

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2009/10

COURSE TITLE: OPERATIONS RESEARCH I

COURSE CODE: ST 307

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS (20 MARKS)

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS

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Question 1

The Feed'N Ship feedlot fattens cattle for local farmers and ships them to meat markets in Manzini and Mbabane. The owners of the feedlot seek to determine the amounts of cattle feed to buy to satisfy minimum nutritional standards and, at the same time minimise total feed cost. Each grain stock contains different amounts of four nutritional ingredients: A, B, C and D. Here are the ingredient contents of each grain, in grams per kilogram of grain.

| Ingredient | Feed | | |
|------------|---------|---------|---------|
| | Stock X | Stock Y | Stock Z |
| A | 3 | 2 | 4 |
| B | 2 | 3 | 1 |
| C | 1 | 0 | 2 |
| D | 6 | 8 | 4 |

The cost per kilogram of grains X, Y and Z is E0.20, E0.40 and E0.25 respectively. The minimum requirement per cow per month is 64 grams of ingredient A, 80 grams of ingredient B, 16 grams of ingredient C and 128 grams of ingredient D. The feedlot faces one additional restriction – it can obtain only 500 grams of stock Z per month from the feed suppliers, regardless of its needs. There are usually 100 cows at the Feed'N Ship feedlot at any given time.

Specify a feed ingredient combination that will satisfy the stated nutritional requirements at a minimum cost level.

(20 marks)

Question 2

Williams Auto Top Carriers currently maintains plants in Atlanta and Tulsa to supply auto top carriers to distribution centres in Los Angeles and New York. Because of expanding demand, Williams has decided to open a third plant and has narrowed the choice to one of two cities – New Orleans and Houston. The below provides pertinent production and distribution costs in dollars (\$) as well as plant capacities and distribution demands. Which of the new locations, in combination with the existing plants and distribution centres, yield lower cost for the firm? Use the Northwest-Corner Rule and the stepping-stone method to answer this question.

| From Plants | To Distribution Centres | | Normal Production | Unit Production Cost |
|--------------------|-------------------------|----------|-------------------|----------------------|
| | Los Angeles | New York | | |
| Existing plants | | | | |
| Atlanta | \$8 | \$5 | 600 | \$6 |
| Tulsa | \$4 | \$7 | 900 | \$5 |
| Proposed locations | | | | |
| New Orleans | \$5 | \$6 | 500 | \$4(anticipated) |
| Houston | \$4 | \$6 | 500 | \$3(anticipated) |
| Forecast demand | 800 | 1,200 | 2,000 | |

(20 marks)

Question 3

(a) The following table contains information on the cost to run three jobs on four available machines. Determine an assignment plan that will minimise costs.

| Job | Machine | | | |
|-----|---------|----|----|----|
| | A | B | C | D |
| 1 | 12 | 16 | 14 | 10 |
| 2 | 9 | 8 | 13 | 7 |
| 3 | 15 | 12 | 9 | 11 |

(5 marks)

(b) Use the assignment method to obtain a plan that will minimise processing costs in the following table under these conditions:

(i) The condition 2-D is undesirable.

(7 marks)

(ii) The combination 1-A and 2-D are undesirable.

(8 marks)

| Job | Machine | | | | |
|-----|---------|----|----|----|----|
| | A | B | C | D | E |
| 1 | 14 | 18 | 20 | 17 | 18 |
| 2 | 14 | 15 | 19 | 16 | 17 |
| 3 | 12 | 16 | 15 | 14 | 17 |
| 4 | 11 | 13 | 14 | 12 | 14 |
| 5 | 10 | 16 | 15 | 14 | 13 |

Question 4

(a) For which decision environment is linear programming most suited?

(2 marks)

(b) Briefly explain these terms:

(i) Range of feasibility

(ii) Range of optimality

(iii) Range of insignificance

(iv) Shadow price

(8 marks)

(c) Solve the following Linear Programming problem by simplex technique:

$$\text{Maximise } G = 10x + 6y + 4z$$

$$\text{Subject to } x + y + z \leq 100$$

$$10x + 4y + 5z \leq 600$$

$$2x + 2y + 6z \leq 300$$

$$x, y, z \geq 0.$$

(10 marks)

Question 5

(a) What is duality? What important conclusions do you draw from the optimal tableaux of primal and dual solutions of a linear programming problem? **(5 marks)**

(b) Construct the dual of the following Linear programming problem and solve both the primal and the dual:

$$\begin{array}{ll} \text{Minimise} & Z = 4x_1 + 2x_2 + 3x_3 \\ \text{Subject to} & 2x_1 + 4x_3 \geq 5 \\ & 2x_1 + 3x_2 + x_3 \geq 4 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

(15 marks)

END OF EXAM!!