UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2008

TITLE OF PAPER	:	NONPARAMETRIC ANALYSIS
COURSE CODE	:	ST 409
TIME ALLOWED	:	2 (TWO) HOURS
REQUIREMENTS	:	STATISTICAL TABLES AND CALCULATOR
INSTRUCTIONS	:	ANSWER ALL QUESTIONS IN SECTION ONE AND ANY THREE FROM SECTION TWO

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SECTION ONE

ANSWER ALL QUESTIONS

QUESTION ONE

[5 marks, 1 mark each]

1) The observed frequencies
   a) Are always whole numbers
   b) Can contain fractions or decimal values
   c) Can contain both positive and negative values
   d) Both B and C
2) The Mann-Whitney U-test provides a nonparametric alternative to the
   a) Single sample t-test
   b) Independent measures t-test
   c) Repeated measures t-test
   d) Pearson correlation
3) A sample of 100 people is classified by gender (male/female) and by whether or not they are registered voters. The sample consists of 60 females, of whom 50 are registered voters, and 40 males of whom 25 are registered voters. If these data were used for a chi-square test for independence, the expected frequency for registered males would be
   a) 15
   b) 25
   c) 30
   d) 45
4) The Wilcoxon signed rank test is an alternative to
   a) Single sample t-test
   b) Independent measures t-test
   c) Pearson correlation
   d) Repeated measures t-test
5) An experimenter wants to simultaneously evaluate two methods for teaching foreign languages and the languages that are being taught, French, German, Spanish and Italian. The results will be obtained through a standardized verbal language test (ratio scale). What is the appropriate test for this data?
   a) Wilcoxon signed rank test
   b) Kruskal-Wallis test
   c) Chi-square test
   d) Pearson's correlation

QUESTION TWO

[10 marks, 1 point each]

Indicate whether the sentence or statement is TRUE or FALSE.

1) Nonparametric tests have no assumptions.
2) The Mann-Whitney test is used for repeated-measures studies.
3) The same formula is used to compute chi-square for goodness-of-fit test and for the test for independence.
4) The null hypothesis for the sign test states that the sample differences will equal zero.
5) A nonparametric test should be used on occasions where it is more sensitive than a comparable parametric test.
6) The p-value is the probability of (only) observing values of the test statistic more extreme than the observed value of the test statistic given that the null hypothesis is true.

7) The McNemar test is used for ordinal data.
8) The Kruskal-Wallis test is used for repeated measures studies.
9) If a given result is significant with a nonparametric test, and can be examined with a parametric test, a significant result will also be obtained with the parametric test.
10) If a significant relationship between two variables exists, the linear correlation coefficient will always detect it.

QUESTION THREE

[25 marks, 5+10+5+5]

Researchers investigated the effect of socio-economic class on physical development of Turkish children. Physical development was classified on a scale of 1 (none) to 5 (fully developed) and the socio-economic class of their parents was assessed on a scale of 1 to 4. The data were as follows:

Socio-economic class of parents	Physical development				
	1	2	3	4	5
1	2	14	28	40	18
2	1	21	25	25	9
3	1	12	12	12	2
4	6	17	34	33	6

a. Plot these data in a meaningful way and report your initial findings.
b. Stating clearly your hypotheses, carry out an analysis to test for a relationship between physical development and socio-economic class using as many different categories of physical development as possible, and report your conclusions.
c. Carry out a further analysis comparing those who are fully developed (stage 5) with those who are not (stages 1-4) and report your conclusion.
d. Provide an explanation for these two conclusions.
(For any test where you detect a relationship, report on the nature of that relationship.)

SECTION TWO

ANSWER ANY TWO QUESTIONS

QUESTION FOUR

[10 marks]

In a diet test, each of four diet programs is applied to a sample of people. At the end of three weeks, the amount of pounds people lost are shown below.

Diet Program			
1	2	3	4
12	19	16	28
6	10	20	17
18	13	26	22
23	20	19	16
	25		20

Test to determine if there is enough evidence at the 1% significance level to infer that at least two population locations differ. State the hypothesis, critical region(s) and conclusions. Show all calculations.

QUESTION FIVE

[10 marks, 3+7]

The following data give the stroke index for 10 patients before and after treatment. We wish to test the hypothesis that the treatment has no effect.

Before	109,57,53,57,68,72,51,65,52,61
After	56,44,55,40,62,46,48,41,56,49

i. Calculate the differences, draw a boxplot of these and comment on why a nonparametric test might be preferred.

ii. Use an appropriate test to see if the treatment has no effect. Use a 5% significance level. State the hypothesis, define the critical region(s) and conclusion. Show all calculations.

QUESTION SIX

[10 marks]

Quality of life scores are often non-normally distributed in healthy populations, because many values tend to be near the upper range of the scale. The SF-36 scale ranges from 0 to 100, with 100 indicating perfect health. In a study measuring the quality of life of patients with wrist fractures, the following SF-36 data are collected:

Patients with wrist fractures	Healthy (unmatched) controls
85	98
87	93
97	99
62	89
75	65
92	100
91	
81	
76	
88	
90	

Perform an appropriate two-sided nonparametric test to examine if subjects with wrist fractures have different quality of life compared to healthy subjects. State the null and alternative hypothesis, critical/rejection region(s) and conclusion. Show all calculations.

TABLE A1 Normal Distribution*

Selected values $z_{0.0001} = -3.7190$ $z_{0.0005} = -3.2905$ $z_{0.025} = -1.9600$ $z_{0.05} = -1.6449$

$z_{0.9999} = 3.7190$ $z_{0.9995} = 3.2905$ $z_{0.975} = 1.9600$ $z_{0.95} = 1.6449$

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00		−3.0902	−2.8782	−2.7478	−2.6521	−2.5758	−2.5121	−2.4573	−2.4089	−2.3656
0.01	−2.3263	−2.2904	−2.2571	−2.2262	−2.1973	−2.1701	−2.1444	−2.1201	−2.0969	−2.0749
0.02	−2.0537	−2.0335	−2.0141	−1.9954	−1.9774	−1.9600	−1.9431	−1.9268	−1.9110	−1.8957
0.03	−1.8808	−1.8663	−1.8522	−1.8384	−1.8250	−1.8119	−1.7991	−1.7866	−1.7744	−1.7624
0.04	−1.7507	−1.7392	−1.7279	−1.7169	−1.7060	−1.6954	−1.6849	−1.6747	−1.6646	−1.6546
0.05	−1.6449	−1.6352	−1.6258	−1.6164	−1.6072	−1.5982	−1.5893	−1.5805	−1.5718	−1.5632
0.06	−1.5548	−1.5464	−1.5382	−1.5301	−1.5220	−1.5141	−1.5063	−1.4985	−1.4909	−1.4833
0.07	−1.4758	−1.4684	−1.4611	−1.4538	−1.4466	−1.4395	−1.4325	−1.4255	−1.4187	−1.4118
0.08	−1.4051	−1.3984	−1.3917	−1.3852	−1.3787	−1.3722	−1.3658	−1.3595	−1.3532	−1.3469
0.09	−1.3408	−1.3346	−1.3285	−1.3225	−1.3165	−1.3106	−1.3047	−1.2988	−1.2930	−1.2873
0.10	−1.2816	−1.2759	−1.2702	−1.2646	−1.2591	−1.2536	−1.2481	−1.2426	−1.2372	−1.2319
0.11	−1.2265	−1.2212	−1.2160	−1.2107	−1.2055	−1.2004	−1.1952	−1.1901	−1.1850	−1.1800
0.12	−1.1750	−1.1700	−1.1650	−1.1601	−1.1552	−1.1503	−1.1455	−1.1407	−1.1359	−1.1311
0.13	−1.1264	−1.1217	−1.1170	−1.1123	−1.1077	−1.1031	−1.0985	−1.0939	−1.0893	−1.0848
0.14	−1.0803	−1.0758	−1.0714	−1.0669	−1.0625	−1.0581	−1.0537	−1.0494	−1.0450	−1.0407
0.15	−1.0364	−1.0322	−1.0279	−1.0237	−1.0194	−1.0152	−1.0110	−1.0069	−1.0027	−0.9986
0.16	−0.9945	−0.9904	−0.9863	−0.9822	−0.9782	−0.9741	−0.9701	−0.9661	−0.9621	−0.9581
0.17	−0.9542	−0.9502	−0.9463	−0.9424	−0.9385	−0.9346	−0.9307	−0.9269	−0.9230	−0.9192
0.18	−0.9154	−0.9116	−0.9078	−0.9040	−0.9002	−0.8965	−0.8927	−0.8890	−0.8853	−0.8816
0.19	−0.8779	−0.8742	−0.8705	−0.8669	−0.8633	−0.8596	−0.8560	−0.8524	−0.8488	−0.8452
0.20	−0.8416	−0.8381	−0.8345	−0.8310	−0.8274	−0.8239	−0.8204	−0.8169	−0.8134	−0.8099
0.21	−0.8064	−0.8030	−0.7995	−0.7961	−0.7926	−0.7892	−0.7858	−0.7824	−0.7790	−0.7756
0.22	−0.7722	−0.7688	−0.7655	−0.7621	−0.7588	−0.7554	−0.7521	−0.7488	−0.7454	−0.7421
0.23	−0.7388	−0.7356	−0.7323	−0.7290	−0.7257	−0.7225	−0.7192	−0.7160	−0.7128	−0.7095
0.24	−0.7063	−0.7031	−0.6999	−0.6967	−0.6935	−0.6903	−0.6871	−0.6840	−0.6808	−0.6776

TABLE A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.25	−0.6745	−0.6713	−0.6682	−0.6651	−0.6620	−0.6588	−0.6557	−0.6526	−0.6495	−0.6464
0.26	−0.6433	−0.6403	−0.6372	−0.6341	−0.6311	−0.6280	−0.6250	−0.6219	−0.6189	−0.6158
0.27	−0.6128	−0.6098	−0.6068	−0.6038	−0.6008	−0.5978	−0.5948	−0.5918	−0.5888	−0.5858
0.28	−0.5828	−0.5799	−0.5769	−0.5740	−0.5710	−0.5681	−0.5651	−0.5622	−0.5592	−0.5563
0.29	−0.5534	−0.5505	−0.5476	−0.5446	−0.5417	−0.5388	−0.5359	−0.5330	−0.5302	−0.5273
0.30	−0.5244	−0.5215	−0.5187	−0.5158	−0.5129	−0.5101	−0.5072	−0.5044	−0.5015	−0.4987
0.31	−0.4959	−0.4930	−0.4902	−0.4874	−0.4845	−0.4817	−0.4789	−0.4761	−0.4733	−0.4705
0.32	−0.4677	−0.4649	−0.4621	−0.4593	−0.4565	−0.4538	−0.4510	−0.4482	−0.4454	−0.4427
0.33	−0.4399	−0.4372	−0.4344	−0.4316	−0.4289	−0.4261	−0.4234	−0.4207	−0.4179	−0.4152
0.34	−0.4125	−0.4097	−0.4070	−0.4043	−0.4016	−0.3989	−0.3961	−0.3934	−0.3907	−0.3880
0.35	−0.3853	−0.3826	−0.3799	−0.3772	−0.3745	−0.3719	−0.3692	−0.3665	−0.3638	−0.3611
0.36	−0.3585	−0.3558	−0.3531	−0.3505	−0.3478	−0.3451	−0.3425	−0.3398	−0.3372	−0.3345
0.37	−0.3319	−0.3292	−0.3266	−0.3239	−0.3213	−0.3186	−0.3160	−0.3134	−0.3107	−0.3081
0.38	−0.3055	−0.3029	−0.3002	−0.2976	−0.2950	−0.2924	−0.2898	−0.2871	−0.2845	−0.2819
0.39	−0.2793	−0.2767	−0.2741	−0.2715	−0.2689	−0.2663	−0.2637	−0.2611	−0.2585	−0.2559
0.40	−0.2533	−0.2508	−0.2482	−0.2456	−0.2430	−0.2404	−0.2378	−0.2353	−0.2327	−0.2301
0.41	−0.2275	−0.2250	−0.2224	−0.2198	−0.2173	−0.2147	−0.2121	−0.2096	−0.2070	−0.2045
0.42	−0.2019	−0.1993	−0.1968	−0.1942	−0.1917	−0.1891	−0.1866	−0.1840	−0.1815	−0.1789
0.43	−0.1764	−0.1738	−0.1713	−0.1687	−0.1662	−0.1637	−0.1611	−0.1586	−0.1560	−0.1535
0.44	−0.1510	−0.1484	−0.1459	−0.1434	−0.1408	−0.1383	−0.1358	−0.1332	−0.1307	−0.1282
0.45	−0.1257	−0.1231	−0.1206	−0.1181	−0.1156	−0.1130	−0.1105	−0.1080	−0.1055	−0.1030
0.46	−0.1004	−0.0979	−0.0954	−0.0929	−0.0904	−0.0878	−0.0853	−0.0828	−0.0803	−0.0778
0.47	−0.0753	−0.0728	−0.0702	−0.0677	−0.0652	−0.0627	−0.0602	−0.0577	−0.0552	−0.0527
0.48	−0.0502	−0.0476	−0.0451	−0.0426	−0.0401	−0.0376	−0.0351	−0.0326	−0.0301	−0.0276
0.49	−0.0251	−0.0226	−0.0201	−0.0175	−0.0150	−0.0125	−0.0100	−0.0075	−0.0050	−0.0025
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231

TABLE A3 (Continued)

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
15	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0037	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0176	0.0063	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0592	0.0255	0.0093	0.0028	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
	5	0.1509	0.0769	0.0338	0.0124	0.0037	0.0008	0.0001	0.0000	0.0000	0.0000
	6	0.3036	0.1818	0.0950	0.0422	0.0152	0.0042	0.0008	0.0001	0.0000	0.0000
	7	0.5000	0.3465	0.2131	0.1132	0.0500	0.0173	0.0042	0.0006	0.0000	0.0000
	8	0.6964	0.5478	0.3902	0.2452	0.1311	0.0566	0.0181	0.0036	0.0003	0.0000
	9	0.8491	0.7392	0.5968	0.4357	0.2784	0.1484	0.0611	0.0168	0.0022	0.0001
	10	0.9408	0.8796	0.7827	0.6481	0.4845	0.3135	0.1642	0.0617	0.0127	0.0006
	11	0.9824	0.9576	0.9095	0.8273	0.7031	0.5387	0.3518	0.1773	0.0556	0.0055
	12	0.9963	0.9893	0.9729	0.9383	0.8732	0.7639	0.6020	0.3958	0.1841	0.0362
	13	0.9995	0.9983	0.9948	0.9858	0.9647	0.9198	0.8329	0.6814	0.4510	0.1710
	14	1.0000	0.9999	0.9995	0.9984	0.9953	0.9866	0.9648	0.9126	0.7941	0.5367
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0106	0.0035	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0384	0.0149	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.1051	0.0486	0.0191	0.0062	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000
	6	0.2272	0.1241	0.0583	0.0229	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000
	7	0.4018	0.2559	0.1423	0.0671	0.0257	0.0075	0.0015	0.0002	0.0000	0.0000
	8	0.5982	0.4371	0.2839	0.1594	0.0744	0.0271	0.0070	0.0011	0.0001	0.0000
	9	0.7728	0.6340	0.4728	0.3119	0.1753	0.0796	0.0267	0.0056	0.0005	0.0000
	10	0.8949	0.8024	0.6712	0.5100	0.3402	0.1897	0.0817	0.0235	0.0033	0.0001
	11	0.9616	0.9147	0.8334	0.7108	0.5501	0.3698	0.2018	0.0791	0.0170	0.0009
	12	0.9894	0.9719	0.9349	0.8661	0.7541	0.5950	0.4019	0.2101	0.0684	0.0070
	13	0.9979	0.9934	0.9817	0.9549	0.9006	0.8029	0.6482	0.4386	0.2108	0.0429
	14	0.9997	0.9990	0.9967	0.9902	0.9739	0.9365	0.8593	0.7161	0.4853	0.1892
	15	1.0000	0.9999	0.9997	0.9990	0.9967	0.9900	0.9719	0.9257	0.8147	0.5599
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

n	y	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902
	7	1.0000	0.9999	0.9983	0.9891	0.9598	0.8954	0.7872	0.6405	0.4743
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9873	0.9617	0.9081	0.8166
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9968	0.9880	0.9652	0.9174
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9970	0.9894	0.9699
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9914
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120
	4	0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.1886	0.0942	0.0411
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3743	0.2258
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9788	0.9424	0.8720
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9938	0.9797	0.9463
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9986	0.9942	0.9817
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
17	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0012	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0064	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0245	0.0086	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0717	0.0301	0.0106	0.0030	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
	6	0.1662	0.0826	0.0348	0.0120	0.0032	0.0006	0.0001	0.0000	0.0000	0.0000
	7	0.3145	0.1834	0.0919	0.0383	0.0127	0.0031	0.0005	0.0000	0.0000	0.0000
	8	0.5000	0.3374	0.1989	0.0994	0.0403	0.0124	0.0026	0.0003	0.0000	0.0000
	9	0.6855	0.5257	0.3595	0.2128	0.1046	0.0402	0.0109	0.0017	0.0001	0.0000
	10	0.8338	0.7098	0.5522	0.3812	0.2248	0.1071	0.0377	0.0083	0.0008	0.0000
	11	0.9283	0.8529	0.7361	0.5803	0.4032	0.2347	0.1057	0.0319	0.0047	0.0001
	12	0.9755	0.9404	0.8740	0.7652	0.6113	0.4261	0.2418	0.0987	0.0221	0.0012
	13	0.9936	0.9816	0.9536	0.8972	0.7981	0.6470	0.4511	0.2444	0.0826	0.0088
	14	0.9988	0.9959	0.9877	0.9673	0.9226	0.8363	0.6904	0.4802	0.2382	0.0503
	15	0.9999	0.9994	0.9979	0.9933	0.9807	0.9499	0.8818	0.7475	0.5182	0.2078
	16	1.0000	1.0000	0.9998	0.9993	0.9977	0.9925	0.9775	0.9369	0.8332	0.5819
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0154	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0481	0.0183	0.0058	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.1189	0.0537	0.0203	0.0062	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000
	7	0.2403	0.1280	0.0576	0.0212	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000
	8	0.4073	0.2527	0.1347	0.0597	0.0210	0.0054	0.0009	0.0001	0.0000	0.0000
	9	0.5927	0.4222	0.2632	0.1391	0.0596	0.0193	0.0043	0.0005	0.0000	0.0000
	10	0.7597	0.6085	0.4366	0.2717	0.1407	0.0569	0.0163	0.0027	0.0002	0.0000
	11	0.8811	0.7742	0.6257	0.4509	0.2783	0.1390	0.0513	0.0118	0.0012	0.0000
	12	0.9519	0.8923	0.7912	0.6450	0.4656	0.2825	0.1329	0.0419	0.0064	0.0002
	13	0.9846	0.9589	0.9058	0.8114	0.6673	0.4813	0.2836	0.1206	0.0282	0.0015
	14	0.9962	0.9880	0.9672	0.9217	0.8354	0.6943	0.4990	0.2798	0.0982	0.0109
	15	0.9993	0.9975	0.9918	0.9764	0.9400	0.8647	0.7287	0.5203	0.2662	0.0581
	16	0.9999	0.9997	0.9987	0.9954	0.9858	0.9605	0.9009	0.7759	0.5497	0.2265
	17	1.0000	1.0000	0.9999	0.9996	0.9984	0.9944	0.9820	0.9464	0.8499	0.6028
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

n	y	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015
	3	0.9868	0.8850	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940
	9	1.0000	1.0000	0.9999	0.9984	0.9911	0.9674	0.9125	0.8139	0.6710
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9653	0.9115	0.8159
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9648	0.9129
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9884	0.9658
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9969	0.9891
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9972
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000
	1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001
	2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009
	3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049
	4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189
	5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553
	6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299
	7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520
	8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143
	9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914
	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507
	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692
	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000
	11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002
	14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020
	15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* Y has the binomial distribution with parameters n and p. The entries are the values of $P(Y \le y) = \sum_{i=0}^{y} \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95.
For n larger than 20, the rth quantile y_r of a binomial random variable may be approximated using $y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the rth quantile of a standard normal random variable, obtained from Table A1.

TABLE A4 Exact Confidence Intervals for the Binomial Parameter p

		90%		95%		99%	
n	Y	Lower	Upper	Lower	Upper	Lower	Upper
1	0	0.000	0.950	0.000	0.975	0.000	0.995
	1	0.050	1.000	0.025	1.000	0.005	1.000
2	0	0.000	0.776	0.000	0.842	0.000	0.929
	1	0.025	0.975	0.013	0.987	0.003	0.997
	2	0.224	1.000	0.158	1.000	0.071	1.000
3	0	0.000	0.632	0.000	0.708	0.000	0.829
	1	0.017	0.865	0.008	0.906	0.002	0.959
	2	0.135	0.983	0.094	0.992	0.041	0.998
	3	0.368	1.000	0.292	1.000	0.171	1.000
4	0	0.000	0.527	0.000	0.602	0.000	0.734
	1	0.013	0.751	0.006	0.806	0.001	0.889
	2	0.098	0.902	0.068	0.932	0.029	0.971
	3	0.249	0.987	0.194	0.994	0.111	0.999
	4	0.473	1.000	0.398	1.000	0.266	1.000
5	0	0.000	0.451	0.000	0.522	0.000	0.653
	1	0.010	0.657	0.005	0.716	0.001	0.815
	2	0.076	0.811	0.053	0.853	0.023	0.917
	3	0.189	0.924	0.147	0.947	0.083	0.977
	4	0.343	0.990	0.284	0.995	0.185	0.999
	5	0.549	1.000	0.478	1.000	0.347	1.000
6	0	0.000	0.393	0.000	0.459	0.000	0.586
	1	0.009	0.582	0.004	0.641	0.001	0.746
	2	0.063	0.729	0.043	0.777	0.019	0.856
	3	0.153	0.847	0.118	0.882	0.066	0.934
	4	0.271	0.937	0.223	0.957	0.144	0.981
	5	0.418	0.991	0.359	0.996	0.254	0.999
	6	0.607	1.000	0.541	1.000	0.414	1.000
7	0	0.000	0.348	0.000	0.410	0.000	0.531
	1	0.007	0.521	0.004	0.579	0.001	0.685
	2	0.053	0.659	0.037	0.710	0.016	0.797
	3	0.129	0.775	0.099	0.816	0.055	0.882
	4	0.225	0.871	0.184	0.901	0.118	0.945
	5	0.341	0.947	0.290	0.963	0.203	0.984
	6	0.479	0.993	0.421	0.996	0.315	0.999
	7	0.652	1.000	0.590	1.000	0.469	1.000

TABLE A7 Quantiles of the Mann–Whitney Test Statistic[a]

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0.001	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	0.005	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4
	0.01	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
	0.025	3	3	3	3	3	3	4	4	4	4	5	5	5	5	5	6	6	6	6
	0.05	3	3	3	4	4	4	5	5	5	5	6	6	7	7	7	7	8	8	8
	0.10	3	4	4	5	5	5	6	6	7	7	8	8	8	9	9	10	10	11	11
3	0.001	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7
	0.005	6	6	6	6	6	6	6	7	7	7	8	8	8	9	9	9	9	10	10
	0.01	6	6	6	6	6	7	7	8	8	8	9	9	9	10	10	11	11	11	12
	0.025	6	6	6	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15
	0.05	6	7	7	8	9	9	10	11	11	12	12	13	14	14	15	16	16	17	18
	0.10	7	8	8	9	10	11	12	12	13	14	15	16	17	17	18	19	20	21	22
4	0.001	10	10	10	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	14
	0.005	10	10	10	10	11	11	12	12	13	13	14	14	15	16	16	17	17	18	19
	0.01	10	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	20	21
	0.025	10	10	11	12	13	14	15	15	16	17	18	19	20	21	22	22	23	24	25
	0.05	10	11	12	13	14	15	16	17	18	19	20	21	22	23	25	26	27	28	29
	0.10	11	12	14	15	16	17	18	20	21	22	23	24	26	27	28	29	31	32	33
5	0.001	15	15	15	15	15	15	16	17	17	18	18	19	19	20	21	21	22	23	23
	0.005	15	15	15	16	17	17	18	19	20	21	22	23	23	24	25	26	27	28	29
	0.01	15	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	0.025	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35	36
	0.05	16	17	18	20	21	22	24	25	27	28	29	31	32	34	35	36	38	39	41
	0.10	17	18	20	21	23	24	26	28	29	31	33	34	36	38	39	41	43	44	46
6	0.001	21	21	21	21	21	21	23	24	25	26	26	27	28	29	30	31	32	33	34
	0.005	21	21	22	23	24	25	26	27	28	29	31	32	33	34	35	37	38	39	40
	0.01	21	21	23	24	25	26	28	29	30	31	33	34	35	37	38	40	41	42	44
	0.025	21	23	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49
	0.05	22	24	26	27	29	30	32	34	36	38	39	41	43	45	47	48	50	52	54
	0.10	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	56	58	60
7	0.001	28	28	28	28	29	30	31	33	34	35	36	37	38	39	40	41	42	44	45
	0.005	28	28	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51	53
	0.01	28	29	30	32	33	35	36	38	40	41	43	45	46	48	50	52	53	55	57
	0.025	28	30	32	34	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
	0.05	29	31	33	35	37	40	42	44	46	48	50	52	54	57	59	62	64	66	68
	0.10	30	32	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75
8	0.001	36	36	36	37	38	39	41	42	43	45	46	48	49	51	52	54	55	57	58
	0.005	36	36	38	39	41	43	44	46	48	50	52	54	55	57	59	61	63	65	67
	0.01	36	37	39	41	43	44	46	48	50	52	54	56	58	61	63	65	67	69	71
	0.025	37	39	41	43	45	47	50	52	54	56	59	61	63	66	68	71	73	75	78
	0.05	38	40	42	45	47	50	52	55	57	60	63	65	68	70	73	76	78	81	84
	0.10	39	42	44	47	50	53	55	58	61	64	67	70	73	76	79	82	85	88	91

TABLE A7 (Continued)

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
9	0.001	45	45	45	47	48	49	51	52	54	56	58	60	61	63	65	67	69	71	72
	0.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	72	75	77	79	82
	0.01	45	47	48	51	53	55	57	60	62	64	67	69	72	74	77	79	82	84	86
	0.025	46	48	50	53	56	58	61	63	66	68	71	74	77	80	83	85	88	91	94
	0.05	47	50	52	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100
	0.10	48	51	55	58	61	64	68	71	74	77	81	84	87	91	94	98	101	104	108
10	0.001	55	55	56	57	59	61	62	64	66	68	70	73	75	77	79	81	83	86	88
	0.005	55	56	58	60	62	65	67	69	72	74	77	80	82	85	87	90	93	95	98
	0.01	55	57	59	62	64	67	69	72	75	78	80	83	86	89	92	94	97	100	103
	0.025	56	59	61	64	67	70	73	76	79	82	85	88	92	95	98	101	104	108	111
	0.05	57	60	63	67	70	73	76	80	83	87	90	93	97	100	104	107	111	114	118
	0.10	59	62	66	69	73	77	80	84	88	92	95	99	103	107	110	114	118	122	126
11	0.001	66	66	67	69	71	73	75	77	79	82	84	87	89	91	94	96	99	101	104
	0.005	66	67	69	72	74	77	80	83	85	88	91	94	97	100	103	106	109	112	115
	0.01	66	68	71	74	76	79	82	85	88	92	95	98	101	104	108	111	114	117	120
	0.025	67	70	73	76	80	83	86	90	93	97	100	104	107	111	114	118	122	125	129
	0.05	68	72	75	79	83	86	90	94	98	101	105	109	113	117	121	124	128	132	136
	0.10	70	74	78	82	86	90	94	98	102	107	111	115	119	124	128	132	136	140	145
12	0.001	78	78	79	81	83	86	88	91	93	96	98	102	104	106	110	113	116	118	121
	0.005	78	80	82	85	88	91	94	97	100	103	106	110	113	116	120	123	126	130	133
	0.01	78	81	84	87	90	93	96	100	103	107	110	114	117	121	125	128	132	135	139
	0.025	80	83	86	90	93	97	101	105	108	112	116	120	124	128	132	136	140	144	148
	0.05	81	84	88	92	96	100	105	109	113	117	121	126	130	134	139	143	147	151	156
	0.10	83	87	91	96	100	105	109	114	118	123	128	132	137	142	146	151	156	160	165
13	0.001	91	91	93	95	97	100	103	106	109	112	115	118	121	124	127	130	134	137	140
	0.005	91	93	95	99	102	105	109	112	116	119	123	126	130	134	137	141	145	149	152
	0.01	92	94	97	101	104	108	112	115	119	123	127	131	135	139	143	147	151	155	159
	0.025	93	96	100	104	108	112	116	120	125	129	133	137	142	146	151	155	159	164	168
	0.05	94	98	102	107	111	116	120	125	129	134	139	143	148	153	157	162	167	172	176
	0.10	96	101	105	110	115	120	125	130	135	140	145	150	155	160	166	171	176	181	186
14	0.001	105	105	107	109	112	115	118	121	125	128	131	135	138	142	145	149	152	156	160
	0.005	105	107	110	113	117	121	124	128	132	136	140	144	148	152	156	160	164	169	173
	0.01	106	108	112	116	119	123	128	132	136	140	144	149	153	157	162	166	171	175	179
	0.025	107	111	115	119	123	128	132	137	142	146	151	156	161	165	170	175	180	184	189
	0.05	109	113	117	122	127	132	137	142	147	152	157	162	167	172	177	183	188	193	198
	0.10	110	116	121	126	131	137	142	147	153	158	164	169	175	180	186	191	197	203	208
15	0.001	120	120	122	125	128	131	135	139	142	146	149	153	157	161	164	168	173	176	180
	0.005	120	123	126	129	133	137	141	145	150	154	158	163	167	172	176	181	185	190	194
	0.01	121	124	128	132	136	140	145	149	154	158	163	168	172	177	182	187	191	196	201
	0.025	122	126	131	135	140	145	150	155	160	165	170	175	180	185	191	196	201	206	211
	0.05	124	128	133	139	144	149	154	160	165	171	176	182	187	193	198	204	209	215	221
	0.10	126	131	137	143	148	154	160	166	172	178	184	189	195	201	207	213	219	225	231
16	0.001	136	136	139	142	145	148	152	156	160	164	168	172	176	180	185	189	193	197	202
	0.005	136	139	142	146	150	155	159	164	168	173	177	182	187	192	197	202	207	211	216
	0.01	137	140	144	149	153	158	163	168	173	178	183	188	193	198	203	208	213	219	224
	0.025	138	143	148	152	158	163	168	174	179	184	190	196	201	207	211	218	223	229	235
	0.05	140	145	151	156	162	167	173	179	185	191	197	202	208	214	220	226	232	238	244
	0.10	142	148	154	160	166	173	179	185	191	198	204	211	217	223	229	236	243	249	256

TABLE A7 (Continued)

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
17	0.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220	224
	0.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235	240
	0.01	154	158	162	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242	247
	0.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253	259
	0.05	157	163	169	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263	269
	0.10	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274	281
18	0.001	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243	248
	0.005	171	174	178	183	188	193	198	203	208	214	219	225	230	236	242	247	253	259	264
	0.01	172	176	181	186	191	196	202	208	213	219	226	231	237	243	248	254	260	266	272
	0.025	174	179	184	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278	284
	0.05	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288	295
	0.10	178	185	192	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300	307
19	0.001	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268	273
	0.005	191	194	198	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284	290
	0.01	192	198	200	206	211	217	223	229	235	241	248	254	260	266	273	279	285	292	298
	0.025	193	198	204	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304	310
	0.05	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314	321
	0.10	198	205	212	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326	334
20	0.001	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293	299
	0.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310	316
	0.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318	325
	0.025	213	219	225	231	238	245	251	259	266	273	280	287	294	301	309	316	323	330	338
	0.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341	349
	0.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354	362

For n or m greater than 20, the pth quantile w_p of the Mann–Whitney test statistic may be approximated by

$$w_p = n(N+1)/2 + z_p\sqrt{nm(N+1)/12}$$

where z_p is the pth quantile of a standard normal random variable, obtained from Table A1, and where $N = m + n$.

[a] The entries in this table are quantiles w_p of the Mann–Whitney test statistic T, given by Equation 5.1.1, for selected values of p. Note that $P(T < w_p) \le p$. Upper quantiles may be found from the equation

$$w_p = n(n + m + 1) - w_{1-p}$$

Critical regions correspond to values of T less than (or greater than) but not equal to the appropriate quantile.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes[a]

Sample Sizes	$w_{0.90}$	$w_{0.95}$	$w_{0.99}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

SOURCE: Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.

[a] The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A10 Quantiles of Spearman's ρ[a]

n	$p = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6536	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p = \frac{z_p}{\sqrt{n-1}}$$

where z_p is the pth quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glasser and Winter (1961), with corrections, with permission from the *Biometrika* Trustees.

[a] The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$p = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5385)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	196 (0.2943)