

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2009

TITLE OF PAPER : MULTIVARIATE ANALYSIS

COURSE CODE : ST410

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

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QUESTION ONE.

[3 + 3 + 4 + 4 + 3 + 8 marks]

The following tables are part of the complete output running SPSS for a set of multivariate variables; not necessarily from the same set of variables. Tables 1-6 are obtained running Factor Analysis and Tables 7-9 are from Discriminant Function Analysis:

Table 1:

Correlation Matrix

	X1	X2	X3	X4	X5	X6	X7
Correlatio X1	1.000	.036	-.671	-.400	-.538	-.645	-.764
X2	.036	1.000	.445	.405	-.026	-.495	-.221
X3	-.671	.445	1.000	.385	.494	.080	.200
X4	-.400	.405	.385	1.000	.060	.199	.185
X5	-.538	-.026	.494	.060	1.000	.271	.210
X6	-.645	-.495	.080	.199	.271	1.000	.424
X7	-.764	-.221	.200	.185	.210	.424	1.000

Table 2:

Component	Eigenvalues
1	2.965
2	1.850
3	0.971
4	0.643
5	0.359
6	0.211
7	0.000

Table 3:

Component Matrix^a

	Component					
	1	2	3	4	5	6
X1	-.986	5.070E-02	-2.47E-02	.113	.102	-2.99E-02
X2	-1.24E-02	.936	7.822E-02	-.109	3.541E-04	.325
X3	.683	.566	-.258	-3.93E-02	-.333	-.186
X4	.483	.481	.590	.327	.241	-.153
X5	.621	4.503E-02	-.677	.186	.344	4.014E-02
X6	.641	-.562	.179	.402	-.186	.212
X7	.705	-.319	.246	-.560	.166	2.041E-02

Extraction Method: Principal Component Analysis.

a. 6 components extracted.

Table 4:

Component Matrix^a

	Component		
	1	2	3
X1	-.986	5.070E-02	-2.47E-02
X2	-1.24E-02	.936	7.822E-02
X3	.683	.566	-.258
X4	.483	.481	.590
X5	.621	4.503E-02	-.677
X6	.641	-.562	.179
X7	.705	-.319	.246

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

Table 5:

Rotated Component Matrix^a

	Component		
	1	2	3
X1	-.764	-.550	-.301
X2	-.515	.195	.761
X3	8.552E-02	.756	.523
X4	.295	-2.72E-02	.851
X5	.185	.892	-.127
X6	.858	8.626E-02	-.124
X7	.790	.141	.120

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 6 iterations.

Table 6:

Component Score Coefficient Matrix

	Component		
	1	2	3
X1	-.270	-.170	-.103
X2	-.263	.078	.433
X3	-.102	.422	.169
X4	.192	-.291	.584
X5	-.113	.658	-.290
X6	.400	-.097	-.063
X7	.365	-.099	.083

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Table 7:

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1 through 2	.147	39.244	12	.000
2	.656	8.655	5	.124

Table 8:

Standardized Canonical Discriminant Function Coefficients

	Function	
	1	2
X1	.268	1.193
X2	.202	-1.065
X3	.412	.938
X4	.573	.478
X5	.129	.097
X6	-1.040	-.249

Table 9:

Canonical Discriminant Function Coefficients

	Function	
	1	2
X1	.019	.083
X2	.305	-1.603
X3	.059	.135
X4	1.512	1.263
X5	.076	.057
X6	-.268	-.064
(Constant)	.217	-3.732

Unstandardized coefficients

- How many variables do you have in your data set? How many principal components will you have? Check Table 1 and Table 2.
- How many factors will you choose if you wish to use factor analysis method? Explain your answer.
- How many factors will you get in your factor model from Table 3? List the last two equations of your model and compute their communalities.
- Suppose the same data were analyzed using with a restriction on the number of factors. How many factors were chosen in Table 4? List the first two equations of your model and compute their communalities.
- List all equations needed to compute factor scores.
- Write all the discriminant functions and test whether each of those are significant at 5% level of significance.

QUESTION TWO.

- 2.1 Define Principal component analysis and discuss its all important properties.
- 2.2 State and discuss the four steps of the procedure for a principal component analysis.
- 2.3 Consider the following table:

Component	Eigenvalue	Eigenvectors				
		X ₁	X ₂	X ₃	X ₄	X ₅
1	2.616	0.452	0.462	0.451	0.471	0.398
2	1.532	-0.051	0.300	0.325	0.185	-0.377
3	0.386	0.691	0.341	-0.455	-0.411	-0.179
4	0.302	-0.420	0.548	-0.606	0.388	0.069
5	0.165	0.374	-0.530	-0.343	0.652	-0.192

- a. How many components will you choose? Explain why.
- b. List those selected components and interpret those in terms of original variables, X_i'^s.

QUESTION THREE.

[3 + 3 + 3 + 4 + 12 marks]

- 3.1 Discuss the purpose of the discriminant function analysis.
- 3.2 State the criteria for selecting canonical discriminant functions and also state the number of functions you can select.
- 3.3 The following table shows the eigenvalues and corresponding eigenvectors of $W^{-1}B$:

Component	Eigenvalue	Eigenvectors			
		X ₁	X ₂	X ₃	X ₄
1	0.437	-0.0107	0.0040	0.0119	-0.0068
2	0.035	0.0031	0.0168	-0.0046	-0.0022
3	0.015	-0.0068	0.0010	0.0000	0.0247
4	0.002	0.0126	-0.0001	0.0112	0.0054

- a. How many groups and variables were considered in this problem?
- b. List all the canonical discriminant functions.
- c. Assuming that the i^{th} sample size, $n_i = 30$ for all $i = 1, 2, 3, 4, 5$; test whether each of these functions varies significantly from group to group.

QUESTION FOUR.

[15 + 10 marks]

- 4.1 Suppose we have two groups, with 10 subjects in each group. The means for the two variables (X_1 and X_2) measures in group A are 10 and 7.5, while the means in group B are 9 and 9.5. The respective pooled sample variances are 9 and 4 for variables X_1 and X_2 , while the pooled correlation is 0.7. Perform Hotellings' T^2 test and also perform univariate t tests for each of these two variables at 5% level of significance. Compare the results.
- 4.2 Suppose we have three variables in each of the 3 groups, with 10 subjects per group. Let the sum of squares matrices are as follows:

$$\mathbf{B} = \begin{bmatrix} 1.68 & 1.38 & -1.26 \\ 1.38 & 1.14 & -1.08 \\ -1.26 & -1.08 & 1.26 \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} 1.24 & 0.06 & 0.56 \\ 0.06 & 1.08 & 0.18 \\ 0.56 & 0.18 & 2.74 \end{bmatrix}, \quad \& \quad \mathbf{T} = \begin{bmatrix} 2.92 & 1.44 & -0.70 \\ 1.44 & 2.22 & -0.90 \\ -0.70 & -0.90 & 4.00 \end{bmatrix}$$

Compute Wilk's Λ statistics and use χ^2 approximation to test the equality of population mean vectors.

QUESTION FIVE.

[8 + 10 + 5 + 2 marks]

- 5.1 Define factor analysis and compare its properties with those of principal component analysis.
- 5.2 Discuss the procedures of principal component factor analysis to determine the final factors.
- 5.3 Using the table given in Question 2.3, write the unrotated factor model.
- 5.4 Compute the communalities.

Table 5
Percentage points of the t distributions

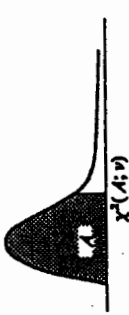


$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df.
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

From "Table of Percentage Points of the t-Distribution,"
Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p.
300. Reproduced by permission of Professor E. S. Pearson.

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.05982	0.05982	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.63	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.63	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrics* 32 (1941), pp. 188-99.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

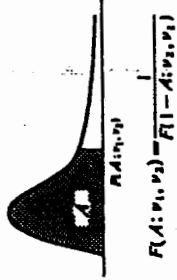


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df								
	1	2	3	4	5	6	7	8	9
1	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.50	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
.95	161	200	216	225	230	237	239	241	
.975	648	800	864	900	922	937	948	957	962
.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.50	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
.95	18.5	19.0	19.2	19.3	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.3	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3	0.385	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.50	5.34	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4	0.349	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.50	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
.995	31.3	26.3*	24.3	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.50	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.50	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.50	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.84	4.72
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.80	6.62
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df								
	10	12	15	20	24	30	60	120	∞
1	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20
.50	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3
.95	242	244	246	248	249	250	252	253	254
.975	969	977	985	993	997	1,001	1,010	1,014	1,018
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464
.999	605,620	610,670	615,760	620,910	623,300	626,100	631,340	633,970	636,620
2	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
.50	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5
.995	199	199	199	199	199	199	199	199	200
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5
3	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27
.50	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5
4	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
.50	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5
.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1
5	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
.50	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1
.999	26.9	26.4	25.9	25.4	25.1	24.8	24.3	24.1	23.8
6	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
.50	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7
7	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
.50	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	4	Numerator df													
		10	12	15	20	24	30	60	120	∞					
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09					
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29					
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93					
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67					
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.93	4.86					
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08					
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16					
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71					
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33					
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31					
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07					
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.12	2.08	2.06					
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54					
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08					
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91					
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06					
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90					
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30					
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72					
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36					
15	.50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05					
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76					
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07					
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40					
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87					
20	.50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03					
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61					
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84					
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09					
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42					
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03					
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53					
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73					
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94					
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21					

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	4	Numerator df													
		1	2	3	4	5	6	7	8	9					
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01					
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56					
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39					
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36					
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91					
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00					
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44					
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18					
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03					
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35					
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992					
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35					
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02					
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78					
	.99	10.0	7.36	6.35	5.99	5.64	5.39	5.20	5.06	4.94					
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981					
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21					
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80					
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44					
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39					
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970					
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09					
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59					
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12					
	.99	8.88	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89					
20	.50	0.472	0.718	0.818	0.868	0.900	0.922	0.938	0.950	0.959					
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96					
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39					
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84					
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46					
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953					
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91					
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30					
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.70	2.60					
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26					

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df											
	4	10	12	15	20	24	30	60	120	∞		
30	.50	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02	1.02	
	.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46		
	.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62		
	.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79		
	.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01		
	.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18		
	.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59		
60	.50	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01		
	.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29		
	.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39		
	.975	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48		
	.99	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60		
	.995	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69		
	.999	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89		
120	.50	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01		
	.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19		
	.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25		
	.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31		
	.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38		
	.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43		
	.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54		
∞	.50	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00		
	.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00		
	.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00		
	.975	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00		
	.99	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00		
	.995	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00		
	.999	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00		

Source: Reprinted from Table 5 of Pearson and Hartley, *Biometric Tables for Statisticians*, Volume 2, 1972, published by the Cambridge University Press, on behalf of The Biometrika Society, by permission of the authors and publishers.

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
30	.50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
	.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	.50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
	.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	.50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
	.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
	.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞	.50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
	.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
	.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
	.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
	.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10