 2.46 | 2.46 | 2.46 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 |  |          |
| 4        | 1.81   | 2.00 | 2.05 | 2.06 | 2.07 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 |  |          |
| 5        | 1.69   | 1.85 | 1.88 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.88 | 1.88 | 1.88 | 1.88 | 1.87 | 1.87 | 1.87 |  |          |
| 6        | 1.62   | 1.76 | 1.78 | 1.79 | 1.79 | 1.78 | 1.78 | 1.78 | 1.77 | 1.77 | 1.77 | 1.76 | 1.76 | 1.75 | 1.75 | 1.75 | 1.74 | 1.74 | 1.74 |  |          |
| 7        | 1.57   | 1.70 | 1.72 | 1.72 | 1.71 | 1.71 | 1.70 | 1.70 | 1.69 | 1.69 | 1.68 | 1.68 | 1.67 | 1.67 | 1.66 | 1.66 | 1.65 | 1.65 | 1.65 |  |          |
| 8        | 1.54   | 1.66 | 1.67 | 1.66 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.62 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.58 |  |          |
| 9        | 1.51   | 1.62 | 1.63 | 1.63 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.54 | 1.54 | 1.53 | 1.53 |  |          |
| 10       | 1.49   | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.52 | 1.51 | 1.51 | 1.50 | 1.49 | 1.48 |  |          |
| 11       | 1.47   | 1.58 | 1.58 | 1.57 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.47 | 1.46 | 1.45 |  |          |
| 12       | 1.46   | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.45 | 1.44 | 1.43 | 1.42 |  |          |
| 13       | 1.45   | 1.55 | 1.55 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.42 | 1.41 | 1.40 |  |          |
| 14       | 1.44   | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.39 | 1.38 |  |          |
| 15       | 1.43   | 1.52 | 1.52 | 1.51 | 1.51 | 1.49 | 1.47 | 1.46 | 1.46 | 1.45 | 1.44 | 1.43 | 1.41 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 |  |          |
| 16       | 1.42   | 1.51 | 1.51 | 1.51 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 |  |          |
| 17       | 1.42   | 1.51 | 1.50 | 1.49 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 |  |          |
| 18       | 1.41   | 1.50 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.42 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 |  |          |
| 19       | 1.41   | 1.49 | 1.49 | 1.47 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.30 |  |          |
| 20       | 1.40   | 1.49 | 1.48 | 1.47 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.29 |  |          |
| 21       | 1.40   | 1.48 | 1.48 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.28 |  |          |
| 22       | 1.40   | 1.48 | 1.47 | 1.45 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 | 1.37 | 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 |  |          |
| 23       | 1.39   | 1.47 | 1.47 | 1.45 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 | 1.30 | 1.28 | 1.27 |  |          |
| 24       | 1.39   | 1.47 | 1.46 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.38 | 1.36 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 |  |          |
| 25       | 1.39   | 1.47 | 1.46 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 |  |          |
| 26       | 1.38   | 1.46 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.37 | 1.35 | 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 | 1.25 |  |          |
| 27       | 1.38   | 1.46 | 1.45 | 1.43 | 1.42 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 | 1.24 |  |          |
| 28       | 1.38   | 1.46 | 1.45 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.34 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 |  |          |
| 29       | 1.38   | 1.45 | 1.45 | 1.43 | 1.41 | 1.40 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.27 | 1.26 | 1.25 | 1.23 |  |          |
| 30       | 1.38   | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.32 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 | 1.24 | 1.23 |  |          |
| 40       | 1.36   | 1.44 | 1.42 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.31 | 1.30 | 1.28 | 1.26 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 |  |          |
| 60       | 1.35   | 1.42 | 1.41 | 1.38 | 1.37 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 | 1.17 | 1.15 |  |          |
| 120      | 1.34   | 1.40 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.26 | 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.16 | 1.13 | 1.10 |  |          |
| $\infty$ | 1.32   | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.16 | 1.14 | 1.12 | 1.08 |  |          |

$\nu_2$  = degrees of freedom

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IV. Percentage Points of the F Distribution (continued)

| $\nu_1$  | Degrees of Freedom for the Numerator ( $\nu_2$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |  |
|----------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|--|
|          | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |  |
| 1        | 647.8  | 799.5 | 864.2 | 899.6 | 921.8 | 937.1 | 948.2 | 956.7 | 963.3 | 968.6 | 976.7 | 984.9 | 993.1 | 997.2 | 1001  | 1006  | 1010  | 1014  | 1018     |  |
| 2        | 38.51  | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 | 39.43 | 39.45 | 39.46 | 39.46 | 39.47 | 39.48 | 39.49 | 39.50    |  |
| 3        | 17.44  | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 | 14.25 | 14.17 | 14.12 | 14.08 | 14.04 | 13.99 | 13.95 | 13.90    |  |
| 4        | 12.22  | 10.65 | 9.98  | 9.60  | 9.36  | 9.20  | 9.07  | 8.98  | 8.90  | 8.84  | 8.75  | 8.66  | 8.56  | 8.51  | 8.46  | 8.41  | 8.36  | 8.31  | 8.26     |  |
| 5        | 10.01  | 8.43  | 7.76  | 7.39  | 7.15  | 6.98  | 6.85  | 6.76  | 6.68  | 6.62  | 6.52  | 6.43  | 6.33  | 6.28  | 6.23  | 6.18  | 6.12  | 6.07  | 6.02     |  |
| 6        | 8.81   | 7.26  | 6.60  | 6.23  | 5.99  | 5.82  | 5.70  | 5.60  | 5.52  | 5.46  | 5.37  | 5.27  | 5.17  | 5.12  | 5.07  | 5.01  | 4.96  | 4.90  | 4.85     |  |
| 7        | 8.07   | 6.54  | 5.89  | 5.52  | 5.29  | 5.12  | 4.99  | 4.90  | 4.82  | 4.76  | 4.67  | 4.57  | 4.47  | 4.42  | 4.36  | 4.31  | 4.25  | 4.20  | 4.14     |  |
| 8        | 7.57   | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  | 4.30  | 4.20  | 4.10  | 4.00  | 3.95  | 3.89  | 3.84  | 3.78  | 3.73  | 3.67     |  |
| 9        | 7.21   | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  | 3.96  | 3.87  | 3.77  | 3.67  | 3.61  | 3.56  | 3.51  | 3.45  | 3.39  | 3.33     |  |
| 10       | 6.94   | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  | 3.72  | 3.62  | 3.52  | 3.42  | 3.37  | 3.31  | 3.26  | 3.20  | 3.14  | 3.08     |  |
| 11       | 6.72   | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  | 3.53  | 3.43  | 3.33  | 3.23  | 3.17  | 3.12  | 3.06  | 3.00  | 2.94  | 2.88     |  |
| 12       | 6.55   | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  | 3.37  | 3.28  | 3.18  | 3.07  | 3.02  | 2.96  | 2.91  | 2.85  | 2.79  | 2.72     |  |
| 13       | 6.41   | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  | 3.25  | 3.15  | 3.05  | 2.95  | 2.89  | 2.84  | 2.78  | 2.72  | 2.66  | 2.60     |  |
| 14       | 6.30   | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  | 3.15  | 3.05  | 2.95  | 2.84  | 2.79  | 2.73  | 2.67  | 2.61  | 2.55  | 2.49     |  |
| 15       | 6.20   | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  | 3.06  | 2.96  | 2.86  | 2.76  | 2.70  | 2.64  | 2.59  | 2.52  | 2.46  | 2.40     |  |
| 16       | 6.12   | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  | 2.99  | 2.89  | 2.79  | 2.68  | 2.63  | 2.57  | 2.51  | 2.45  | 2.38  | 2.32     |  |
| 17       | 6.04   | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  | 2.92  | 2.82  | 2.72  | 2.62  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.25     |  |
| 18       | 5.98   | 4.56  | 3.95  | 3.61  | 3.38  | 3.22  | 3.10  | 3.01  | 2.93  | 2.87  | 2.77  | 2.67  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.26  | 2.19     |  |
| 19       | 5.92   | 4.51  | 3.90  | 3.56  | 3.33  | 3.17  | 3.05  | 2.96  | 2.88  | 2.82  | 2.72  | 2.62  | 2.51  | 2.45  | 2.39  | 2.33  | 2.27  | 2.20  | 2.13     |  |
| 20       | 5.87   | 4.46  | 3.86  | 3.51  | 3.29  | 3.13  | 3.01  | 2.91  | 2.84  | 2.77  | 2.68  | 2.57  | 2.46  | 2.41  | 2.35  | 2.29  | 2.22  | 2.16  | 2.09     |  |
| 21       | 5.83   | 4.42  | 3.82  | 3.48  | 3.25  | 3.09  | 2.97  | 2.87  | 2.80  | 2.73  | 2.64  | 2.53  | 2.42  | 2.37  | 2.31  | 2.25  | 2.18  | 2.11  | 2.04     |  |
| 22       | 5.79   | 4.38  | 3.78  | 3.44  | 3.22  | 3.05  | 2.93  | 2.84  | 2.76  | 2.70  | 2.60  | 2.50  | 2.39  | 2.33  | 2.27  | 2.21  | 2.14  | 2.08  | 2.00     |  |
| 23       | 5.75   | 4.35  | 3.75  | 3.41  | 3.18  | 3.02  | 2.90  | 2.81  | 2.73  | 2.67  | 2.57  | 2.47  | 2.36  | 2.30  | 2.24  | 2.18  | 2.11  | 2.04  | 1.97     |  |
| 24       | 5.72   | 4.32  | 3.72  | 3.38  | 3.15  | 2.99  | 2.87  | 2.78  | 2.70  | 2.64  | 2.54  | 2.44  | 2.33  | 2.27  | 2.21  | 2.15  | 2.08  | 2.01  | 1.94     |  |
| 25       | 5.69   | 4.29  | 3.69  | 3.35  | 3.13  | 2.97  | 2.85  | 2.75  | 2.68  | 2.61  | 2.51  | 2.41  | 2.30  | 2.24  | 2.18  | 2.12  | 2.05  | 1.98  | 1.91     |  |
| 26       | 5.66   | 4.27  | 3.67  | 3.33  | 3.10  | 2.94  | 2.82  | 2.73  | 2.65  | 2.59  | 2.49  | 2.39  | 2.28  | 2.22  | 2.16  | 2.09  | 2.03  | 1.95  | 1.88     |  |
| 27       | 5.63   | 4.24  | 3.65  | 3.31  | 3.08  | 2.92  | 2.80  | 2.71  | 2.63  | 2.57  | 2.47  | 2.36  | 2.25  | 2.19  | 2.13  | 2.07  | 2.00  | 1.93  | 1.85     |  |
| 28       | 5.61   | 4.22  | 3.63  | 3.29  | 3.06  | 2.90  | 2.78  | 2.69  | 2.61  | 2.55  | 2.45  | 2.34  | 2.23  | 2.17  | 2.11  | 2.05  | 1.98  | 1.91  | 1.83     |  |
| 29       | 5.59   | 4.20  | 3.61  | 3.27  | 3.04  | 2.88  | 2.76  | 2.67  | 2.59  | 2.53  | 2.43  | 2.32  | 2.21  | 2.15  | 2.09  | 2.03  | 1.96  | 1.89  | 1.81     |  |
| 30       | 5.57   | 4.18  | 3.59  | 3.25  | 3.03  | 2.87  | 2.75  | 2.65  | 2.57  | 2.51  | 2.41  | 2.31  | 2.20  | 2.14  | 2.07  | 2.01  | 1.94  | 1.87  | 1.79     |  |
| 40       | 5.42   | 4.05  | 3.46  | 3.13  | 2.90  | 2.74  | 2.62  | 2.53  | 2.45  | 2.39  | 2.29  | 2.18  | 2.07  | 2.01  | 1.94  | 1.88  | 1.80  | 1.72  | 1.64     |  |
| 60       | 5.29   | 3.93  | 3.34  | 3.01  | 2.79  | 2.63  | 2.51  | 2.41  | 2.33  | 2.27  | 2.17  | 2.06  | 1.94  | 1.88  | 1.82  | 1.74  | 1.67  | 1.58  | 1.48     |  |
| 120      | 5.15   | 3.80  | 3.23  | 2.89  | 2.67  | 2.52  | 2.39  | 2.30  | 2.22  | 2.16  | 2.05  | 1.94  | 1.82  | 1.76  | 1.69  | 1.61  | 1.53  | 1.43  | 1.31     |  |
| $\infty$ | 5.02   | 3.69  | 3.12  | 2.79  | 2.57  | 2.41  | 2.29  | 2.19  | 2.11  | 2.05  | 1.94  | 1.83  | 1.71  | 1.64  | 1.57  | 1.48  | 1.39  | 1.27  | 1.00     |  |

Degrees of Freedom for the Denominator ( $\nu_2$ )

IV. Percentage Points of the F Distribution (continued)

| $\nu_1$  | Degrees of Freedom for the Numerator ( $\nu_2$ ) |        |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |  |
|----------|--|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|--|
|          | 1  | 2      | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |  |
| 1        | 4052   | 4999.5 | 5403  | 5625  | 5764  | 5859  | 5928  | 5982  | 6022  | 6056  | 6106  | 6157  | 6209  | 6235  | 6261  | 6287  | 6313  | 6339  | 6366     |  |
| 2        | 98.50  | 99.00  | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50    |  |
| 3        | 34.12  | 30.82  | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.69 | 26.00 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13    |  |
| 4        | 21.20  | 18.00  | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46    |  |
| 5        | 16.26  | 13.27  | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89  | 9.72  | 9.55  | 9.47  | 9.38  | 9.29  | 9.20  | 9.11  | 9.02     |  |
| 6        | 13.75  | 10.92  | 9.78  | 9.15  | 8.75  | 8.47  | 8.26  | 8.10  | 7.98  | 7.87  | 7.72  | 7.56  | 7.40  | 7.31  | 7.23  | 7.14  | 7.06  | 6.97  | 6.88     |  |
| 7        | 12.25  | 9.55   | 8.45  | 7.85  | 7.46  | 7.19  | 6.99  | 6.84  | 6.72  | 6.62  | 6.47  | 6.31  | 6.16  | 6.07  | 5.99  | 5.91  | 5.82  | 5.74  | 5.65     |  |
| 8        | 11.26  | 8.65   | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  | 5.81  | 5.67  | 5.52  | 5.36  | 5.28  | 5.20  | 5.12  | 5.03  | 4.95  | 4.86     |  |
| 9        | 10.56  | 8.02   | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  | 5.26  | 5.11  | 4.96  | 4.81  | 4.73  | 4.65  | 4.57  | 4.48  | 4.40  | 4.31     |  |
| 10       | 10.04  | 7.56   | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  | 4.85  | 4.71  | 4.56  | 4.41  | 4.33  | 4.25  | 4.17  | 4.08  | 4.00  | 3.91     |  |
| 11       | 9.65   | 7.21   | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  | 4.54  | 4.40  | 4.25  | 4.10  | 4.02  | 3.94  | 3.86  | 3.78  | 3.69  | 3.60     |  |
| 12       | 9.33   | 6.93   | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  | 4.30  | 4.16  | 4.01  | 3.86  | 3.78  | 3.70  | 3.62  | 3.54  | 3.45  | 3.36     |  |
| 13       | 9.07   | 6.70   | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  | 4.10  | 3.96  | 3.82  | 3.66  | 3.59  | 3.51  | 3.43  | 3.34  | 3.25  | 3.17     |  |
| 14       | 8.86   | 6.51   | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  | 3.94  | 3.80  | 3.66  | 3.51  | 3.43  | 3.35  | 3.27  | 3.18  | 3.09  | 3.00     |  |
| 15       | 8.68   | 6.36   | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  | 3.80  | 3.67  | 3.52  | 3.37  | 3.29  | 3.21  | 3.13  | 3.05  | 2.96  | 2.87     |  |
| 16       | 8.53   | 6.23   | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  | 3.69  | 3.55  | 3.41  | 3.26  | 3.18  | 3.10  | 3.02  | 2.93  | 2.84  | 2.75     |  |
| 17       | 8.40   | 6.11   | 5.18  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  | 3.59  | 3.46  | 3.31  | 3.16  | 3.08  | 3.00  | 2.92  | 2.83  | 2.75  | 2.65     |  |
| 18       | 8.29   | 6.01   | 5.09  | 4.58  | 4.25  | 4.01  | 3.84  | 3.71  | 3.60  | 3.51  | 3.37  | 3.23  | 3.08  | 3.00  | 2.92  | 2.84  | 2.75  | 2.66  | 2.57     |  |
| 19       | 8.18   | 5.93   | 5.01  | 4.50  | 4.17  | 3.94  | 3.77  | 3.63  | 3.52  | 3.43  | 3.30  | 3.15  | 3.00  | 2.92  | 2.84  | 2.76  | 2.67  | 2.58  | 2.49     |  |
| 20       | 8.10   | 5.85   | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  | 3.37  | 3.23  | 3.09  | 2.94  | 2.86  | 2.78  | 2.69  | 2.61  | 2.52  | 2.42     |  |
| 21       | 8.02   | 5.78   | 4.87  | 4.37  | 4.04  | 3.81  | 3.64  | 3.51  | 3.40  | 3.31  | 3.17  | 3.03  | 2.88  | 2.80  | 2.72  | 2.64  | 2.55  | 2.46  | 2.36     |  |
| 22       | 7.95   | 5.72   | 4.82  | 4.31  | 3.99  | 3.76  | 3.59  | 3.45  | 3.35  | 3.26  | 3.12  | 2.98  | 2.83  | 2.75  | 2.67  | 2.58  | 2.50  | 2.40  | 2.31     |  |
| 23       | 7.88   | 5.66   | 4.76  | 4.26  | 3.94  | 3.71  | 3.54  | 3.41  | 3.30  | 3.21  | 3.07  | 2.93  | 2.78  | 2.70  | 2.62  | 2.54  | 2.45  | 2.35  | 2.26     |  |
| 24       | 7.82   | 5.61   | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  | 3.17  | 3.03  | 2.89  | 2.74  | 2.66  | 2.58  | 2.49  | 2.40  | 2.31  | 2.21     |  |
| 25       | 7.77   | 5.57   | 4.68  | 4.18  | 3.85  | 3.63  | 3.46  | 3.32  | 3.22  | 3.13  | 2.99  | 2.85  | 2.70  | 2.62  | 2.54  | 2.45  | 2.36  | 2.27  | 2.17     |  |
| 26       | 7.72   | 5.53   | 4.64  | 4.14  | 3.82  | 3.59  | 3.42  | 3.29  | 3.18  | 3.09  | 2.96  | 2.81  | 2.66  | 2.58  | 2.50  | 2.42  | 2.33  | 2.23  | 2.13     |  |
| 27       | 7.68   | 5.49   | 4.60  | 4.11  | 3.78  | 3.56  | 3.39  | 3.26  | 3.15  | 3.06  | 2.93  | 2.78  | 2.63  | 2.55  | 2.47  | 2.38  | 2.29  | 2.20  | 2.10     |  |
| 28       | 7.64   | 5.45   | 4.57  | 4.07  | 3.75  | 3.53  | 3.36  | 3.23  | 3.12  | 3.03  | 2.90  | 2.75  | 2.60  | 2.52  | 2.44  | 2.34  | 2.26  | 2.17  | 2.06     |  |
| 29       | 7.60   | 5.42   | 4.54  | 4.04  | 3.73  | 3.50  | 3.33  | 3.20  | 3.09  | 3.00  | 2.87  | 2.73  | 2.57  | 2.49  | 2.41  | 2.33  | 2.23  | 2.14  | 2.03     |  |
| 30       | 7.56   | 5.39   | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  | 2.98  | 2.84  | 2.70  | 2.55  | 2.47  | 2.39  | 2.30  | 2.21  | 2.11  | 2.01     |  |
| 40       | 7.31   | 5.18   | 4.31  | 3.83  | 3.51  | 3.29  | 3.12  | 2.99  | 2.89  | 2.80  | 2.66  | 2.52  | 2.37  | 2.29  | 2.20  | 2.11  | 2.02  | 1.92  | 1.80     |  |
| 60       | 7.08   | 4.98   | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  | 2.63  | 2.50  | 2.35  | 2.20  | 2.12  | 2.03  | 1.94  | 1.84  | 1.73  | 1.60     |  |
| 120      | 6.85   | 4.79   | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  | 2.47  | 2.34  | 2.19  | 2.03  | 1.95  | 1.86  | 1.76  | 1.66  | 1.53  | 1.38     |  |
| $\infty$ | 6.63   | 4.61   | 3.78  | 3.32  | 3.02  | 2.80  | 2.64  | 2.51  | 2.41  | 2.32  | 2.18  | 2.04  | 1.88  | 1.79  | 1.70  | 1.59  | 1.47  | 1.32  | 1.00     |  |

Degrees of Freedom for the Denominator ( $\nu_2$ )

VIII. Critical Values for Dunnett's Test for Comparing Treatments with a Control (continued)  
 $d_{0.05}(a-1, f)$   
 One-Sided Comparisons

| <i>f</i> | <i>a</i> - 1 = Number of Treatment Means (excluding control) |      |      |      |      |      |      |      |      |
|----------|--|------|------|------|------|------|------|------|------|
|          | 1  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 5        | 2.02   | 2.44 | 2.68 | 2.85 | 2.98 | 3.08 | 3.16 | 3.24 | 3.30 |
| 6        | 1.94   | 2.34 | 2.56 | 2.71 | 2.83 | 2.92 | 3.00 | 3.07 | 3.12 |
| 7        | 1.89   | 2.27 | 2.48 | 2.62 | 2.73 | 2.82 | 2.89 | 2.95 | 3.01 |
| 8        | 1.86   | 2.22 | 2.42 | 2.55 | 2.66 | 2.74 | 2.81 | 2.87 | 2.92 |
| 9        | 1.83   | 2.18 | 2.37 | 2.50 | 2.60 | 2.68 | 2.75 | 2.81 | 2.86 |
| 10       | 1.81   | 2.15 | 2.34 | 2.47 | 2.56 | 2.64 | 2.70 | 2.76 | 2.81 |
| 11       | 1.80   | 2.13 | 2.31 | 2.44 | 2.53 | 2.60 | 2.67 | 2.72 | 2.77 |
| 12       | 1.78   | 2.11 | 2.29 | 2.41 | 2.50 | 2.58 | 2.64 | 2.69 | 2.74 |
| 13       | 1.77   | 2.09 | 2.27 | 2.39 | 2.48 | 2.55 | 2.61 | 2.66 | 2.71 |
| 14       | 1.76   | 2.08 | 2.25 | 2.37 | 2.46 | 2.53 | 2.59 | 2.64 | 2.69 |
| 15       | 1.75   | 2.07 | 2.24 | 2.36 | 2.44 | 2.51 | 2.57 | 2.62 | 2.67 |
| 16       | 1.75   | 2.06 | 2.23 | 2.34 | 2.43 | 2.50 | 2.56 | 2.61 | 2.65 |
| 17       | 1.74   | 2.05 | 2.22 | 2.33 | 2.42 | 2.49 | 2.54 | 2.59 | 2.64 |
| 18       | 1.73   | 2.04 | 2.21 | 2.32 | 2.41 | 2.48 | 2.53 | 2.58 | 2.62 |
| 19       | 1.73   | 2.03 | 2.20 | 2.31 | 2.40 | 2.47 | 2.52 | 2.57 | 2.61 |
| 20       | 1.72   | 2.03 | 2.19 | 2.30 | 2.39 | 2.46 | 2.51 | 2.56 | 2.60 |
| 24       | 1.71   | 2.01 | 2.17 | 2.28 | 2.36 | 2.43 | 2.48 | 2.53 | 2.57 |
| 30       | 1.70   | 1.99 | 2.15 | 2.25 | 2.33 | 2.40 | 2.45 | 2.50 | 2.54 |
| 40       | 1.68   | 1.97 | 2.13 | 2.23 | 2.31 | 2.37 | 2.42 | 2.47 | 2.51 |
| 60       | 1.67   | 1.95 | 2.10 | 2.21 | 2.28 | 2.35 | 2.39 | 2.44 | 2.48 |
| 120      | 1.66   | 1.93 | 2.08 | 2.18 | 2.26 | 2.32 | 2.37 | 2.41 | 2.45 |
| ∞        | 1.64   | 1.92 | 2.06 | 2.16 | 2.23 | 2.29 | 2.34 | 2.38 | 2.42 |

$d_{0.01}(a-1, f)$   
 One-Sided Comparisons

| <i>f</i> | <i>a</i> - 1 = Number of Treatment Means (excluding control) |      |      |      |      |      |      |      |      |
|----------|--|------|------|------|------|------|------|------|------|
|          | 1  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 5        | 3.37   | 3.90 | 4.21 | 4.43 | 4.60 | 4.73 | 4.85 | 4.94 | 5.03 |
| 6        | 3.14   | 3.61 | 3.88 | 4.07 | 4.21 | 4.33 | 4.43 | 4.51 | 4.59 |
| 7        | 3.00   | 3.42 | 3.66 | 3.83 | 3.96 | 4.07 | 4.15 | 4.23 | 4.30 |
| 8        | 2.90   | 3.29 | 3.51 | 3.67 | 3.79 | 3.88 | 3.96 | 4.03 | 4.09 |
| 9        | 2.82   | 3.19 | 3.40 | 3.55 | 3.66 | 3.75 | 3.82 | 3.89 | 3.94 |
| 10       | 2.76   | 3.11 | 3.31 | 3.45 | 3.56 | 3.64 | 3.71 | 3.78 | 3.83 |
| 11       | 2.72   | 3.06 | 3.25 | 3.38 | 3.48 | 3.56 | 3.63 | 3.69 | 3.74 |
| 12       | 2.68   | 3.01 | 3.19 | 3.32 | 3.42 | 3.50 | 3.56 | 3.62 | 3.67 |
| 13       | 2.65   | 2.97 | 3.15 | 3.27 | 3.37 | 3.44 | 3.51 | 3.56 | 3.61 |
| 14       | 2.62   | 2.94 | 3.11 | 3.23 | 3.32 | 3.40 | 3.46 | 3.51 | 3.56 |
| 15       | 2.60   | 2.91 | 3.08 | 3.20 | 3.29 | 3.36 | 3.42 | 3.47 | 3.52 |
| 16       | 2.58   | 2.88 | 3.05 | 3.17 | 3.26 | 3.33 | 3.39 | 3.44 | 3.48 |
| 17       | 2.57   | 2.86 | 3.03 | 3.14 | 3.23 | 3.30 | 3.36 | 3.41 | 3.45 |
| 18       | 2.55   | 2.84 | 3.01 | 3.12 | 3.21 | 3.27 | 3.33 | 3.38 | 3.42 |
| 19       | 2.54   | 2.83 | 2.99 | 3.10 | 3.18 | 3.25 | 3.31 | 3.36 | 3.40 |
| 20       | 2.53   | 2.81 | 2.97 | 3.08 | 3.17 | 3.23 | 3.29 | 3.34 | 3.38 |
| 24       | 2.49   | 2.77 | 2.92 | 3.03 | 3.11 | 3.17 | 3.22 | 3.27 | 3.31 |
| 30       | 2.46   | 2.72 | 2.87 | 2.97 | 3.05 | 3.11 | 3.16 | 3.21 | 3.24 |
| 40       | 2.42   | 2.68 | 2.82 | 2.92 | 2.99 | 3.05 | 3.10 | 3.14 | 3.18 |
| 60       | 2.39   | 2.64 | 2.78 | 2.87 | 2.94 | 3.00 | 3.04 | 3.08 | 3.12 |
| 120      | 2.36   | 2.60 | 2.73 | 2.82 | 2.89 | 2.94 | 2.99 | 3.03 | 3.06 |
| ∞        | 2.33   | 2.56 | 2.68 | 2.77 | 2.84 | 2.89 | 2.93 | 2.97 | 3.00 |

VIII. Critical Values for Dunnett's Test for Comparing Treatments with a Control<sup>a</sup>
 $d_{0.05}(a-1, f)$   
Two-Sided Comparisons

| $a - 1 = \text{Number of Treatment Means (excluding control)}$ |      |      |      |      |      |      |      |      |      |
|--|------|------|------|------|------|------|------|------|------|
| $f$  | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 5  | 2.57 | 3.03 | 3.29 | 3.48 | 3.62 | 3.73 | 3.82 | 3.90 | 3.97 |
| 6  | 2.45 | 2.86 | 3.10 | 3.26 | 3.39 | 3.49 | 3.57 | 3.64 | 3.71 |
| 7  | 2.36 | 2.75 | 2.97 | 3.12 | 3.24 | 3.33 | 3.41 | 3.47 | 3.53 |
| 8  | 2.31 | 2.67 | 2.88 | 3.02 | 3.13 | 3.22 | 3.29 | 3.35 | 3.41 |
| 9  | 2.26 | 2.61 | 2.81 | 2.95 | 3.05 | 3.14 | 3.20 | 3.26 | 3.32 |
| 10   | 2.23 | 2.57 | 2.76 | 2.89 | 2.99 | 3.07 | 3.14 | 3.19 | 3.24 |
| 11   | 2.20 | 2.53 | 2.72 | 2.84 | 2.94 | 3.02 | 3.08 | 3.14 | 3.19 |
| 12   | 2.18 | 2.50 | 2.68 | 2.81 | 2.90 | 2.98 | 3.04 | 3.09 | 3.14 |
| 13   | 2.16 | 2.48 | 2.65 | 2.78 | 2.87 | 2.94 | 3.00 | 3.06 | 3.10 |
| 14   | 2.14 | 2.46 | 2.63 | 2.75 | 2.84 | 2.91 | 2.97 | 3.02 | 3.07 |
| 15   | 2.13 | 2.44 | 2.61 | 2.73 | 2.82 | 2.89 | 2.95 | 3.00 | 3.04 |
| 16   | 2.12 | 2.42 | 2.59 | 2.71 | 2.80 | 2.87 | 2.92 | 2.97 | 3.02 |
| 17   | 2.11 | 2.41 | 2.58 | 2.69 | 2.78 | 2.85 | 2.90 | 2.95 | 3.00 |
| 18   | 2.10 | 2.40 | 2.56 | 2.68 | 2.76 | 2.83 | 2.89 | 2.94 | 2.98 |
| 19   | 2.09 | 2.39 | 2.55 | 2.66 | 2.75 | 2.81 | 2.87 | 2.92 | 2.96 |
| 20   | 2.09 | 2.38 | 2.54 | 2.65 | 2.73 | 2.80 | 2.86 | 2.90 | 2.95 |
| 24   | 2.06 | 2.35 | 2.51 | 2.61 | 2.70 | 2.76 | 2.81 | 2.86 | 2.90 |
| 30   | 2.04 | 2.32 | 2.47 | 2.58 | 2.66 | 2.72 | 2.77 | 2.82 | 2.86 |
| 40   | 2.02 | 2.29 | 2.44 | 2.54 | 2.62 | 2.68 | 2.73 | 2.77 | 2.81 |
| 60   | 2.00 | 2.27 | 2.41 | 2.51 | 2.58 | 2.64 | 2.69 | 2.73 | 2.77 |
| 120  | 1.98 | 2.24 | 2.38 | 2.47 | 2.55 | 2.60 | 2.65 | 2.69 | 2.73 |
| $\infty$   | 1.96 | 2.21 | 2.35 | 2.44 | 2.51 | 2.57 | 2.61 | 2.65 | 2.69 |

 $d_{0.01}(a-1, f)$   
Two-Sided Comparisons

| $a - 1 = \text{Number of Treatment Means (excluding control)}$ |      |      |      |      |      |      |      |      |      |
|--|------|------|------|------|------|------|------|------|------|
| $f$  | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 5  | 4.03 | 4.63 | 4.98 | 5.22 | 5.41 | 5.56 | 5.69 | 5.80 | 5.89 |
| 6  | 3.71 | 4.21 | 4.51 | 4.71 | 4.87 | 5.00 | 5.10 | 5.20 | 5.28 |
| 7  | 3.50 | 3.95 | 4.21 | 4.39 | 4.53 | 4.64 | 4.74 | 4.82 | 4.89 |
| 8  | 3.36 | 3.77 | 4.00 | 4.17 | 4.29 | 4.40 | 4.48 | 4.56 | 4.62 |
| 9  | 3.25 | 3.63 | 3.85 | 4.01 | 4.12 | 4.22 | 4.30 | 4.37 | 4.43 |
| 10   | 3.17 | 3.53 | 3.74 | 3.88 | 3.99 | 4.08 | 4.16 | 4.22 | 4.28 |
| 11   | 3.11 | 3.45 | 3.65 | 3.79 | 3.89 | 3.98 | 4.05 | 4.11 | 4.16 |
| 12   | 3.05 | 3.39 | 3.58 | 3.71 | 3.81 | 3.89 | 3.96 | 4.02 | 4.07 |
| 13   | 3.01 | 3.33 | 3.52 | 3.65 | 3.74 | 3.82 | 3.89 | 3.94 | 3.99 |
| 14   | 2.98 | 3.29 | 3.47 | 3.59 | 3.69 | 3.76 | 3.83 | 3.88 | 3.93 |
| 15   | 2.95 | 3.25 | 3.43 | 3.55 | 3.64 | 3.71 | 3.78 | 3.83 | 3.88 |
| 16   | 2.92 | 3.22 | 3.39 | 3.51 | 3.60 | 3.67 | 3.73 | 3.78 | 3.83 |
| 17   | 2.90 | 3.19 | 3.36 | 3.47 | 3.56 | 3.63 | 3.69 | 3.74 | 3.79 |
| 18   | 2.88 | 3.17 | 3.33 | 3.44 | 3.53 | 3.60 | 3.66 | 3.71 | 3.75 |
| 19   | 2.86 | 3.15 | 3.31 | 3.42 | 3.50 | 3.57 | 3.63 | 3.68 | 3.72 |
| 20   | 2.85 | 3.13 | 3.29 | 3.40 | 3.48 | 3.55 | 3.60 | 3.65 | 3.69 |
| 24   | 2.80 | 3.07 | 3.22 | 3.32 | 3.40 | 3.47 | 3.52 | 3.57 | 3.61 |
| 30   | 2.75 | 3.01 | 3.15 | 3.25 | 3.33 | 3.39 | 3.44 | 3.49 | 3.52 |
| 40   | 2.70 | 2.95 | 3.09 | 3.19 | 3.26 | 3.32 | 3.37 | 3.41 | 3.44 |
| 60   | 2.66 | 2.90 | 3.03 | 3.12 | 3.19 | 3.25 | 3.29 | 3.33 | 3.37 |
| 120  | 2.62 | 2.85 | 2.97 | 3.06 | 3.12 | 3.18 | 3.22 | 3.26 | 3.29 |
| $\infty$   | 2.58 | 2.79 | 2.92 | 3.00 | 3.06 | 3.11 | 3.15 | 3.19 | 3.22 |

 $f$  = degrees of freedom.

<sup>a</sup>Reproduced with permission from C. W. Dunnett, "New Tables for Multiple Comparison with a Control," *Biometrics*, Vol. 20, No. 3, 1964, and from C. W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," *Journal of the American Statistical Association*, Vol. 50, 1955.