

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2010

TITLE OF PAPER : **LINEAR STATISTICAL METHODS**

COURSE CODE : **ST204**

TIME ALLOWED : **2 (TWO) HOURS**

REQUIRMENTS : **STATISTICAL TABLES
AND CALCULATORS**

INSTRUCTIONS : **1. THIS PAPER CONSISTS OF FIVE
QUESTIONS.
2. ANSWER ANY FOUR QUESTIONS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[12 + 13 marks]

- State the Simple Linear Regression Model and discuss its important features.
- Find the point estimators of the regression coefficients of the model stated in part (a), using the method of least squares.

QUESTION TWO.

[25 marks]

A researcher wishes to try three different techniques to lower the blood pressure of individuals diagnosed with high blood pressure. The subjects are randomly assigned to three groups; the first group takes medication, the second group exercises, and the third group follows a special diet. After four weeks, the reduction in each person's blood pressure is recorded.

Medication	Exercise	Diet
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4

- What is the dependent variable here?
- Identify the factor studied. What are the factor levels?
- Is the factor quantitative or qualitative? An experimental or a classification factor?
- Obtain the fitted values.
- Obtain the residuals. Do they sum to zero in accord with the property of residuals?
- Obtain the analysis of variance table.

QUESTION THREE.

[25 marks]

A researcher collects the following data and determines that there is a significant relationship between the age of a copy machine and its monthly maintenance cost.

Machine	Age (years)	Monthly Cost (Emalangeni)
A	1	62
B	4	93
C	3	70
D	6	103
E	2	78
F	4	90

- Obtain the least square estimates of β_0 and β_1 , and state the estimated regression function.
- Interpret β_0 and β_1 in your estimated regression function.
- Obtain a point estimate of the mean Monthly Cost for a 5 years old machine.
- What is the point estimate of the change in the mean response when the age of the machine increases by one year?

QUESTION FOUR.

[25 marks]

The following information was obtained from running the model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ using SPSS:

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2964.587	1	2964.587	3.407	.114
	Residual	5221.413	6	870.235		
	Total	8186.000	7			

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	110.115	59.108		1.863	.112
	x	12.784	6.926	.602	1.846	.114

- State the fitted regression line. Interpret the estimated values of β_1 and β_2 .
- Perform the F-test and clearly state the conclusion.
- Test $\beta_1 = 15$ against $\beta_1 < 15$ at $\alpha = 0.01$.
- Construct a 98% confidence interval for β_0 .
- Compute coefficient of correlation, r and explain the nature and strength of the relationship between dependent and independent variables.

QUESTION FIVE.

[25 marks]

A company wishes to test the effectiveness of its advertising. A product is selected, and two types of ads are written; one is serious and one is humorous. Also the ads run on both medium of advertising; television and radio. Sixteen potential customers are selected and assigned randomly to one of the four groups. After seeing or listening to the ad, each customer is asked to rate its effectiveness on a scale of 1 to 20 and the data was analyzed using SPSS. The following ANOVA table is a part of the output from that analysis:

ANOVA TABLE

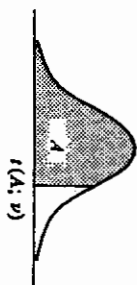
Source of Variation	Sum of Squares	df	Mean Square	F
Between treatments	186.189			
Factor A	10.563			
Factor B	175.563			
A X B	0.063			
Within treatments	66.250			
Total	252.439			

Complete the ANOVA Table

- The size of the sample used in this experiment?
- What are the treatments in this experiment?
- Which one is the Factor A?
- Which one is the Factor B?
- Using 5% level of significance, describe only the conclusions (based on F-test) in terms of effectiveness of the types of ad, effectiveness of the medium of advertising and effectiveness of their interaction.

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



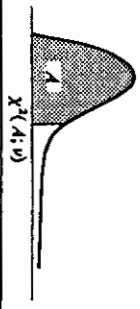
ν	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.503
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE A.2 (concluded) Percentiles of the *t* Distribution

ν	A									
	.98	.985	.99	.9925	.995	.9975	.9995	.9995	.9995	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590			
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598			
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924			
4	2.999	3.298	3.747	4.088	4.604	5.988	8.610			
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869			
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959			
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408			
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041			
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781			
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587			
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437			
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318			
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221			
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140			
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073			
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015			
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965			
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922			
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883			
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849			
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819			
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792			
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768			
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745			
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725			
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707			
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690			
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674			
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659			
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646			
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551			
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460			
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373			
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291			

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.004393	0.0157	0.00982	0.02393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.23	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.53	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.33	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-99.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

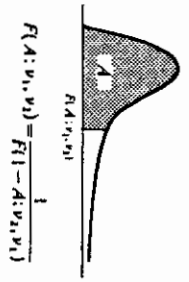


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
1	.50	1.00	1.30	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.3
.95	161	200	216	225	230	234	237	239	241	241
.975	648	800	864	900	922	937	948	957	963	963
.99	4,052	5,000	5,403	5,623	5,764	5,859	5,928	5,981	6,022	6,022
.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	602,280
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.38
.95	18.5	19.0	19.2	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.2	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.9
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	48.5
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.85	4.77	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	27.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.95	2.95
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.4
.999	33.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	18.7
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.99	8.89	8.81	8.81
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df												
	10	12	15	20	24	30	60	120	∞				
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20			
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	63.3			
.95	242	244	246	248	250	252	253	254	254	254			
.975	969	977	985	993	997	1,001	1,010	1,014	1,018	1,018			
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366	6,366			
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	25,464			
.999	605,620	610,670	615,760	620,910	626,100	631,340	636,620	641,940	647,300	652,620			
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44			
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	9.49			
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5			
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5			
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5			
.995	199	199	199	199	199	199	199	199	199	199			
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	999.5			
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27			
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13	5.13			
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53	8.53			
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9	13.9			
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1	26.1			
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8	41.8			
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	123.5			
4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19			
.90	3.90	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76	3.76			
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63	5.63			
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	8.26			
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	13.5			
.995	21.0	20.7	20.4	20.2	20.0	19.9	19.8	19.7	19.3	19.3			
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	44.1			
5	.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15			
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11	3.11			
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37	4.37			
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	6.02			
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	9.02			
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	12.1			
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	23.8			
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12			
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72	2.72			
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67	3.67			
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	4.85			
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88	6.88			
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	8.88			
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	15.7			
7	.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10			
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47	2.47			
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23	3.23			
.975	4.62	4.57	4.57	4.47	4.47	4.42	4.25	4.20	4.14	4.14			
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	5.65			
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	7.08			
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	11.7			

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	3.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	3.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.06	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.53	5.99	5.64	5.39	5.20	5.06	4.94
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.35	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.58	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.38	3.26

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09	
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29	
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93	
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67	
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08	
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07	
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06	
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54	
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06	
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72	
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	
15	.50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05	
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87	
20	.50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03	
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09	
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03	
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.999	13.1	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.995	8.18	5.34	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df											
	10	12	15	20	24	30	60	120	∞			
30	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02			
.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46			
.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62			
.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79			
.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01			
.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18			
.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59			
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01			
.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29			
.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39			
.975	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48			
.99	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60			
.995	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69			
.999	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89			
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01			
.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19			
.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25			
.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31			
.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38			
.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43			
.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54			
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00			
.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00			
.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00			
.975	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00			
.99	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00			
.995	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00			
.999	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00			

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